

Constructing Triangles (SSS, SAS, ASA Conditions)

Name: _____

Date: _____

Score: _____ / 12

Can you build a triangle from a set of measurements, or is there not enough information? That is exactly what this topic answers! Three side lengths (**SSS**), two sides and the included angle (**SAS**), or two angles and the included side (**ASA**) each lock in exactly one triangle. Other combos might give you one triangle, more than one, or none at all. Learning these conditions helps you understand *why* some drawings are fixed and others are flexible—real geometric thinking!



SSS
3 sides known



SAS
2 sides + included angle



ASA
2 angles + included side

Key Concepts & Quick Review

SSS: Give all three side lengths \Rightarrow unique triangle.

SAS: Two sides and the angle *between* them \Rightarrow unique triangle.

ASA: Two angles and the side *between* them \Rightarrow unique triangle.

Note: Sum of any two sides must exceed the third (Triangle Inequality). If not, no triangle is possible.

Examples

① Can a triangle be built with sides 5 cm , 8 cm , and 12 cm ? What condition does this satisfy?

Think It Through: To see whether a triangle is possible, check the triangle inequality: the sum of any two sides must be greater than the third. Here, $5+8 = 13 > 12$, $5+12 = 17 > 8$, and $8+12 = 20 > 5$, so all three checks work. Since all three side lengths are given, the triangle is specified by SSS, which gives one unique triangle.

Answer: Yes, valid SSS triangle

② A triangle is specified by two angles 55° and 70° , with the side between them 6 cm . Which condition is this? Find the third angle.

Think It Through: Two angles together with the side between them is the ASA condition. To find the third angle, subtract the known angles from 180° : $180^\circ - 55^\circ - 70^\circ = 55^\circ$. Because two angles are equal, the triangle is isosceles. ASA determines one unique triangle because both the shape and the size are pinned down.

Answer: ASA; third angle = 55° (isosceles)



 **Practice Problems**

Identify the construction condition (SSS, SAS, ASA), check validity, or find the missing part.

1. A triangle has side lengths 3, 4, and 5 *cm*. Identify the construction condition. _____
2. A triangle has side lengths 7, 7, and 7 *cm*. Identify the construction condition and classify the triangle. _____
3. Can side lengths 2, 3, and 6 *cm* form a valid triangle? Explain using the triangle inequality. _____
4. Can side lengths 5, 12, and 13 *cm* form a valid triangle? If yes, identify the construction condition. _____
5. A triangle is given by sides 6 *cm* and 9 *cm* with included angle 45° . Identify the construction condition. _____
6. A triangle is given by angles 60° and 80° with included side 5 *cm*. Identify the construction condition. _____
7. A triangle has sides 4 *cm* and 4 *cm* with included angle 90° . Identify the condition and find the hypotenuse. _____
8. A triangle has angles 30° and 110° with included side 8 *cm*. Find the third angle. _____
9. Two sides of a triangle are 10 and 6. Find the possible range for the third side. _____
10. A triangle has side lengths a , a , and a . Classify the triangle and give its angle measures. _____

Study Tips

-  **Triangle Inequality:** the sum of any two sides must be *strictly greater than* the third. Equal does not count — the shape would be flat.
-  **AAA** alone does *not* guarantee a unique triangle — the triangles could be similar but different sizes. You need at least one side.
-  For SAS and ASA, the angle or side must be *included* (between the two sides or two angles). An angle not between the two sides is SSA — not guaranteed unique.

 **Word Problems**

11. An engineer is designing a triangular support bracket with three steel rods. The rods have lengths 9 *cm*, 12 *cm*, and 15 *cm*. Is a valid triangle possible? Which condition specifies it? Is this bracket a right triangle? (Check using the Pythagorean relationship.) _____
12. A land surveyor marks triangle PQR . She knows that $\angle P = 42^\circ$, $\angle Q = 76^\circ$, and the distance from P to Q is 80 *m*. Which condition specifies this triangle (SSS, SAS, or ASA)? Find $\angle R$. If a second surveyor claims the same triangle using only the three angle measures without a side length, can they draw the same unique triangle? _____



Answer Keys

- | | |
|--|--|
| <p>1) SSS; valid right triangle</p> <p>2) SSS; equilateral</p> <p>3) not valid</p> <p>4) valid; SSS; right triangle</p> <p>5) SAS</p> <p>6) ASA</p> <p>7) SAS; $4\sqrt{2}$ cm</p> | <p>8) 40°</p> <p>9) $4 < x < 16$</p> <p>10) equilateral; 60° each</p> <p>11) Valid; SSS; right triangle</p> <p>12) ASA; $\angle R = 62^\circ$; no, AAA gives infinitely many</p> |
|--|--|

Step-by-Step Explanations

Strategy: For Solving Equations with the Distributive Property, distribute before combining like terms so the equation is genuinely simpler before solving. This equation page rewards patience: expand first, then solve the simpler line.

Practice 1: $2(x+3)=14$ **Answer:** 4

At the beginning of the practice, distribute first so each side of the equation is ready to simplify.

Practice 15: $2(5m-4)+6=20$ **Answer:** 2

For the second model problem, collect variable terms on one side and constants on the other before dividing.

Word-problem notes:

16. Answer: $h = 14$ per hour; 6 friends: $6(14 + 5) = 6(19) = \$114$.

The expression $4(h + 5) = 76$ means each of the four friends pays the same rink-time cost h plus the \$5 rental fee. Distribute first: $4h + 20 = 76$. Subtract 20 to get $4h = 56$, then divide by 4 to find $h = 14$. For six friends, use the same per-person cost: $6(14 + 5) = 6(19) = 114$. So six friends would pay \$114 total.

17. Answer: Plot A perimeter: $4(2x + 3) = 8x + 12$; Plot B: $2(x + 5) + 2(4) = 2x + 18$; $8x + 12 = 2x + 18 \Rightarrow x = 1$; perimeter: 20 ft each.

Write an expression for each perimeter. The square has four equal sides, so Plot A's perimeter is $4(2x + 3) = 8x + 12$. The rectangle has two lengths and two widths, so Plot B's perimeter is $2(x + 5) + 2(4) = 2x + 18$. Set them equal because the fencing lengths are the same: $8x + 12 = 2x + 18$. Subtract $2x$ and then subtract 12 to get $6x = 6$, so $x = 1$. Substituting into either perimeter expression gives 20 feet.



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