

# Constructing Triangles (SSS, SAS, ASA Conditions)

Name: \_\_\_\_\_

Date: \_\_\_\_\_

Score: \_\_\_\_\_ / 12

Can you build a triangle from a set of measurements, or is there not enough information? That is exactly what this topic answers! Three side lengths (**SSS**), two sides and the included angle (**SAS**), or two angles and the included side (**ASA**) each lock in exactly one triangle. Other combos might give you one triangle, more than one, or none at all. Learning these conditions helps you understand *why* some drawings are fixed and others are flexible—real geometric thinking!



**SSS**  
3 sides known



**SAS**  
2 sides + included angle



**ASA**  
2 angles + included side

## Key Concepts & Quick Review

**SSS:** Give all three side lengths  $\Rightarrow$  unique triangle.

**SAS:** Two sides and the angle *between* them  $\Rightarrow$  unique triangle.

**ASA:** Two angles and the side *between* them  $\Rightarrow$  unique triangle.

**Note:** Sum of any two sides must exceed the third (Triangle Inequality). If not, no triangle is possible.

## Examples

① Can a triangle be built with sides  $5\text{ cm}$ ,  $8\text{ cm}$ , and  $12\text{ cm}$ ? What condition does this satisfy?

**Think It Through:** To see whether a triangle is possible, check the triangle inequality: the sum of any two sides must be greater than the third. Here,  $5+8 = 13 > 12$ ,  $5+12 = 17 > 8$ , and  $8+12 = 20 > 5$ , so all three checks work. Since all three side lengths are given, the triangle is specified by SSS, which gives one unique triangle.

**Answer:** Yes, valid SSS triangle

② A triangle is specified by two angles  $55^\circ$  and  $70^\circ$ , with the side between them  $6\text{ cm}$ . Which condition is this? Find the third angle.

**Think It Through:** Two angles together with the side between them is the ASA condition. To find the third angle, subtract the known angles from  $180^\circ$ :  $180^\circ - 55^\circ - 70^\circ = 55^\circ$ . Because two angles are equal, the triangle is isosceles. ASA determines one unique triangle because both the shape and the size are pinned down.

**Answer:** ASA; third angle =  $55^\circ$  (isosceles)



 **Practice Problems**

Identify the construction condition (SSS, SAS, ASA), check validity, or find the missing part.

1. A triangle has side lengths 3, 4, and 5 *cm*. Identify the construction condition. \_\_\_\_\_
2. A triangle has side lengths 7, 7, and 7 *cm*. Identify the construction condition and classify the triangle. \_\_\_\_\_
3. Can side lengths 2, 3, and 6 *cm* form a valid triangle? Explain using the triangle inequality. \_\_\_\_\_
4. Can side lengths 5, 12, and 13 *cm* form a valid triangle? If yes, identify the construction condition. \_\_\_\_\_
5. A triangle is given by sides 6 *cm* and 9 *cm* with included angle  $45^\circ$ . Identify the construction condition. \_\_\_\_\_
6. A triangle is given by angles  $60^\circ$  and  $80^\circ$  with included side 5 *cm*. Identify the construction condition. \_\_\_\_\_
7. A triangle has sides 4 *cm* and 4 *cm* with included angle  $90^\circ$ . Identify the condition and find the hypotenuse. \_\_\_\_\_
8. A triangle has angles  $30^\circ$  and  $110^\circ$  with included side 8 *cm*. Find the third angle. \_\_\_\_\_
9. Two sides of a triangle are 10 and 6. Find the possible range for the third side. \_\_\_\_\_
10. A triangle has side lengths  $a$ ,  $a$ , and  $a$ . Classify the triangle and give its angle measures. \_\_\_\_\_

**Study Tips**

-  **Triangle Inequality:** the sum of any two sides must be *strictly greater than* the third. Equal does not count — the shape would be flat.
-  **AAA** alone does *not* guarantee a unique triangle — the triangles could be similar but different sizes. You need at least one side.
-  For SAS and ASA, the angle or side must be *included* (between the two sides or two angles). An angle not between the two sides is SSA — not guaranteed unique.

 **Word Problems**

11. An engineer is designing a triangular support bracket with three steel rods. The rods have lengths 9 *cm*, 12 *cm*, and 15 *cm*. Is a valid triangle possible? Which condition specifies it? Is this bracket a right triangle? (Check using the Pythagorean relationship.) \_\_\_\_\_
12. A land surveyor marks triangle  $PQR$ . She knows that  $\angle P = 42^\circ$ ,  $\angle Q = 76^\circ$ , and the distance from  $P$  to  $Q$  is 80 *m*. Which condition specifies this triangle (SSS, SAS, or ASA)? Find  $\angle R$ . If a second surveyor claims the same triangle using only the three angle measures without a side length, can they draw the same unique triangle? \_\_\_\_\_



## Answer Keys

- |  |  |
|--|--|
| <p>1) SSS; valid right triangle</p> <p>2) SSS; equilateral</p> <p>3) not valid</p> <p>4) valid; SSS; right triangle</p> <p>5) SAS</p> <p>6) ASA</p> <p>7) SAS; <math>4\sqrt{2}</math> cm</p> | <p>8) <math>40^\circ</math></p> <p>9) <math>4 &lt; x &lt; 16</math></p> <p>10) equilateral; <math>60^\circ</math> each</p> <p>11) Valid; SSS; right triangle</p> <p>12) ASA; <math>\angle R = 62^\circ</math>; no, AAA gives infinitely many</p> |
|--|--|

### Step-by-Step Explanations

**Strategy:** For Solving Equations with the Distributive Property, distribute before combining like terms so the equation is genuinely simpler before solving. This equation page rewards patience: expand first, then solve the simpler line.

**Practice 1:**  $2(x+3)=14$  **Answer:** 4

At the beginning of the practice, distribute first so each side of the equation is ready to simplify.

**Practice 15:**  $2(5m-4)+6=20$  **Answer:** 2

For the second model problem, collect variable terms on one side and constants on the other before dividing.

#### Word-problem notes:

**16. Answer:**  $h = 14$  per hour; 6 friends:  $6(14 + 5) = 6(19) = \$114$ .

The expression  $4(h + 5) = 76$  means each of the four friends pays the same rink-time cost  $h$  plus the \$5 rental fee. Distribute first:  $4h + 20 = 76$ . Subtract 20 to get  $4h = 56$ , then divide by 4 to find  $h = 14$ . For six friends, use the same per-person cost:  $6(14 + 5) = 6(19) = 114$ . So six friends would pay \$114 total.

**17. Answer:** Plot A perimeter:  $4(2x + 3) = 8x + 12$ ; Plot B:  $2(x + 5) + 2(4) = 2x + 18$ ;  $8x + 12 = 2x + 18 \Rightarrow x = 1$ ; perimeter: 20 ft each.

Write an expression for each perimeter. The square has four equal sides, so Plot A's perimeter is  $4(2x + 3) = 8x + 12$ . The rectangle has two lengths and two widths, so Plot B's perimeter is  $2(x + 5) + 2(4) = 2x + 18$ . Set them equal because the fencing lengths are the same:  $8x + 12 = 2x + 18$ . Subtract  $2x$  and then subtract 12 to get  $6x = 6$ , so  $x = 1$ . Substituting into either perimeter expression gives 20 feet.



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