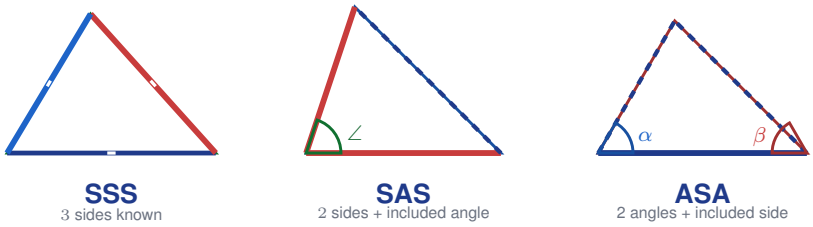
Constructing Triangles (SSS, SAS, ASA Conditions)

Name: _____	Date: _____	Score: _____ / 12

Can you build a triangle from a set of measurements, or is there not enough information? That is exactly what this topic answers! Three side lengths (**SSS**), two sides and the included angle (**SAS**), or two angles and the included side (**ASA**) each lock in exactly one triangle. Other combos might give you one triangle, more than one, or none at all. Learning these conditions helps you understand *why* some drawings are fixed and others are flexible—real geometric thinking!



SSS
3 sides known

SAS
2 sides + included angle

ASA
2 angles + included side

Key Concepts & Quick Review

SSS: Give all three side lengths ⇒ unique triangle.
SAS: Two sides and the angle *between* them ⇒ unique triangle.
ASA: Two angles and the side *between* them ⇒ unique triangle.
Note: Sum of any two sides must exceed the third (Triangle Inequality). If not, no triangle is possible.

Examples

① Can a triangle be built with sides $5\ cm$, $8\ cm$, and $12\ cm$? What condition does this satisfy?

Think It Through: To see whether a triangle is possible, check the triangle inequality: the sum of any two sides must be greater than the third. Here, $5+8=13>12$, $5+12=17>8$, and $8+12=20>5$, so all three checks work. Since all three side lengths are given, the triangle is specified by SSS, which gives one unique triangle.

Answer: *Yes, valid SSS triangle*

② A triangle is specified by two angles $55°$ and $70°$, with the side between them $6\ cm$. Which condition is this? Find the third angle.

Think It Through: Two angles together with the side between them is the ASA condition. To find the third angle, subtract the known angles from $180°$: $180° - 55° - 70° = 55°$. Because two angles are equal, the triangle is isosceles. ASA determines one unique triangle because both the shape and the size are pinned down.

Answer: *ASA; third angle* $= 55°$ *(isosceles)*

Practice Problems

Identify the construction condition (SSS, SAS, ASA), check validity, or find the missing part.

1. A triangle has side lengths 3, 4, and $5\ cm$. Identify the construction condition. _____
2. A triangle has side lengths 7, 7, and $7\ cm$. Identify the construction condition and classify the triangle. _____
3. Can side lengths 2, 3, and $6\ cm$ form a valid triangle? Explain using the triangle inequality. _____
4. Can side lengths 5, 12, and $13\ cm$ form a valid triangle? If yes, identify the construction condition. _____
5. A triangle is given by sides $6\ cm$ and $9\ cm$ with included angle $45°$. Identify the construction condition. _____
6. A triangle is given by angles $60°$ and $80°$ with included side $5\ cm$. Identify the construction condition. _____
7. A triangle has sides $4\ cm$ and $4\ cm$ with included angle $90°$. Identify the condition and find the hypotenuse. _____
8. A triangle has angles $30°$ and $110°$ with included side $8\ cm$. Find the third angle. _____
9. Two sides of a triangle are 10 and 6. Find the possible range for the third side. _____
10. A triangle has side lengths a, a, and a. Classify the triangle and give its angle measures. _____

Study Tips

- **Triangle Inequality:** the sum of any two sides must be *strictly greater than* the third. Equal does not count — the shape would be flat.
- **AAA** alone does *not* guarantee a unique triangle — the triangles could be similar but different sizes. You need at least one side.
- For SAS and ASA, the angle or side must be *included* (between the two sides or two angles). An angle not between the two sides is SSA — not guaranteed unique.

Word Problems

11. An engineer is designing a triangular support bracket with three steel rods. The rods have lengths $9\ cm$, $12\ cm$, and $15\ cm$. Is a valid triangle possible? Which condition specifies it? Is this bracket a right triangle? (Check using the Pythagorean relationship.) _____

12. A land surveyor marks triangle PQR. She knows that $\angle P = 42°$, $\angle Q = 76°$, and the distance from P to Q is $80\ m$. Which condition specifies this triangle (SSS, SAS, or ASA)? Find $\angle R$. If a second surveyor claims the same triangle using only the three angle measures without a side length, can they draw the same unique triangle? _____

Answer Keys

1) SSS; valid right triangle
2) SSS; equilateral
3) not valid
4) valid; SSS; right triangle
5) SAS
6) ASA
7) SAS; $4\sqrt{2}\,cm$
8) $40°$
9) $4 < x < 16$
10) equilateral; $60°$ each
11) Valid; SSS; right triangle
12) ASA; $\angle R = 62°$; no, AAA gives infinitely many

Step-by-Step Explanations

Strategy: For Solving Equations with the Distributive Property, distribute before combining like terms so the equation is genuinely simpler before solving. This equation page rewards patience: expand first, then solve the simpler line.

Practice 1: 2(x+3)=14 **Answer:** 4
At the beginning of the practice, distribute first so each side of the equation is ready to simplify.

Practice 15: 2(5m-4)+6=20 **Answer:** 2
For the second model problem, collect variable terms on one side and constants on the other before dividing.

Word-problem notes:

16. Answer: $h = 14$ per hour; 6 friends: $6(14 + 5) = 6(19) = \$114$.
The expression $4(h + 5) = 76$ means each of the four friends pays the same rink-time cost h plus the $\$5$ rental fee. Distribute first: $4h + 20 = 76$. Subtract 20 to get $4h = 56$, then divide by 4 to find $h = 14$. For six friends, use the same per-person cost: $6(14+5) = 6(19) = 114$. So six friends would pay $114 total.

17. Answer: Plot A perimeter: $4(2x + 3) = 8x + 12$; Plot B: $2(x + 5) + 2(4) = 2x + 18$; $8x + 12 = 2x + 18 \Rightarrow x = 1$; perimeter: $20\,ft$ each.
Write an expression for each perimeter. The square has four equal sides, so Plot A's perimeter is $4(2x + 3) = 8x + 12$. The rectangle has two lengths and two widths, so Plot B's perimeter is $2(x+5)+2(4) = 2x+18$. Set them equal because the fencing lengths are the same: $8x+12 = 2x+18$. Subtract $2x$ and then subtract 12 to get $6x = 6$, so $x = 1$. Substituting into either perimeter expression gives 20 feet.

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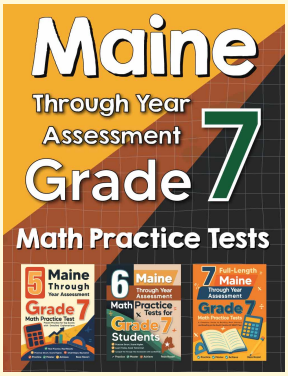

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