

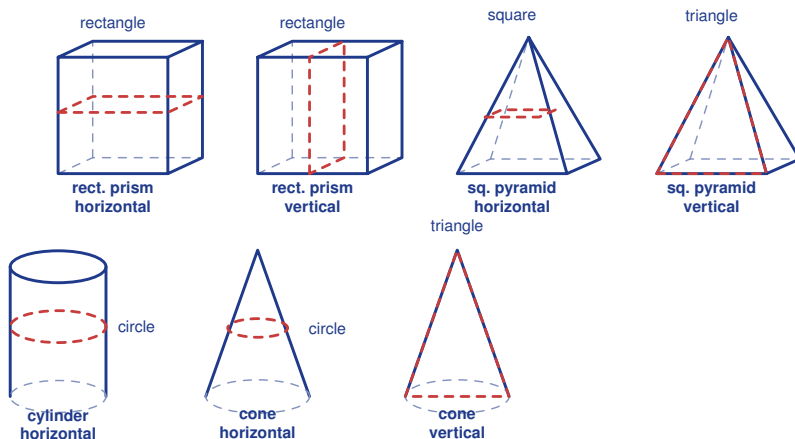
Cross Sections of 3-D Figures

Name: _____

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Score: _____ / 17

Imagine slicing a 3-D solid with a perfectly flat plane—the shape you see on the cut surface is called a **cross section**. The fascinating part is that the *same* solid can produce different cross sections depending on how you make the cut! A horizontal slice, a vertical slice, and a diagonal slice through a cylinder, for example, give very different shapes. Cross sections are a wonderfully visual way to connect 2-D and 3-D geometry.



Key Concepts & Quick Review

- **Rectangular prism:** horizontal → rectangle; vertical (parallel to face) → rectangle; diagonal → rectangle or parallelogram.
- **Square pyramid:** horizontal → square (smaller toward apex); vertical through apex → triangle.
- **Cylinder:** horizontal → circle; vertical → rectangle; diagonal → ellipse.
- **Cone:** horizontal → circle; vertical through apex → triangle (isosceles).
- **Sphere:** any cut through centre → circle (great circle); off-centre → smaller circle.

Examples

① A rectangular prism ($10\text{ cm} \times 6\text{ cm} \times 4\text{ cm}$) is sliced horizontally halfway up. Describe the cross section and find its area.

Think It Through: A horizontal cut is parallel to the base, so the cross section has the same shape as the base. The base of this prism is a 10 cm by 6 cm rectangle, so the slice is also a rectangle with those dimensions. Its area is therefore $10 \times 6 = 60\text{ cm}^2$. The height of the cut does not change the shape or area in a prism like this.

Answer: *Rectangle* 10×6 ; *area* $= 60\text{ cm}^2$



② A square pyramid with base 8 cm is cut by a plane parallel to the base at half the vertical height. What shape is the cross section and what is its side length?

Think It Through: A cut parallel to the base of a pyramid makes a smaller, similar copy of the base, so the cross section is a square. Because the cut is halfway from the apex to the base, every linear measurement is scaled by $\frac{1}{2}$. That makes the side length $\frac{1}{2} \times 8 = 4\text{ cm}$. Once the side is known, the area is $4^2 = 16\text{ cm}^2$.

Answer: Square with side 4 cm ; area = 16 cm^2

Practice Problems

Name the shape of each cross section or find the area/dimensions asked.

1. A cube is sliced by a horizontal plane parallel to its base. What is the cross-section shape? _____
2. A cube is sliced by a vertical plane parallel to one face. What is the cross-section shape? _____
3. A cube is sliced by a diagonal plane that cuts across opposite faces. What is the cross-section shape? _____
4. A square pyramid is sliced by a horizontal plane parallel to its base. What is the cross-section shape? _____
5. A square pyramid is sliced by a vertical plane through its apex. What is the cross-section shape? _____
6. A cylinder is sliced by a horizontal plane parallel to its circular bases. What is the cross-section shape? _____
7. A cylinder is sliced by a vertical plane through its height. What is the cross-section shape? _____



8. A cylinder is sliced by a diagonal plane that is not parallel to the base. What is the cross-section shape? _____



9. A cone is sliced by a horizontal plane parallel to its base. What is the cross-section shape? _____



10. A cone is sliced by a vertical plane through its apex. What is the cross-section shape? _____





11. A sphere is sliced by any flat plane through its center. What is the cross-section shape? _____
12. A rectangular prism measures 10 by 8 by 6. A horizontal cut is parallel to the 10 by 8 base. Find the cross-section area. _____
13. A square pyramid has base side length 12. A horizontal cut is made one-third of the way up from the base. Find the side length of the cross section. _____
14. A square pyramid has base side length 9. A horizontal cut is made two-thirds of the way up from the base. Find the side length of the cross section. _____
15. A cylinder has radius 5. A horizontal cut is parallel to its base. Find the cross-section area. _____

Study Tips

- Direction determines shape.** A cut parallel to the base gives a scaled copy of the base. A cut through the apex gives a triangle.
- For a pyramid cut at fraction f of the height from the apex, every linear dimension of the cross section is f times the base dimension.
- Real-world cross sections appear in MRI/CT scans, sawing timber, cooking (slicing fruit), and engineering blueprints — recognising them is a genuinely useful skill!

Word Problems

16. A block of cheese is shaped like a rectangular prism, 24 *cm* long, 12 *cm* wide, and 8 *cm* tall. A chef slices it with three different cuts: (a) horizontally at half-height, (b) vertically along the length, (c) diagonally from one top edge to the opposite bottom edge. Describe the shape and dimensions of each cross section, and find its area. _____
17. A traffic cone is a solid cone with base radius 15 *cm* and height 40 *cm*. A road worker cuts it with a horizontal plane 10 *cm* from the top. What shape is the cross section? Using similar triangles, find the radius of the cross section. Find the area of the cross section. _____



Answer Keys

- | | |
|---|---|
| <p>1) square</p> <p>2) rectangle</p> <p>3) rectangle or parallelogram</p> <p>4) square</p> <p>5) triangle</p> <p>6) circle</p> <p>7) rectangle</p> <p>8) ellipse</p> <p>9) circle</p> | <p>10) triangle</p> <p>11) circle</p> <p>12) 80</p> <p>13) 8</p> <p>14) 3</p> <p>15) 78.5</p> <p>16) (a) 288 cm^2; (b) 192 cm^2; (c) rectangle, about 345.6 cm^2</p> <p>17) Circle; radius 3.75 cm; area about 44.2 cm^2</p> |
|---|---|

Step-by-Step Explanations

Strategy: For Area of Circles, use radius for circle area every time, dividing the diameter by two first if the problem gives diameter. For circle area, mark the radius first; the formula depends on it.

Practice 1: Find the area of a circle with radius 4 cm . **Answer:** 50.24 cm^2

For the first worked item, use A equals pi r squared, making sure r is the radius.

Practice 15: Find the exact area of a circle with radius 3 . **Answer:** 9π

Near the end of this topic, use A equals pi r squared, making sure r is the radius.

Word-problem notes:

16. Answer: A: 254.34 m^2 ; B: 452.16 m^2 ; difference: 197.82 m^2 ; A: $\approx 5.1 \text{ hr}$; B: $\approx 9.0 \text{ hr}$.

Find each pond area with $A = \pi r^2$. Pond A has area $\pi(9)^2 = 81\pi \approx 254.34 \text{ m}^2$, and Pond B has area $\pi(12)^2 = 144\pi \approx 452.16 \text{ m}^2$. Subtract to compare them: $452.16 - 254.34 = 197.82 \text{ m}^2$. To find the treatment time, divide each area by the crew's rate of 50 m^2 per hour. That gives about 5.1 hours for Pond A and 9.0 hours for Pond B.

17. Answer: Whole: $\pi(14)^2 \approx 615.44 \text{ cm}^2$; slice: $\approx 76.93 \text{ cm}^2$; volume: $\approx 384.6 \text{ cm}^3$.

Start by converting the diameter to radius: $28 \div 2 = 14 \text{ cm}$. The whole cake area is $\pi(14)^2 = 196\pi \approx 615.44 \text{ cm}^2$. Since the cake is cut into 8 equal slices, divide by 8 to get one slice area: about 76.93 cm^2 . The slice volume is approximately area times height, so $76.93 \times 5 \approx 384.6 \text{ cm}^3$.



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