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Reza Nazari

GRADE 9 ALGEBRA I FOR BEGINNERS

A Comprehensive Review and Step-by-Step Guide to Mastering Algebra 1

Answers and Solutions

By

Reza Nazari

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CHAPTER

1 Exponents and Variables

Math topics in this chapter:

- ☑ Multiplication Property of Exponents
- ☑ Division Property of Exponents
- ☑ Powers of Products and Quotients
- ☑ Zero and Negative Exponents
- ☑ Negative Exponents and Negative Bases
- ☑ Scientific Notation
- ☑ Addition and Subtraction in Scientific Notation
- ☑ Multiplication and division in Scientific Notation

Practices

Find the products.

1) $x^2 \times 4xy^2 =$
2) $3x^2y \times 5x^3y^2 =$
3) $6x^4y^2 \times x^2y^3 =$
4) $7xy^3 \times 2x^2y =$
5) $-5x^5y^5 \times x^3y^2 =$
6) $-8x^3y^2 \times 3x^3y^2 =$
7) $-6x^2y^6 \times 5x^4y^2 =$
8) $-3x^3y^3 \times 2x^3y^2 =$
9) $-6x^5y^3 \times 4x^4y^3 =$
10) $-2x^4y^3 \times 5x^6y^2 =$
11) $-7y^6 \times 3x^6y^3 =$
12) $-9x^4 \times 2x^4y^2 =$

Simplify.

13) $\frac{5^3 \times 5^4}{5^9 \times 5} =$
14) $\frac{3^3 \times 3^2}{7^2 \times 7} =$
15) $\frac{15x^5}{5x^3} =$
16) $\frac{16x^3}{4x^5} =$
17) $\frac{72y^2}{8x^3y^6} =$
18) $\frac{10x^3y^4}{50x^2y^3} =$
19) $\frac{13y^2}{52x^4y^4} =$
20) $\frac{50xy^3}{200x^3y^4} =$
21) $\frac{48x^2}{56x^2y^2} =$
22) $\frac{81y^6x}{54x^4y^3} =$

Solve.

23) $(x^3y^3)^2 =$
24) $(3x^3y^4)^3 =$
25) $(4x \times 6xy^3)^2 =$
26) $(5x \times 2y^3)^3 =$
27) $\left(\frac{9x}{x^3}\right)^2 =$
28) $(\frac{3y}{18y^2})^2 =$
29) $\left(\frac{3x^2y^3}{24x^4y^2}\right)^3 =$
30) $\left(\frac{26x^5y^3}{52x^3y^5}\right)^2 =$
31) $\left(\frac{18x^7y^4}{72x^5y^2}\right)^2 =$
32) $\left(\frac{12x^6y^4}{48x^5y^3}\right)^2 =$

Evaluate each expression.

33) $(\frac{1}{4})^{-2} =$

34) $(\frac{1}{3})^{-2} =$

35) $(\frac{1}{7})^{-3} =$

36) $(\frac{2}{5})^{-3} =$

37) $(\frac{2}{3})^{-3} =$

38) $(\frac{3}{5})^{-4} =$

Write each expression with positive exponents.

39) $x^{-7} =$

40) $3y^{-5} =$

41) $15y^{-3} =$

42) $-20x^{-4} =$

43) $12a^{-3}b^{5} =$

44) $25a^{3}b^{-4}c^{-3} =$

45) $-4x^{5}y^{-3}z^{-6} =$

46) $\frac{18y}{x^{3}y^{-2}} =$

47) $\frac{20a^{-2}b}{-12c^{-4}} =$

Write each number in scientific notation.

48) $0.00412 =$

49) $0.000053 =$

50) $66{,}000 =$

51) $72{,}000{,}000 =$

Write the answer in scientific notation.

52) $6 \times 10^{4} + 10 \times 10^{4} =$ ________

53) $7.2 \times 10^{6} - 3.3 \times 10^{6} =$ __________

54) $2.23 \times 10^{7} + 5.2 \times 10^{7} =$ ______

55) $8.3 \times 10^{9} - 5.6 \times 10^{8} =$ _____

56) $1.4 \times 10^{2} + 7.4 \times 10^{5} =$ _____

57) $9.6 \times 10^{6} - 3 \times 10^{4} =$ ______

Simplify. Write the answer in scientific notation.

58) $(5.6 \times 10^{12})(3 \times 10^{-7}) =$ ____

59) $(3 \times 10^{-8})(7 \times 10^{10}) =$ _____

60) $(9 \times 10^{-3})(4.2 \times 10^{6}) =$ ____

61) $\frac{125\times10^{9}}{50\times10^{12}} =$ ____

62) $\frac{2.8\times10^{12}}{0.4\times10^{20}} =$ ____

63) $\frac{9\times10^{8}}{3\times10^{7}} =$ ____

Answers

1) $4x^3y^2$
2) $15x^5y^3$
3) $6x^6y^5$
4) $14x^3y^4$
5) $-5x^8y^7$
6) $-24x^6y^4$
7) $-30x^6y^8$
8) $-6x^6y^5$
9) $-24x^9y^6$
10) $-10x^{10}y^5$
11) $-21x^6y^9$
12) $-18x^8y^2$
13) $\frac{1}{125}$
14) $\frac{243}{343}$
15) $3x^2$
16) $\frac{4}{x^2}$
17) $\frac{9}{x^3y^4}$
18) $\frac{xy}{5}$
19) $\frac{1}{4x^4y^2}$
20) $\frac{1}{4x^2y}$
21) $\frac{6}{7y^2}$
22) $\frac{3y^3}{2x^3}$
23) x^6y^6
24) $27x^9y^{12}$
25) $576x^4y^6$
26) $1{,}000x^3y^9$
27) $\frac{81}{x^4}$
28) $\frac{1}{36y^2}$
29) $\frac{y^3}{512x^6}$
30) $\frac{x^4}{4y^4}$
31) $\frac{x^4y^4}{16}$
32) $\frac{x^2y^2}{16}$
33) 16
34) 9
35) 343
36) $\frac{125}{8}$
37) $\frac{27}{8}$
38) $\frac{625}{81}$
39) $\frac{1}{x^7}$
40) $\frac{3}{y^5}$
41) $\frac{15}{y^3}$
42) $-\frac{20}{x^4}$
43) $\frac{12b^5}{a^3}$
44) $\frac{25a^3}{b^4c^3}$
45) $-\frac{4x^5}{y^3z^6}$
46) $\frac{18y^3}{x^3}$
47) $-\frac{5bc^4}{3a^2}$
48) 4.12×10^{-3}
49) 5.3×10^{-5}
50) 6.6×10^4
51) 7.2×10^7
52) 1.6×10^5
53) 3.9×10^6
54) 7.43×10^7
55) 7.74×10^9
56) 7.4014×10^5
57) 9.57×10^6
58) 1.68×10^6
59) 2.1×10^3
60) 3.78×10^4
61) 2.5×10^{-3}
62) 7×10^{-8}
63) 3×10^1

Answers and Explanations

1. Multiply the coefficients and then apply the product of powers rule for each term. Coefficients: $1 \times 4 = 4$. For x: $x^2 \times x = x^{2+1} = x^3$. Result: $4x^3y^2$.

2. Coefficients: $3 \times 5 = 15$. For x: $x^2 \times x^3 = x^{2+3} = x^5$. For y: $y \times y^2 = y^{1+2} = y^3$. Answer: $15x^5y^3$.

3. Coefficients remain as 6 (since multiplying by 1). For x: $x^4 \times x^2 = x^{4+2} = x^6$. For y: $y^2 \times y^3 = y^{2+3} = y^5$. Result: $6x^6y^5$.

4. Coefficients: $7 \times 2 = 14$. For x: $x \times x^2 = x^{1+2} = x^3$. For y: $y^3 \times y = y^{3+1} = y^4$. Result: $14x^3y^4$.

5. Coefficients: $-5 \times 1 = -5$. For x: $x^5 \times x^3 = x^{5+3} = x^8$. For y: $y^5 \times y^2 = y^{5+2} = y^7$. Answer: $-5x^8y^7$.

6. Numerically, -8 times 3 is -24. x^3 terms combine to x^6. y^2 terms stay as y^4. The product is $-24x^6y^4$.

7. For the numbers, -6 times 5 is -30. Combining x terms, x^2 and x^4 yield x^6. For y, y^6 and y^2 produce y^8. It's $-30x^6y^8$.

8. -3 multiplied by 2 gives -6. The x^3 terms result in x^6. For y, y^3 and y^2 become y^5. The answer is $-6x^6y^5$.

9. -6 times 4 is -24. Combining x's, $x^5$and x^4 produce x^9. y^3's yield y^6. Hence, $-24x^9y^6$.

10. Numerically, -2 times 5 is -10. x^4 and x^6 combine to x^{10}. y^3 and y^2 become y^5. It's $-10x^{10}y^5$.

11. -7 times 3 results in -21. The x term is just x^6. Combining y's, y^6 and y^3 give y^9. The answer is $-21x^6y^9$.

12. -9 times 2 is -18. For x, x^4 times x^4 yields x^8. The y term remains y^2. The result is $-18x^8y^2$.

13. To simplify this expression, you can combine the exponents of like bases by adding them when multiplying and subtracting them when dividing. Here, $5^3 \times 5^4$ becomes $5^{3+4} = 5^7$ and $5^9 \times 5$ becomes $5^{9+1} = 5^{10}$. Then you divide 5^7 by 5^{10}, which is $5^{7-10} = 5^{-3}$. This is the same as $\frac{1}{5^3}$, which simplifies to $\frac{1}{125}$.

14. Combine the exponents for the number 3: $3^3 \times 3^2 = 3^{3+2} = 3^5$. For the number 7, it's $7^2 \times 7 = 7^{2+1} = 7^3$. Now, divide 3^5 by 7^3. Since these are different bases, you cannot simplify further and are left with $\frac{3^5}{7^3}$ or $\frac{243}{343}$.

15. Divide the numerical coefficients: $\frac{15}{5} = 3$, hen, for the variable $\frac{x^5}{x^3} = x^{5-3} = x^2$. Combining these gives you $3x^2$.

16. First, divide the numbers: $\frac{16}{4} = 4$. For $\frac{x^3}{x^5}$ you subtract the exponents (since you're dividing like bases), which gives you $x^{3-5} = x^{-2}$. This means $\frac{4}{x^2}$.

17. Divide the numerical coefficients: $\frac{72}{8} = 9$. For $\frac{y^2}{y^6}$, subtract the exponents, giving $y^{2-6} = y^4$, which is $\frac{1}{y^4}$. Since the x^3 in the denominator does not have a corresponding x in the numerator, the final answer is $\frac{9}{x^3y^4}$.

18. Divide the numbers: $\frac{10}{50} = \frac{1}{5}$. For, $\frac{x^3}{x^2}$, subtract the exponents: $x^{3-2} = x$. Do the same for $\frac{y^4}{y^3} = y^{4-3} = y$. The final answer is $\frac{xy}{5}$.

19. Here, divide the coefficients: $\frac{13}{52} = \frac{1}{4}$. For the variables, $\frac{y^2}{y^4} = y^{2-4} = y^{-2}$, which is $\frac{1}{y^2}$. There is no x in the numerator to cancel out the x^4 in the denominator, so the answer is $\frac{1}{4x^4y^2}$.

20. Divide $\frac{50}{200}$ to get $\frac{1}{4}$. Now, $\frac{xy^3}{x^3y^4}$ means you have to deal with the exponents separately for x and y. For x, there's no exponent in the numerator, so $\frac{x}{x^3} = x^{1-3} = x^{-2}$. For y, it's $\frac{y^3}{y^4} = y^{3-4} = y^{-1}$. Combined, you have $\frac{1}{4x^2y}$.

21. First, divide $\frac{48}{56}$ which simplifies to $\frac{6}{7}$ when reduced. For the x^2 terms, since the exponents are the same, $\frac{x^2}{x^2}$ cancels out to 1. The y^2 in the denominator remains, so the final expression is $\frac{6}{7y^2}$.

22. Divide the numerical coefficients $\frac{81}{54}$, which reduces to $\frac{3}{2}$. Then for the variables: $\frac{x}{x^4} = \frac{1}{x^3}$, and $\frac{y^6}{y^3} = y^{6-3} = y^3$. Putting it all together gives $\frac{3y^3}{2x^3}$.

23. When raising a power to a power, you multiply the exponents. For x^3 raised to the power of 2, you multiply the exponents: $3 \times 2 = 6$ resulting in x^6. Similarly, for y^3 raised to the power of 2, you get y^6. The result is x^6y^6.

24. Start by raising the coefficient 3 to the power of 3, which is 27. Then, raise x^3 the power of 3 to get x^9. Finally, y^4 to the power of 3 gives y^{12}. Combined, you get $27x^9y^{12}$.

25. First, multiply the coefficients: $4 \times 6 = 24$. Then multiply $x \times x$ to get x^2. Finally, there's a y^3 term. Combining these gives $24x^2y^3$. Now, square the entire expression to get $576x^4y^6$.

26. Multiply the numbers first: $5 \times 2 = 10$. Combine x and y^3 to get $10xy^3$. Now, raise the entire expression to the third power. This results in $1{,}000x^3y^9$.

27. Here, divide $9x$ by x^3. The x's will reduce to x^{-2}. When you square the result, you get $\frac{81}{x^4}$.

28. Divide 3 by 18 to get $\frac{1}{6}$. Then, simplify $\frac{y}{y^2}$ to get y^{-1}. Squaring the result, you have $\frac{1}{36y^2}$.

29. Begin by simplifying the terms inside the parentheses before taking the cube. For the constants, 3 divided by 24 is $\frac{1}{8}$. For the terms involving x, x^2 divided by x^4 is x^{-2}. For the terms involving y, y^3 divided by y^2 is y. So, before taking the cube, the expression is $\frac{y}{8x^2}$. When you cube this, you get $\frac{y^3}{512x^6}$.

30. First, divide the coefficients, 26 by 52, which is $\frac{1}{2}$. Then, for x: $\frac{x^5}{x^3}$ results in x^2. For y: $\frac{y^3}{y^5}$ results in y^2. Combining these gives $\frac{x^2}{2y^2}$. Squaring this entire expression provides $\frac{x^4}{4y^4}$.

31. Dividing the numbers, 18 by 72 becomes $\frac{1}{4}$. Simplifying x: $\frac{x^7}{x^5}$ becomes x^2. Simplifying y: $\frac{y^4}{y^2}$ becomes y^2. Combining these results, you have $\frac{x^2y^2}{4}$. When squared, this gives $\frac{x^4y^4}{16}$.

32. Start by dividing the coefficients, 12 by 48, which results in $\frac{1}{4}$. For the terms involving x, $\frac{x^6}{x^5}$ is x. For y, $\frac{y^4}{y^3}$ becomes y. Combining them, the expression is $\frac{xy}{4}$. Squaring this entire expression, you get $\frac{x^2y^2}{16}$.

33. A negative exponent means to take the reciprocal of the base and then raise it to the positive value of that exponent. For $\frac{1}{4}$ the reciprocal is $\frac{4}{1}$ or just 4. Raising 4 to the power of 2 (because of the -2 exponent) results in $4^2 = 16$.

34. Take the reciprocal of $\frac{1}{3}$, which is 3 (or $\frac{3}{1}$). Now, square 3 (due to the -2 exponent), which equals $3^2 = 9$.

35. First, find the reciprocal of $\frac{1}{7}$, which is 7. Now, raise 7 to the power of 3 (because of the -3 exponent) to get $7^3 = 343$.

36. For $\frac{2}{5}$, the reciprocal is $\frac{5}{2}$. Raise this fraction to the power of 3. That means you'll cube both the numerator and the denominator separately: $5^3 = 125$, and $2^3 = 8$. Thus, the result is $\frac{125}{8}$.

37. The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$. Cube both the numerator and the denominator separately: $3^3 = 27$, and $2^3 = 8$. So, the answer is $\frac{27}{8}$.

38. For $\frac{3}{5}$, the reciprocal is $\frac{5}{3}$. When raised to the power of 4, this means you'll raise both the numerator and denominator to the 4th power:

$5^4 = 625$, and $3^4 = 81$. The final result is $\frac{625}{81}$.

39. A negative exponent means you'll take the reciprocal. In terms of x, this means you'll move it from the numerator to the denominator. So, x^{-7} becomes $\frac{1}{x^7}$.

40. The negative exponent on y means we'll place y in the denominator. The coefficient 3 remains in the numerator. Thus, it becomes $\frac{3}{y^5}$.

41. The y term, because of its negative exponent, will go to the denominator. This gives $\frac{15}{y^3}$.

42. The x term moves to the denominator due to its negative exponent. This results in $\frac{-20}{x^4}$.

43. The a term's negative exponent means it will be placed in the denominator, while the b^5 remains in the numerator. This results in $\frac{12b^5}{a^3}$.

44. Here, both b and c have negative exponents, so they'll move to the denominator. a^3 remains in the numerator. The expression becomes $\frac{25a^3}{b^4c^3}$.

45. Both y and z will move to the denominator because of their negative exponents, while x^5 remains in the numerator. This gives $\frac{-4\,x^5}{y^3z^6}$.

46. The negative exponent on y means it moves to the numerator, thus multiplying with the already existing y in the numerator. This results in $\frac{18y^3}{x^3}$.

47. The negative exponent on a sends it to the denominator, while b remains in the numerator. The c term's negative exponent brings it to the numerator. Combining these adjustments gives $\frac{-20bc^4}{12a^2}$. Simplifying further, this results in $-\frac{5bc^4}{3a^2}$.

48. Scientific notation involves representing a number as a product of two numbers: a coefficient between 1 and 10, and a power of 10. To express 0.00412 in scientific notation, we shift the decimal point two places to the right to make it 4.12. In doing so, we're multiplying by 10^3 or 1000. However, since the original number was smaller, we must divide by 10^3 to balance the operation, resulting in the exponent being -3. Hence, 0.00412 in scientific notation is 4.12×10^{-3}.

49. Shift the decimal point five places to the right to get 5.3. This is equivalent to multiplying by 10^5. But since the original number was smaller, our exponent will be -5. Thus, the scientific notation for 0.000053 is 5.3×10^{-5}.

50. Move the decimal point from the end of the number four places to the left to make it 6.6. In this process, you're effectively dividing the number by 10^4 or 10,000. To balance the operation, we multiply by 10^4. Therefore, 66,000 in scientific notation is 6.6×10^4.

51. Shift the decimal seven places to the left to get 7.2. We've divided the number by 10^7 or 10,000,000. To counteract that division, we multiply by 10^7. Hence, 72,000,000 in scientific notation is 7.2×10^7.

52. When adding numbers in scientific notation with the same exponent, simply add their coefficients. So, $6 + 10 = 16$. Your answer is 1.6×10^5 .

53. For numbers with the same exponent, subtract their coefficients.

$7.2 - 3.3 = 3.9$. The result is 3.9×10^6.

54. Combine the coefficients by adding them: $2.23 + 5.2 = 7.43$. So, your answer is 7.43×10^7.

55. To subtract these, you need to express them with the same exponent. Rewrite 5.6×10^8 as 0.56×10^9. Then subtract: $8.3 - 0.56 = 7.74$. The answer is 7.74×10^9.

56. The exponents are different, so focus on the one with the larger exponent. You can rewrite 1.4×10^2 as 0.0014×10^5. Now, add the coefficients:

$0.0014 + 7.4 = 7.4014$. The result is 7.4014×10^5.

57. Rewrite 3×10^4 as 0.03×10^6. Subtracting the coefficients gives $9.6 - 0.03 = 9.57$. The result is 9.57×10^6.

58. When you multiply numbers in scientific notation, you multiply the coefficients (the numbers in front) together, and then you add the exponents. Coefficient multiplication: $5.6 \times 3 = 16.8$. Exponent addition:

$12 + (-7) = 5$. Answer: 1.68×10^6.

59. Multiply the coefficients and add the exponents. Coefficient multiplication: $3 \times 7 = 21$.

Exponent addition: $(-8) + 10 = 2$. Answer: 2.1×10^3.

60. Coefficient multiplication: $9 \times 4.2 = 37.8$. Exponent addition: $(-3) + 6 = 3$. Answer: 3.78×10^4.

61. For division, you divide the coefficients and subtract the second exponent from the first. Coefficient division: $125 \div 50 = 2.5$. Exponent subtraction:

$9 - 12 = -3$. Answer: 2.5×10^{-3}.

62. Divide the coefficients and subtract the exponents. Coefficient division:

$2.8 \div 0.4 = 7$. Exponent subtraction: $12 - 20 = -8$. Answer: 7×10^{-8}.

63. Coefficient division: $9 \div 3 = 3$. Exponent subtraction: $8 - 7 = 1$.

Answer: 3×10^1.

CHAPTER

2 Expressions and Equations

Math topics in this chapter:

- ☑ Simplifying Variable Expressions
- ☑ Evaluating One Variable
- ☑ Evaluating Two Variables
- ☑ One–Step Equations
- ☑ Multi–Step Equations
- ☑ Rearrange Multi-Variable Equations
- ☑ Finding Midpoint
- ☑ Finding the Distance between Two Points

Practices

Simplify each expression.

1) $(3 + 4x - 1) =$
2) $(-5 - 2x + 7) =$
3) $(12x - 5x - 4) =$
4) $(-16x + 24x - 9) =$
5) $(6x + 5 - 15x) =$
6) $2 + 5x - 8x - 6 =$
7) $5x + 10 - 3x - 22 =$
8) $-5 - 3x^2 - 6 + 4x =$
9) $-6 + 9x^2 - 3 + x =$
10) $5x^2 + 3x - 10x - 3 =$
11) $4x^2 - 2x - 6x + 5 - 8 =$
12) $3x^2 - 5x - 7x + 2 - 4 =$
13) $9x^2 - x - 5x + 3 - 9 =$
14) $2x^2 - 7x - 3x^2 + 4x + 6 =$

Evaluate each expression using the value given.

15) $x = 4 \rightarrow 10 - x =$ ____
16) $x = 6 \rightarrow x + 8 =$ ____
17) $x = 3 \rightarrow 2x - 6 =$ ____
18) $x = 2 \rightarrow 10 - 4x =$ ____
19) $x = 7 \rightarrow 8x - 3 =$ ____
20) $x = 9 \rightarrow 20 - 2x =$ ____
21) $x = 5 \rightarrow 10x - 30 =$ ___
22) $x = -6 \rightarrow 5 - x =$ ____
23) $x = -3 \rightarrow 22 - 3x =$ ____
24) $x = -7 \rightarrow 10 - 9x =$ ____
25) $x = -10 \rightarrow 40 - 3x =$ ____
26) $x = -2 \rightarrow 20x - 5 =$ ____
27) $x = -5 \rightarrow -10x - 8 =$ ___
28) $x = -4 \rightarrow -1 - 4x =$ ___

Evaluate each expression using the values given.

29) $x = 2, y = 1 \rightarrow 2x + 7y =$ ______
30) $a = 3, b = 5 \rightarrow 3a - 5b =$ ______
31) $x = 6, y = 2 \rightarrow 3x - 2y + 8 =$ ______
32) $a = -2, b = 3 \rightarrow -5a + 2b + 6 =$ ______
33) $x = -4, y = -3 \rightarrow -4x + 10 - 8y =$ ______

Solve each equation.

34) $x + 6 = 3 \rightarrow x =$ ____
35) $5 = 11 - x \rightarrow x =$ ____
36) $-3 = 8 + x \rightarrow x =$ ____
37) $x - 2 = -7 \rightarrow x =$ ____
38) $-15 = x + 6 \rightarrow x =$ ____
39) $10 - x = -2 \rightarrow x =$ ____
40) $22 - x = -9 \rightarrow x =$ ____
41) $-4 + x = 28 \rightarrow x =$ ____
42) $11 - x = -7 \rightarrow x =$ ____
43) $35 - x = -7 \rightarrow x =$ ____

Solve each equation.

44) $4(x + 2) = 12 \rightarrow x =$ ____
45) $-6(6 - x) = 12 \rightarrow x =$ ____
46) $5 = -5(x + 2) \rightarrow x =$ ____
47) $-10 = 2(4 + x) \rightarrow x =$ ____
48) $4(x + 2) = -12 \rightarrow x =$ ____
49) $-6(3 + 2x) = 30 \rightarrow x =$ ____
50) $-3(4 - x) = 12 \rightarrow x =$ ____
51) $-4(6 - x) = 16 \rightarrow x =$ ____

Solve.

52) $q = 2l + 2w$ for w.
53) $x = 2yw$ for w.
54) $pv = nRT$ for T.
55) $a = b + c + d$ for d.

Find the midpoint of the line segment with the given endpoints.

56) $(5,0), (1,4)$
57) $(2,3), (4,7)$
58) $(8,1), (2,5)$
59) $(5,10), (3,6)$
60) $(4,-1), (-2,7)$
61) $(2,-5), (4,1)$
62) $(7,6), (-5,2)$
63) $(-2,8), (4,-6)$

Find the distance between each pair of points.

64) $(-2,8), (-6,8)$
65) $(4,-4), (14,20)$
66) $(-1,9), (-5,6)$
67) $(0,3), (4,3)$
68) $(0,-2), (5,10)$
69) $(4,3), (7,-1)$
70) $(2,6), (10,-9)$
71) $(3,3), (6,-1)$
72) $(-2,-12), (14,18)$
73) $(2,-2), (12,22)$

Answers

1) $4x+2$
2) $-2x+2$
3) $7x-4$
4) $8x-9$
5) $-9x+5$
6) $-3x-4$
7) $2x-12$
8) $-3x^2+4x-11$
9) $9x^2+x-9$
10) $5x^2-7x-3$
11) $4x^2-8x-3$
12) $3x^2-12x-2$
13) $9x^2-6x-6$
14) $-x^2-3x+6$
15) 6
16) 14
17) 0
18) 2
19) 53
20) 2
21) 20
22) 11
23) 31
24) 73
25) 70
26) -45
27) 42
28) 15
29) 11
30) -16
31) 22
32) 22
33) 50
34) -3
35) 6
36) -11
37) -5
38) -21
39) 12
40) 31
41) 32
42) 18
43) 42
44) 1
45) 8
46) -3
47) -9
48) -5
49) -4
50) 8
51) 10
52) $\frac{1}{2}q-l=w$
53) $w=\frac{x}{2y}$
54) $T=\frac{PV}{nR}$
55) $d=a-b-c$
56) $(3,2)$
57) $(3,5)$
58) $(5,3)$
59) $(4,8)$
60) $(1,3)$
61) $(3,-2)$
62) $(1,4)$
63) $(1,1)$
64) 4
65) 26
66) 5
67) 4
68) 13
69) 5
70) 17
71) 5
72) 34
73) 26

Answers and Explanations

1. Combine like terms. There are no like terms to the variable x and the constants can be combined. Answer: $4x + 2$.

2. Combining the constants, $-5 + 7$ gives 2. The term $-2x$ remains as it is. Result: $-2x + 2$.

3. Combine the like terms (terms that have x). $12x$ minus $5x$ equals $7x$. The constant -4 remains unchanged. Simplified: $7x - 4$.

4. When we combine the x terms, $-16x + 24x$ gives $8x$. The constant -9 is unaltered. Answer: $8x - 9$.

5. Combine the x coefficients. $6x$ minus $15x$ is $-9x$. The constant 5 remains the same. Simplified: $-9x + 5$.

6. Combine the x coefficients: $5x$ minus $8x$ results in $-3x$. Next, combine the constants: 2 minus 6 is -4. Answer: $-3x - 4$.

7. $5x$ minus $3x$ gives $2x$. And 10 minus 22 equals -12. Answer: $2x - 12$.

8. The term $-3x^2$ remains as it is. The x term, $4x$, remains unchanged. Combining the constants, $-5 - 6$ gives -11. Simplified: $-3x^2 + 4x - 11$.

9. The term $9x^2$ stands alone. Combining the x terms, x remains unchanged.

-6 minus 3 equals -9. Simplified: $9x^2 + x - 9$.

10. The term $5x^2$ is unaltered. $3x$ minus $10x$ is $-7x$. The constant -3 remains the same. Answer: $5x^2 - 7x - 3$.

11. The term $4x^2$ stands alone. $-2x$ minus $6x$ results in $-8x$. 5 minus 8 gives -3. Result: $4x^2 - 8x - 3$.

12. The term $3x^2$ is unaltered. $-5x$ minus $7x$ equals $-12x$. 2 minus 4 results in -2. Result: $3x^2 - 12x - 2$.

13. The term $9x^2$ is unchanged. $-x$ minus $5x$ gives $-6x$. 3 minus 9 is -6. Answer: $9x^2 - 6x - 6$.

14. Combine the x^2 terms: $2x^2$ minus $3x^2$ equals $-x^2$. $-7x$ plus $4x$ is $-3x$. The constant 6 remains unchanged. Result: $-x^2 - 3x + 6$.

15. Subtract the value of x (which is 4) from 10 to get the result, 6.

16. Plugging in 6 for x: $6 + 8 = 14$. Hence, with x as 6, $x + 8$ is 14.

17. Multiply 2 by 3 and then subtract 6: $(2 \times 3) - 6 = 6 - 6 = 0$. Thus, if x is 3, $2x - 6$ is 0.

18. Multiply 4 by 2 and subtract from 10: $10 - (4 \times 2) = 10 - 8 = 2$. Here, with x being 2, $10 - 4x$ equals 2.

19. Multiply 8 by 7 and then subtract 3: $(8 \times 7) - 3 = 56 - 3 = 53$. So, if x is 7, $8x - 3$ becomes 53.

20. Multiplying 2 by 9 and subtracting from 20: $20 - (2 \times 9) = 20 - 18 = 2$. This means, for x as 9, $20 - 2x$ is 2.

21. By multiplying 10 by 5 and then subtracting 30:

$(10 \times 5) - 30 = 50 - 30 = 20$. Thus, when x is 5, $10x - 30$ gives 20.

22. Subtracting -6 from 5 gives: $5 - (-6) = 5 + 6 = 11$. So, with x as -6, $5 - x$ results in 11.

23. Multiplying 3 by -3 and adding to 22: $22 - (3 \times (-3)) = 22 + 9 = 31$. Hence, for x equal to -3, $22 - 3x$ equals 31.

24. Multiply 9 by -7 and add to 10: $10 - (9 \times (-7)) = 10 + 63 = 73$. Thus, when x is -7, $10 - 9x$ is 73.

25. Multiply 3 by -10 and add to 40: $40 - (3 \times (-10)) = 40 + 30 = 70$. With x as -10, $40 - 3x$ results in 70.

26. Multiply 20 by -2 and subtract 5: $(20 \times (-2)) - 5 = -40 - 5 = -45$. Hence, for x being -2, $20x - 5$ is -45.

27. Multiply -10 by -5 and subtract 8: $(-10 \times (-5)) - 8 = 50 - 8 = 42$. So, if x equals -5, $-10x - 8$ gives 42.

28. Multiplying 4 by -4 and subtracting from -1:

$-1 - (4 \times (-4)) = -1 + 16 = 15$. With x as -4, $-1 - 4x$ results in 15.

29. For this expression, replace x with 2 and y with 1: $2(2) + 7(1) = 4 + 7 = 11$. With the values provided, the expression evaluates to 11.

30. Insert the values for a and b into the equation: $3(3) - 5(5) = 9 - 25 = -16$. Using the assigned values, the expression calculates to -16.

31. Place the given values of x and y into the equation:

$3(6) - 2(2) + 8 = 18 - 4 + 8 = 22$. By substituting the values for x and y, we get the result as 22.

32. Add the given values of a and b to the formula:

$-5(-2) + 2(3) + 6 = 10 + 6 + 6 = 22$. By inputting -2 for a and 3 for b, the sum becomes 22.

33. Plug the provided x and y into the equation:

$-4(-4) + 10 - 8(-3) = 16 + 10 + 24 = 50$. Employing the given values for x and y results in a sum of 50.

34. To solve for x, you need to isolate x on one side of the equation. First, subtract 6 from both sides of the equation to get: $x + 6 - 6 = 3 - 6$, $x = -3$.

35. To solve for x in this equation, you want to isolate x on one side. Begin by subtracting 11 from both sides: $5 - 11 = 11 - x - 11 \rightarrow -6 = -x$. Now, to find x, multiply both sides by -1 to get: $-1 \times (-6) = -1 \times (-x) \rightarrow 6 = x$.

36. Start by subtracting 8 from both sides of the equation:

$-3 - 8 = 8 + x - 8 \rightarrow -11 = x$.

37. To solve for x, add 2 to both sides of the equation:

$x - 2 + 2 = -7 + 2 \rightarrow x = -5$.

38. To isolate x, subtract 6 from both sides of the equation:

$-15 - 6 = x + 6 - 6 \rightarrow -21 = x$.

39. Start by adding x to both sides of the equation:

$10 - x + x = -2 + x \rightarrow 10 = x - 2$. Now, add 2 to both sides:

$10 + 2 = x - 2 + 2 \rightarrow 12 = x$.

40. Add x to both sides of the equation: $22 - x + x = -9 + x \rightarrow 22 = -9 + x$. Now, subtract -9 from both sides: $22 + 9 = -9 + x + 9 \rightarrow 31 = x$.

41. To solve for x, add 4 to both sides of the equation:

$-4 + x + 4 = 28 + 4 \rightarrow x = 32$.

42. Add x to both sides of the equation: $11 - x + x = -7 + x \rightarrow 11 = -7 + x$. Now, add 7 to both sides: $11 + 7 = -7 + x + 7 \rightarrow 18 = x$.

43. Add x to both sides of the equation: $35 - x + x = -7 + x \rightarrow 35 = -7 + x$. Now, add 7 to both sides: $35 + 7 = -7 + x + 7 \rightarrow 42 = x$.

44. Multiply out the brackets: $4x + 8 = 12$. Subtract 8 from both sides:$4x = 4$. Divide by 4:$x = 1$.

45. Distribute the -6: $-36+6x=12$. Add 36 to both sides: $6x=48$. Divide by 6: $x=8$.

46. Multiply out the brackets: $5=-5x-10$. Add $5x$ to both sides: $5x+5=-10$. Subtract 5 from both sides: $5x=-15$. Divide by 5: $x=-3$.

47. Distribute the 2: $-10=8+2x$. Subtract 8 from both sides: $-18=2x$. Divide by 2: $x=-9$.

48. Multiply out the brackets: $4x+8=-12$. Subtract 8 from both sides:

$4x=-20$. Divide by 4: $x=-5$.

49. Distribute the -6: $-18-12x=30$. Add 18 to both sides: $-12x=48$. Divide by -12: $x=-4$.

50. Distribute the -3: $-12+3x=12$. Add 12 to both sides: $3x=24$. Divide by 3: $x=8$.

51. Distribute the -4: $-24+4x=16$. Add 24 to both sides: $4x=40$. Divide by 4: $x=10$.

52. First, isolate the terms with w by subtracting $2l$ from both sides: $q-2l=2w$. Now, divide both sides by 2 to solve for w. $w=\frac{q-2l}{2}$. When simplified further:

$$w=\frac{1}{2}q-l.$$

53. Initially, to free w from being multiplied, divide both sides by $2y$: $w=\frac{x}{2y}$.

54. Begin by isolating T. To do this, divide both sides by nR: $T=\frac{pv}{nR}$.

55. To get d on its own, subtract both b and c from each side: $d=a-b-c$.

56. The x-coordinate of the midpoint is the average of 5 and 1, which is $\frac{5+1}{2}=3$. Similarly, the y-coordinate of the midpoint is the average of 0 and 4, which is $\frac{0+4}{2}=2$. Midpoint: $(3,2)$.

57. For the x-values: $\frac{2+4}{2}=3$. For the y-values: $\frac{3+7}{2}=5$. Midpoint: $(3,5)$.

58. The central x-coordinate is $\frac{8+2}{2}=5$ and the y-coordinate is $\frac{1+5}{2}=3$. Midpoint: $(5,3)$.

59. For x, you have $\frac{5+3}{2}=4$. For y, $\frac{10+6}{2}=8$. Midpoint: $(4,8)$.

60. Taking the middle for x: $\frac{4+(-2)}{2}=1$ and for y: $\frac{-1+7}{2}=3$. Midpoint: $(1,3)$.

61. Average the x-values: $\frac{2+4}{2}=3$. For y: $\frac{-5+1}{2}=-2$. Midpoint: $(3,-2)$.

62. The x's midpoint is $\frac{7-5}{2}=1$. For y's, it's $\frac{6+2}{2}=4$. Midpoint: $(1,4)$.

63. For the x-axis: $\frac{-2+4}{2}=1$. For the y-axis: $\frac{8-6}{2}=1$. Midpoint: $(1,1)$.

64. Use the distance formula: $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{(-6-(-2)^2-(8-8)^2}=4$

65. Calculate the differences in x and y, square them, and add. Then take the square root: $d=\sqrt{(14-4)^2-(20-(-4))^2}=\sqrt{100+576}=\sqrt{676}$. $d=26$.

66. Find the differences in x and y, square, add, and root:

$d=\sqrt{(-5-(-1))^2+(6-9)^2}=\sqrt{16+9}=\sqrt{25}=5$.

67. Use the distance formula: $d=\sqrt{(4-0)^2+(3-3)^2}=4$

68. Calculate the x and y differences, square, add, and root:

$d=\sqrt{(5-0)^2+(10-(2))^2}=\sqrt{25+144}=\sqrt{169}=13$.

69. Find the x and y differences, square, add, and root:

$d=\sqrt{(7-4)^2+(-1-3)^2}=\sqrt{9+16}=\sqrt{25}=5$.

70. To calculate the distance, we use the distance formula:

$d=\sqrt{(10-2)^2+(-9-6)^2}=\sqrt{64+225}=\sqrt{289}=17$.

71. Calculate x and y differences, square, add, and root:

$d=\sqrt{(6-3)^2+(-1-3)^2}=\sqrt{9+16}=\sqrt{25}=5$.

72. Find the x and y differences, square, add, and root:

$d=\sqrt{(14-(-2))^2+(18-(-12))^2}=\sqrt{256+900}=\sqrt{1{,}156}=34$.

73. Find x and y differences, square, add, and root:

$d=\sqrt{(12-2)^2+(22-(-2))^2}=\sqrt{100+576}=\sqrt{676}=26$.

Chapter 3 Linear Functions

Math topics in this chapter:

- ☑ Finding Slope
- ☑ Writing Linear Equations
- ☑ Graphing Linear Inequalities
- ☑ Write an Equation from a Graph
- ☑ Slope-intercept Form and Point-slope Form
- ☑ Write a Point-slope Form Equation from a Graph
- ☑ Find $x-$ and $y-$intercepts in the Standard Form of Equation
- ☑ Graph an Equation in the Standard Form
- ☑ Equations of Horizontal and Vertical Lines
- ☑ Graph a Horizontal or Vertical Line
- ☑ Graph an Equation in Point-Slope Form
- ☑ Equation of Parallel or Perpendicular Lines
- ☑ Compare Linear Function's Graph and Linear Equations
- ☑ Graphing Absolute Value Equations
- ☑ Two-variable Linear Equation Word Problems

Practices

Find the slope of each line.

1) $y = x - 5$
2) $y = 2x + 6$
3) $y = -5x - 8$
4) Line through $(2,6)$ and $(5,0)$
5) Line through $(8,0)$ and $(-4,3)$
6) Line through $(-2,-4)$ and $(-4,8)$

Solve.

7) What is the equation of a line with slope 4 and intercept 16? ________________

8) What is the equation of a line with slope 3 and passes through point $(1,5)$? ________________

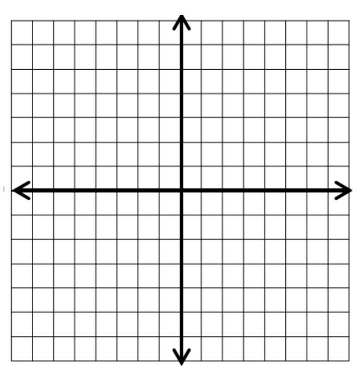
9) What is the equation of a line with slope -5 and passes through point $(-2,7)$? ________________

10) The slope of a line is -4 and it passes through point $(-6,2)$. What is the equation of the line? ________________

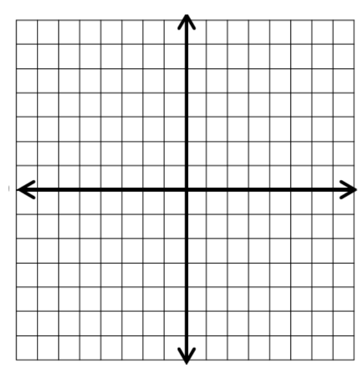
11) The slope of a line is -3 and it passes through point $(-3,-6)$. What is the equation of the line? ________________

Sketch the graph of each linear inequality.

12) $y > 4x + 2$

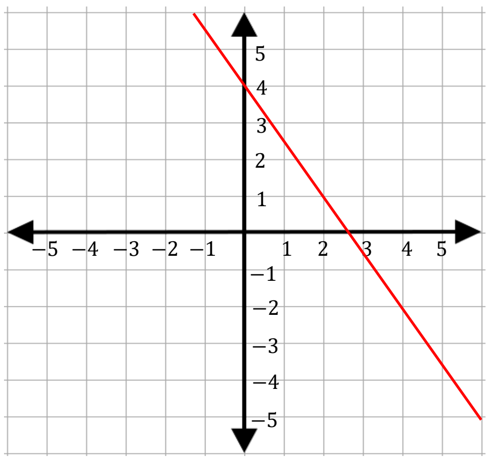


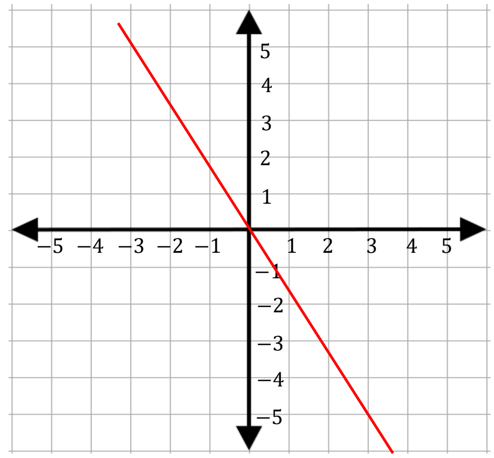
13) $y < -2x + 5$



Write an equation of each of the following lines in slope-intercept from.

14) __________ 15) __________





Find the equation of each line.

16) Through: $(6, -6)$, slope $= -2$
Point-slope form: _________
Slope-intercept form: _________

17) Through: $(-7,7)$, slope $= 4$
Point-slope form: __________
Slope-intercept form: ________

Find the x − intercept of each line.

18) $21x - 3y = -18$

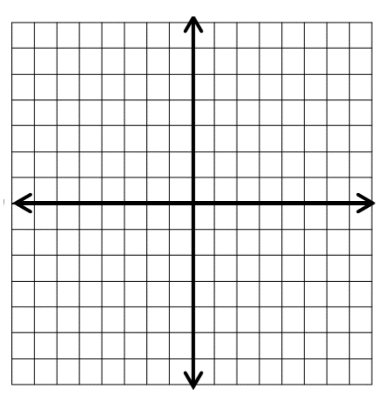
19) $20x + 20y = -10$

20) $8x + 6y = 16$

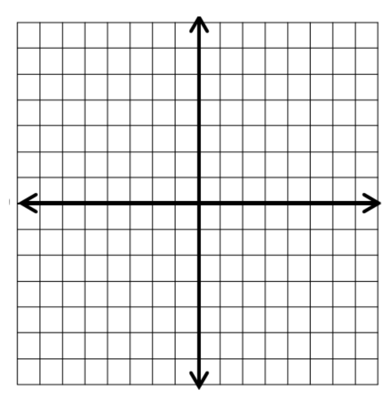
21) $2x - 4y = -12$

Graph each equation.

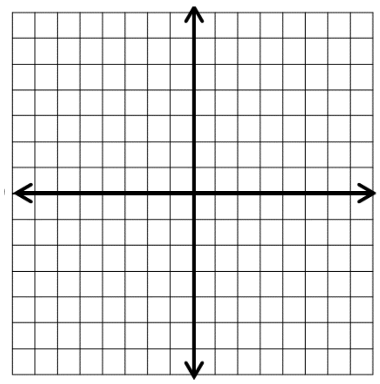
22) $4x - 5y = 40$



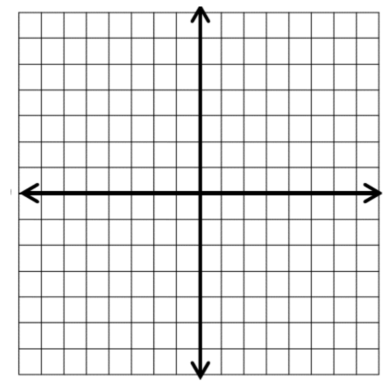
23) $9x - 8y = -72$



24) $y + 3 = -\frac{1}{2}(x - 8)$

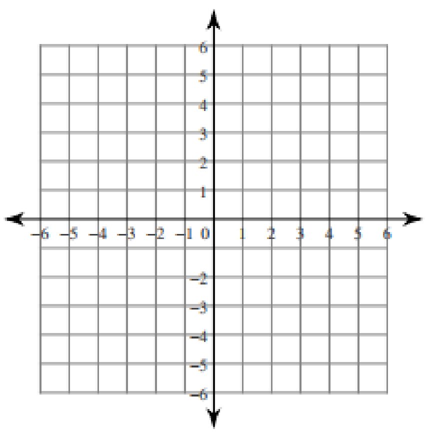


25) $y - 8 = -2(x - 1)$

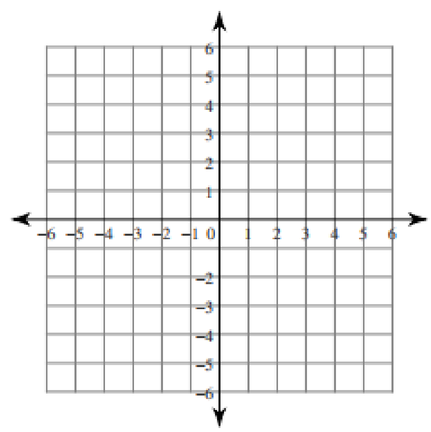


Sketch the graph of each line.

26) Vertical line that passes through (2,6).



27) Horizontal line that passes through (5,3).



Find the equation of each line with the given information.

28) Through: $(4, 4)$

Parallel to $y = -6x + 5$

Equation: _________

29) Through: $(7,1)$

Perp. to $y = -\frac{1}{2}x - 4$

Equation: ________

30) Through: $(2,0)$

Parallel to $y = x$

Equation: _________

31) Through: $(0, -4)$

Perp. to $y = 2x + 3$

Equation: _________

32) Through: $(-1,1)$

Parallel to $y = 2$

Equation: _________

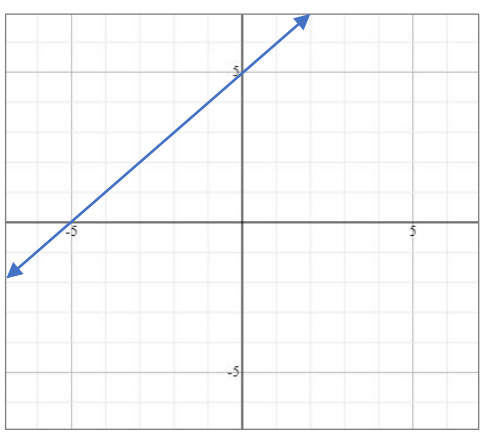
33) Through: $(3,4)$

Perp. to $y = -x$

Equation: _________

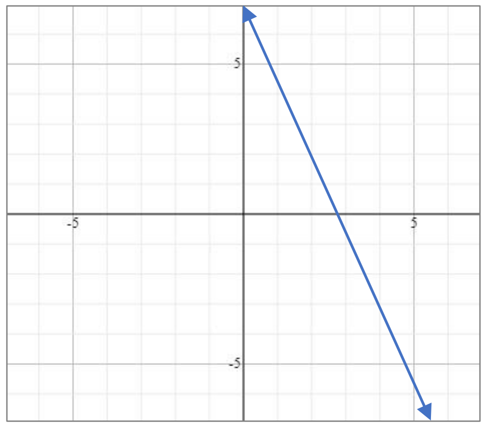
Compare the slope of the function A and function B.

34) Function A:



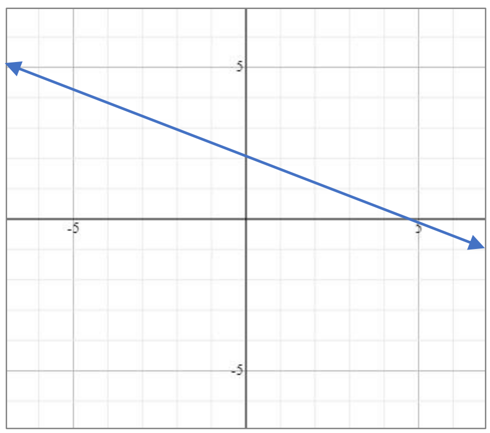
Function B: $y = 6x - 3$

35) Function A:



Function B: $y = -2.5x - 1$

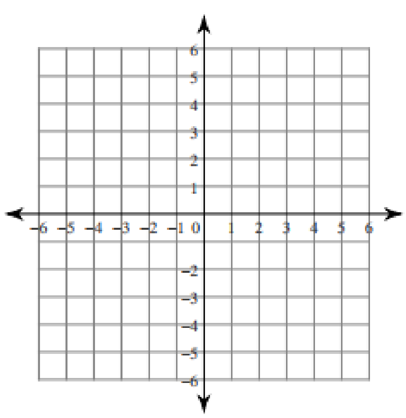
36) Function A:



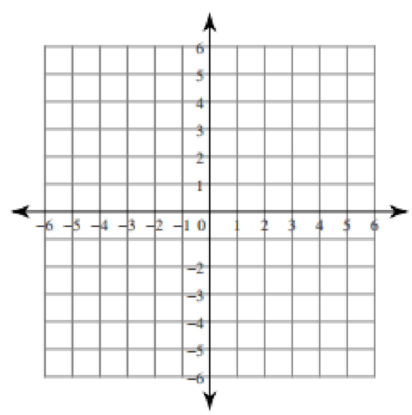
Function B: $y = 2x - 1$

Graph each equation.

37) $y = -|x| - 1$



38) $y = -|x - 3|$



Solve.

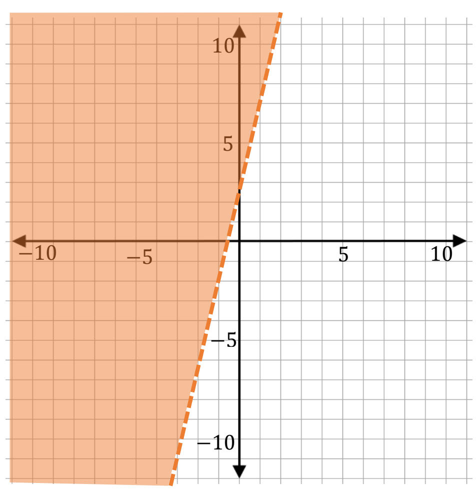
39) John has an automated hummingbird feeder. He fills it to capacity, 8 fluid ounces. It releases 1 fluid ounce of nectar every day. Write an equation that shows how the number of fluid ounces of nectar left, y, depends on the number of days John has filled it, x.

40) The entrance fee to Park City is $9. Additionally, skate rentals cost $4 per hour. Write an equation that shows how the total cost, y, depends on the length of the rental in hours, x.

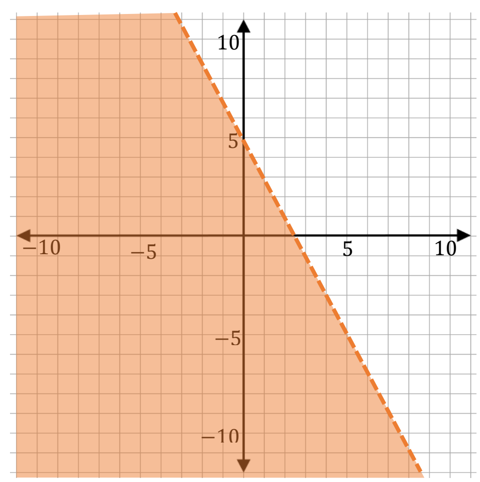
Answers

1) 1
2) 2
3) -5
4) -2
5) $-\frac{1}{4}$
6) -6
7) $y = 4x + 16$
8) $y = 3x + 2$
9) $y = -5x - 3$
10) $y = -4x - 22$
11) $y = -3x - 15$

12) $y > 4x + 2$



13) $y < -2x + 5$



14) $y = -\frac{3}{2}x + 4$

15) $y = -\frac{5}{3}x$

16) Point-slope form: $y + 6 = -2(x - 6)$

Slope-intercept form: $y = -2x + 6$

17) Point-slope form: $y - 7 = 4(x + 7)$

Slope-intercept form: $y = 4x + 35$

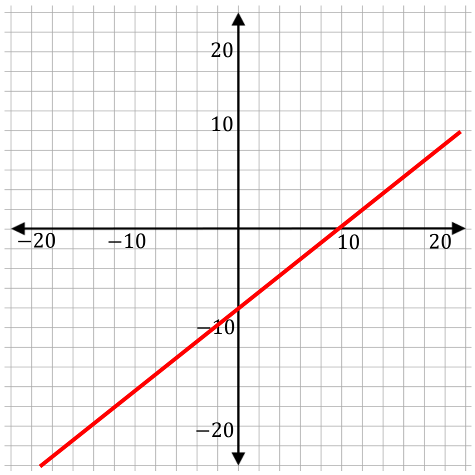
18) $-\frac{6}{7}$

19) $-\frac{1}{2}$

20) 2

21) -6

22)



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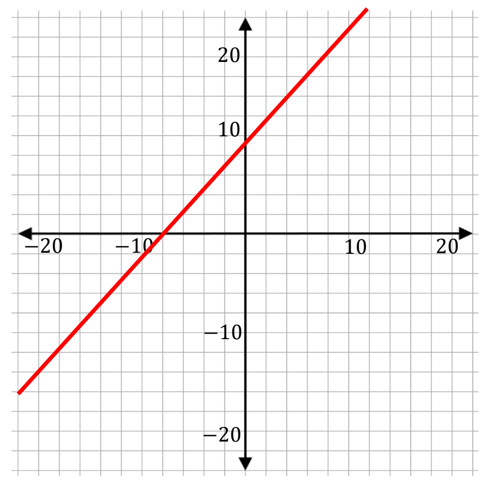

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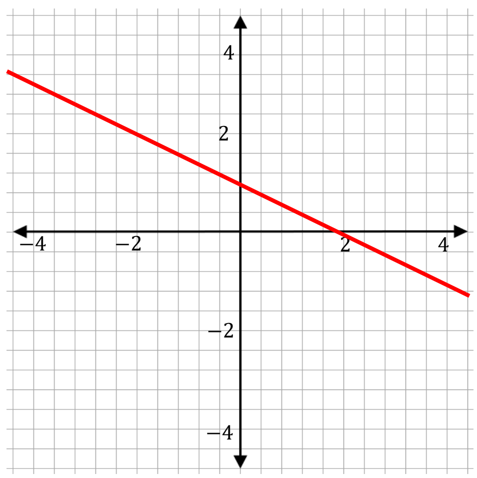

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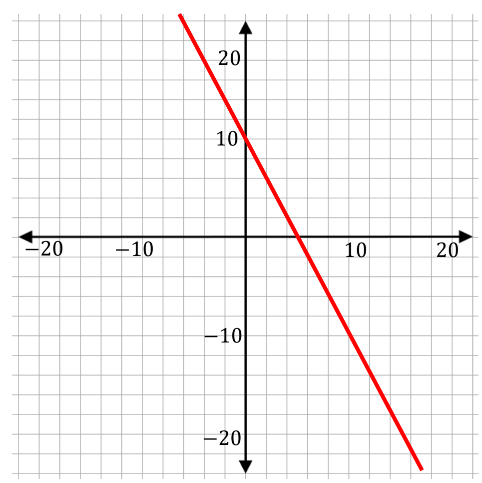

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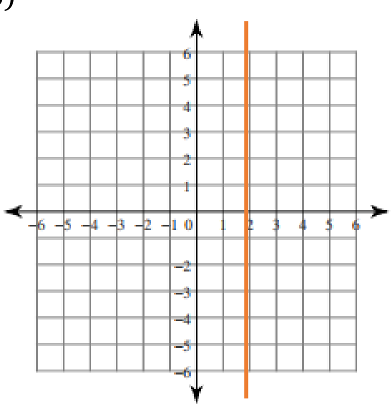

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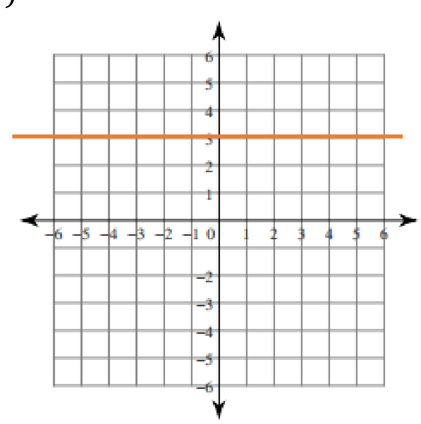


28) $y = -6x + 28$

29) $y = 2x - 13$

30) $y = x - 2$

31) $y = -\frac{1}{2}x - 4$

32) $y = 1$

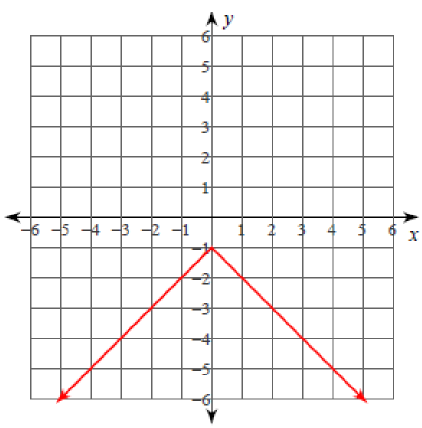
33) $y = x + 1$

34) The slope of function A is 1 and is lower that than the slope of function B (6).

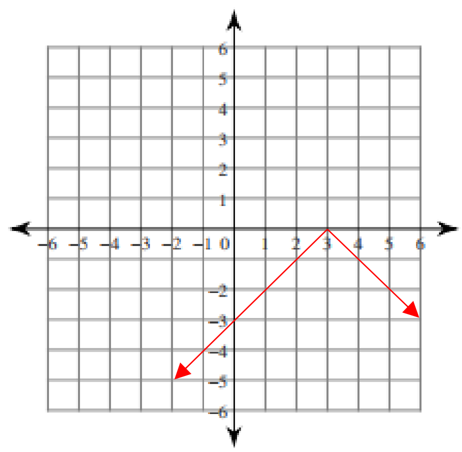
35) Two functions are parallel.

36) Two functions are intersecting.

37) $y = -|x| - 1$



38) $y = -|x - 3|$



39) $y = -x + 8$

40) $y = 4x + 9$

Answers and Explanations

1. This equation is already in the slope-intercept form, which is $y = mx + b$, where m is the slope. The slope here is the coefficient of x, so the slope is 1.

2. Similarly, this is in slope-intercept form. The number in front of x tells you how steep the line is and in which direction it goes. Here, the slope is 2, meaning for every step right on the x-axis, we go 2 steps up on the y-axis.

3. In the slope-intercept form, the slope is the number before x. This time it's -5, indicating a steep decline; for each step right, it goes 5 steps down.

4. Here we have two points, so we'll use the slope formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$. That gives us $m = \frac{0-6}{5-2} = -\frac{6}{3} = -2$.

5. Here, we find the slope with the formula: $= \frac{3-0}{-4-8} = \frac{3}{-12} = -\frac{1}{4}$.

6. Using the slope formula again, we calculate: $m = \frac{8-(-4)}{-4-(-2)} = \frac{12}{-2} = -6$.

7. Plugging these values into the slope-intercept form $y = mx + b$, where m is the slope and b is the y-intercept, we get $y = 4x + 16$.

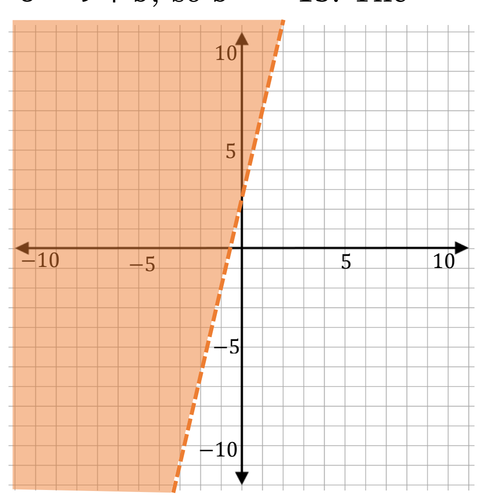
8. First, use the slope and point to find the y-intercept (b) with the formula $y = mx + b$. Plug in the values: $5 = 3(1) + b$, so $b = 2$. The equation is $y = 3x + 2$.

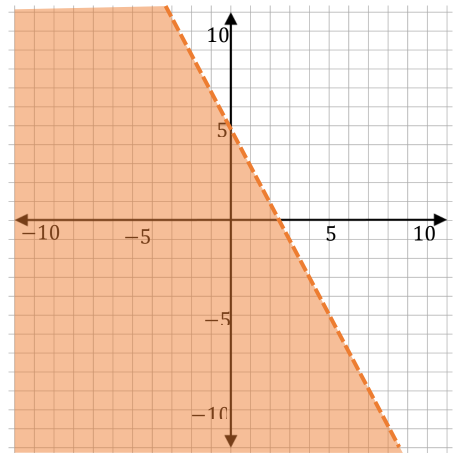
9. We apply the point to find b: $7 = -5(-2) + b$, simplifying to $7 = 10 + b$, which means $b = -3$. The line's equation is $y = -5x - 3$.

10. Insert the point into the formula: $2 = -4(-6) + b$ gives us $2 = 24 + b$, leading to $b = -22$. The line's equation is $y = -4x - 22$.

11. Use the point for b: $-6 = -3(-3) + b$, which is $-6 = 9 + b$, so $b = -15$. The equation of this line is $y = -3x - 15$.

12. Start with the related equation $y = 4x + 2$ to find the intercepts. The y-intercept is where $x = 0$, which is $(0, 2)$. The x-intercept is where $y = 0$; set $y = 0$ and solve for x to get $x = -0.5$ (or $-\frac{1}{2}$). Now, plot these two points on your graph. Draw a dashed line through them to represent that points on the line aren't included (since it's a 'greater than' inequality). Then, shade above the dashed line because y is greater than $4x + 2$ on that side.





13. Begin with $y = -2x + 5$. The y-intercept is $(0, 5)$, and for the x-intercept, set y to 0 and solve for x, giving $x = 2.5$ (or $\frac{5}{2}$). Plot these points and draw a dashed line through them because it's a 'less than' inequality, and we don't include the line itself. Then shade below the dashed line. This shading indicates where y is less than $-2x + 5$.

14. To determine the slope, pick two points on the line. Let's choose the points $(0, 4)$ and $(\frac{8}{3}, 0)$. Use the formula: $m = \frac{0-4}{\frac{8}{3}-0} = -4 \times \frac{3}{8} = -\frac{3}{2}$. The y-intercept is the point where the line crosses the y-axis. For the given line, the y-intercept is the y-coordinate of the point $(0, 4)$. So, $b = 4$. Using the slope and y-intercept, the equation of line is: $y = -\frac{3}{2}x + 4$.

15. To determine the slope, pick two points on the line. Let's choose the points $(0, 0)$ and $(-5, 3)$. Use the formula: $m = \frac{0-(-5)}{0-3} = -\frac{5}{3}$. The y-intercept is the point where the line crosses the y-axis. For the given line, Only at the origin y is equal to 0. So b is equal to 0. Using the slope and y-intercept, the equation of line is: $y = -\frac{5}{3}x$.

16. Using the formula $y - y_1 = m(x - x_1)$, where m is the slope and (x_1, y_1) is the given point. $y + 6 = -2(x - 6)$. To convert to slope-intercept form (which is of the form $y = mx + b$), we can simplify the point-slope form equation: $y = -2x + 6$.

17. Applying the same formula and using the point $(-7,7)$ and slope 4:

$y - 7 = 4(x + 7)$. To rewrite in slope-intercept form, simplify the point-slope form equation: $y = 4x + 35$.

18. To find the x-intercept, set $y = 0$ and solve for x.

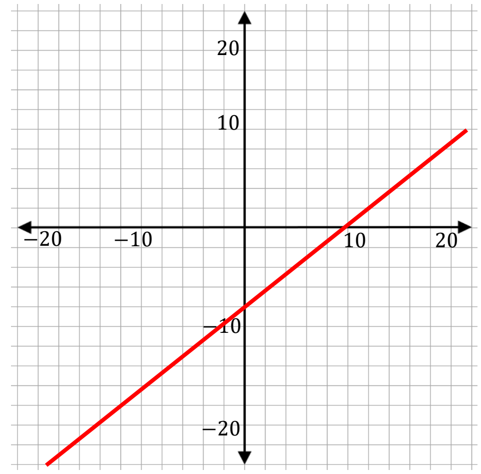
$21x = -18 \rightarrow x = \frac{-18}{21} \rightarrow x = -\frac{6}{7}$.

19. For the x-intercept, make $y = 0$ and rearrange to get x on its own. $20x = -10$. Divide both sides by 20: $x = -\frac{10}{20}$. $x = -0.5$.

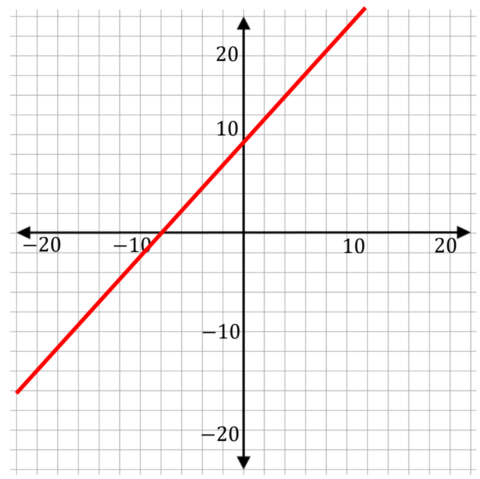
20. Finding the x-intercept requires letting y be zero. $8x = 16$. Divide each side by 8: $x = 2$.

21. For the x-intercept, substitute y with zero. $2x = -12$. Now, divide through by 2: $x = -6$.

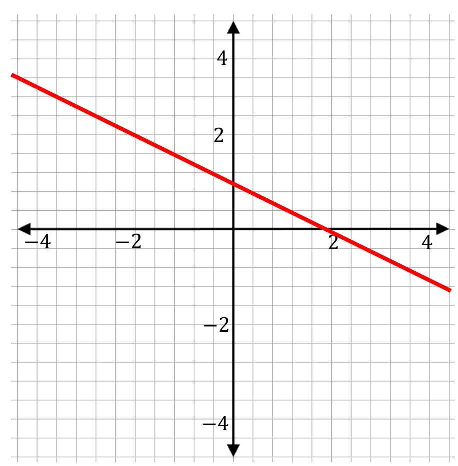
22. To graph the equation $4x - 5y = 40$, let's find two points by choosing values for x and solving for y, and then plot these points on a graph. First point: Let $x = 0$, then the equation becomes $-5y = 40$. Solving for y gives $y = -8$. So, our first point is $(0, -8)$. Second point: Let $x = 10$, then the equation becomes $40 - 5y = 40$. This simplifies to $-5y = 0$, hence $y = 0$. Our second point is $(10, 0)$. Plot these points $(0, -8)$ and $(10, 0)$ on a graph, then draw a straight line through them. This line represents all solutions to the equation $4x - 5y = 40$.



23. For the equation $9x - 8y = -72$, we'll also find two points: First point: Let $x = 0$, then the equation becomes $-8y = -72$. Solving for y gives $y = 9$. So, our first point is $(0, 9)$. Second point: Let $y = 0$, then the equation becomes $9x - 0 = -72$. This simplifies to $9x = -72$, hence $y = -8$. Our second point is $(-8, 0)$. Plot these points $(0, 9)$ and $(-8, 0)$ on a graph, and draw a line through them. This line represents all solutions to the equation $9x - 8y = -72$.



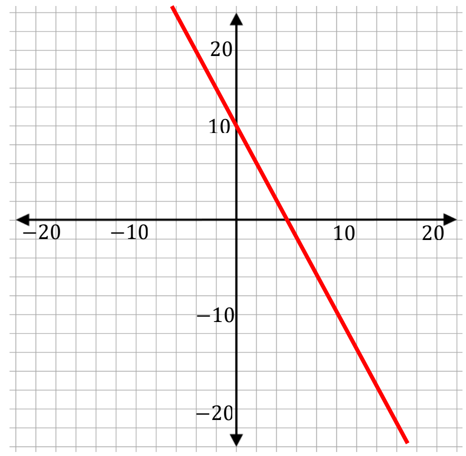
24. To graph the equations, we need to rewrite them in a more familiar format and then plot points or use key features like slope and y-intercept.
For $y + 3 = -\frac{1}{2}(x - 8)$. First, simplify the equation. Expand the right-hand side: $-\frac{1}{2}(x - 8) = -\frac{1}{2}x + 4$ (since $-\frac{1}{2}$ times -8 is 4). Next, bring the equation to the form $y = mx + b$ by isolating y: $y = -\frac{1}{2}x + 4 - 3$ (subtract 3 from both sides). So, $y = -\frac{1}{2}x + 1$. Plot the y-intercept (0,1) on the graph. The x-intercept is (2,0). Draw the line through these points.



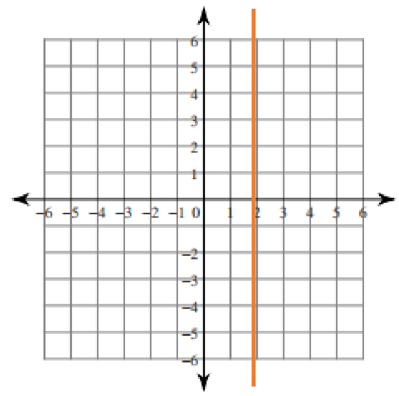
25. Begin by expanding the equation: $-2(x-1) = -2x + 2$. Rearrange it to get y on one side:

$y = -2x + 2 + 8$. Simplify it to $y = -2x + 10$. This is again in

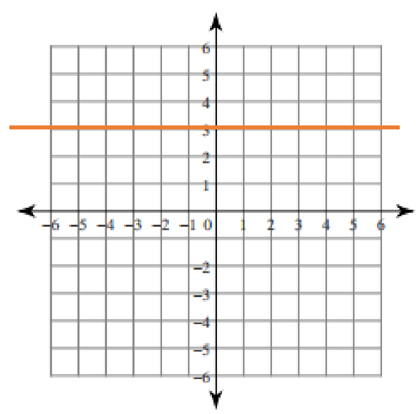
$y = mx + b$ form, with a slope (m) of -2 and a y-intercept (b) of 10. Plot the y-intercept (0,10). Then, plot the x-intercept (5,0). Draw the line through these points.



26. To sketch the vertical line that passes through (2,6), first understand that a vertical line means the x-coordinate is constant. For this line, x is always 2, no matter what y is. This is represented by the equation $x = 2$. To draw this, find the point on the x-axis where x is 2. From this point, draw a straight line straight up and down (perpendicular to the x-axis). This line doesn't depend on the y-coordinate given in the point (2,6); it will be the same vertical line whether it passes through (2,6) or $(2, -3)$ or any point where x is 2.



27. For the horizontal line passing through (5,3), you need to know that in a horizontal line, the y-coordinate is the same everywhere. Here, y is always 3, which is shown in the equation $y = 3$. Start by locating the point on the y-axis where y is 3. From there, draw a straight line that goes left and right (parallel to the x-axis). This line remains at height 3 above the x-axis all along, showing that for any x-coordinate, whether it's 5 or -2 or any other number, y is always 3. The fact that the given point is (5,3) simply confirms that this point lies on the line; the line itself extends infinitely in both directions.



28. Parallel lines have the same slope. The slope of $y = -6x + 5$ is -6. Using the point (4,4) and the slope -6 in the point-slope form: $y - 4 = -6(x - 4)$. Simplify to get the equation: $y = -6x + 28$.

29. Perpendicular lines have slopes that are negative reciprocals. The negative reciprocal of $-\frac{1}{2}$ is 2. Use the point (7,1) and the slope 2: $y - 1 = 2(x - 7)$. Simplify to get the equation: $y = 2x - 13$.

30. The slope of $y = x$ is 1. Parallel lines have the same slope. Using the point (2,0) and the slope 1: $y - 0 = 1(x - 2)$. Simplify this equation: $y = x - 2$.

31. The negative reciprocal of 2 (the slope of the given line) is $-\frac{1}{2}$. Use the point $(0, -4)$ and the slope $-\frac{1}{2}$: $y + 4 = -\frac{1}{2}\ (x - 0)$. Simplify to get the equation:

$y = -\frac{1}{2}x - 4$.

32. The slope of $y = 2$ (a horizontal line) is 0. Parallel lines have the same slope. Since the slope is 0, the line is horizontal, and the equation is simply $y = 1$.

33. The negative reciprocal of -1 (the slope of the given line) is 1. Use the point (3,4) and the slope 1: $y - 4 = 1(x - 3)$. Simplify this equation. $y = x + 1$.

34. For function A, we can determine the slope by finding two points on the line where it crosses the grid lines exactly and then calculate the rise over run. Two points are $(-5, 0)$ and $(0, 5)$. So, slope is: $m = \frac{5-0}{0-(-5)} = 1$. For function B, the equation is given as $y = 6x - 3$. In this linear equation format, $y = mx + b$, the coefficient of x is the slope. Therefore, the slope of function B is 6. Comparing the slopes of function A and function B shows that function B has a steeper slope than function A. The slope of function A is 1, while the slope of function B is 6

35. Let's calculate the slope of function A by choosing two points where the line crosses the grid exactly, then use the rise over run formula to find the slope. Two points are (2.4,0) and (0,6). So, slope is: $m = \frac{6-0}{0-2.4} = -2.5$. From the equation of function B, $y = -2.5x - 1$, we know that the slope is -2.5. The slopes are the same, so the two functions are parallel.

36. For Function A, the slope can be calculated by finding two points on the line and using the formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$

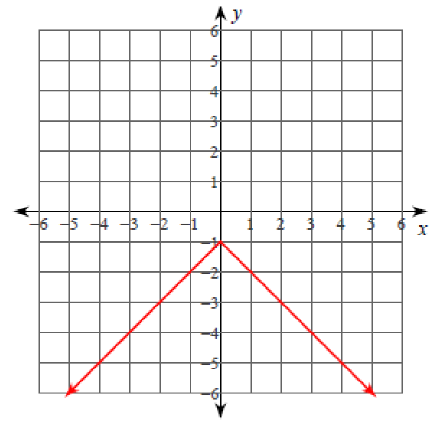
Let's choose the points $(-2, 3)$ and $(0, 2)$. Calculate the slope m using the formula:

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-3}{0-(-2)} = -\frac{1}{2}$.

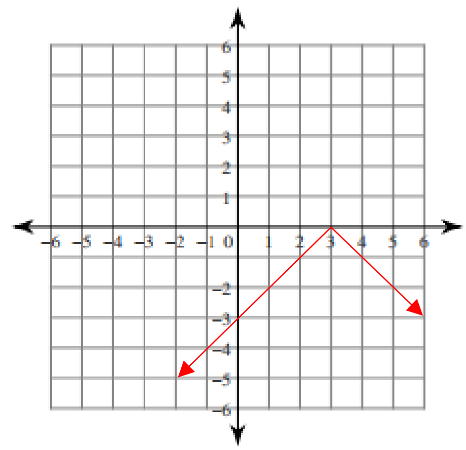
The slope (m) of a linear function in the form $y = mx + b$ is the coefficient of x, which represents the rate of change of y with respect to x. For Function B, the slope is 2, since that's the coefficient of x in the equation.

Considering that the product of the slope of function A and the slope of function B is equal to 1, then the two functions are perpendicular to each other.

37. Start by understanding the absolute value function $|x|$. This function takes the input x and gives its positive magnitude. Now, let's consider the negative sign in front of the absolute value: $-|x|$. This means that whatever positive value the absolute value function returns, we make it negative. So, for $x > 0$, $-|x|$ will be negative, and for $x < 0, -|x|$ will be positive. The -1 at the end of the equation means that we shift the entire graph downward by 1 unit. To graph this equation, start by drawing the graph of $y = |x|$, which is a V-shaped graph centered at the origin (0,0). Then, for all points above the x-axis, make the y-values negative (reflect them below the x-axis) and shift the entire graph down by 1 unit. Your final graph should resemble an upside-down V shape with its vertex at $(0, -1)$.



38. Start by understanding the absolute value function $|x - 3|$. This function takes the input x and subtracts 3 before taking its positive magnitude. So, if $x = 5$, then $|x - 3| = |5 - 3| = |2| = 2$. Now, let's consider the negative sign in front of the absolute value: $-|x - 3|$. Just like in the previous equation, this means that whatever positive value the absolute value function returns, we make it negative. To graph this equation, start with the graph of $y = |x - 3|$. This is also a V-shaped graph, but it will be centered at $x = 3$ because of the "$x - 3$" inside the absolute value. Now, for all points to the right of $x = 3$, make the y-values negative (reflect them below the x-axis). Your final graph should look like an upside-down V shape, centered at $x = 3$, with its vertex at $(3, 0)$, and extending to the left side of the graph.



39. We know that the feeder starts with 8 fluid ounces of nectar, and each day, 1 fluid ounce is released. So, as the number of days, x, increases, the amount of nectar left, y, decreases. We can represent this relationship with the equation: $y = -x + 8$.

40. The total cost, y, depends on two factors: the entrance fee, which is a fixed cost of \$9, and the skate rental cost, which is \$4 per hour. As the length of the rental in hours, x, increases, the rental cost increases proportionally. We can represent this relationship with the equation: $y = 4x + 9$.

CHAPTER

4 Inequalities and System of Equations

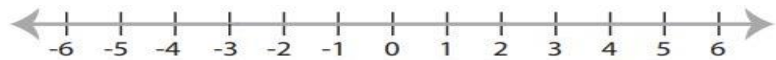
Math topics in this chapter:

- ☑ One–Step Inequalities
- ☑ Multi–Step Inequalities
- ☑ Compound Inequalities
- ☑ Write a Linear Inequality from a Graph
- ☑ Graph Solutions to Linear Inequalities
- ☑ Solve Advanced Linear Inequalities in Two-Variables
- ☑ Graph Solutions to Advanced Linear Inequalities
- ☑ Absolute Value Inequalities
- ☑ Systems of Equations
- ☑ Find the Number of Solutions to a Linear Equation
- ☑ Write a System of Equations Given a Graph
- ☑ Systems of Equations Word Problems
- ☑ Solve Linear Equations Word Problems
- ☑ Systems of Linear Inequalities
- ☑ Write Two-variable Inequalities Word Problems

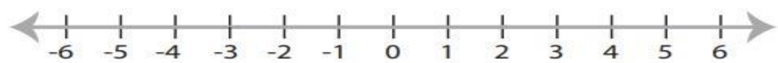
Practices

Solve each inequality and graph it.

1) $x - 2 \geq -2$



2) $2x - 3 < 9$



Solve each inequality.

3) $x + 13 > 4$

4) $x + 6 > 5$

5) $-12 + 2x \leq 26$

6) $-2 + 8x \leq 14$

7) $6 + 4x \leq 18$

8) $4(x + 3) \geq -12$

9) $2(6 + x) \geq -12$

10) $3(x - 5) < -6$

11) $10 + 5x < -15$

12) $6(6 + x) \geq -18$

13) $2(x - 5) \geq -14$

14) $6(x + 4) < -12$

15) $3(x - 8) \geq -48$

16) $-(6 - 4x) > -30$

17) $2(2 + 2x) > -60$

18) $-3(4 + 2x) > -24$

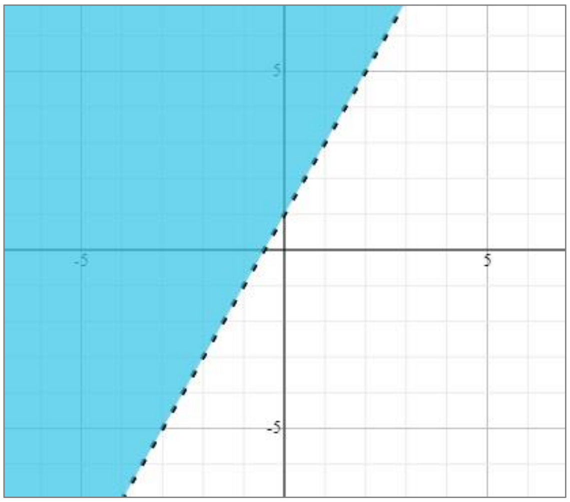
Solve each inequality.

19) $5x \leq 45$ and $x - 11 > -21$

20) $-7 < x - 9 < 8$

Write the slope-intercept from equation of the following graph.

21)



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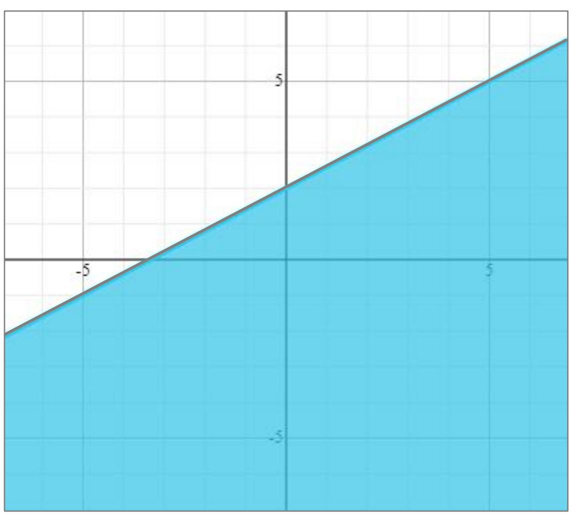

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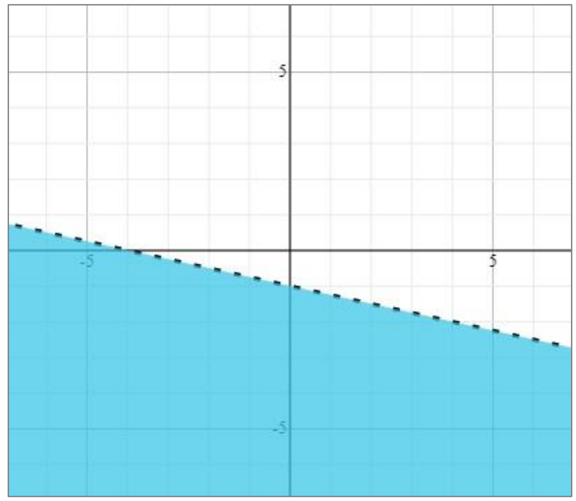

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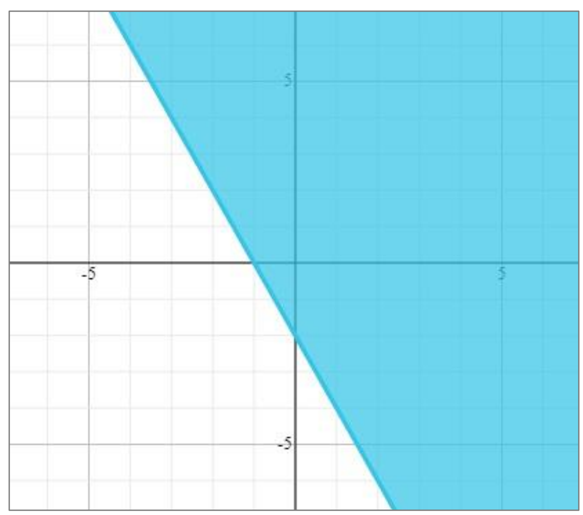


Solve the following inequality and graph the solution.

25) $10 + 6p \le -2$

26) $-r + 8 \le 4$

27) $-2f + 10 \ge 6$

28) $1 + 3p > 7$

Solve each inequality.

29) $8x - 3 \ge 4y + 2$

30) $4x - 3 \ge 5y + x$

31) $y \le \frac{3}{2}x + 4$

32) $5x - 2y \le 10$

Graph the solution of each inequality.

33) $7x + 3 \ge 1 - 2x$

34) $\frac{x+4}{-4} > 8x + 2$

Solve each inequality.

35) $|x| - 4 < 17$

36) $6 + |x - 8| > 15$

37) $\left|\frac{x}{2} + 3\right| > 6$

38) $\left|\frac{x+5}{4}\right| < 7$

Solve each system of equations.

39) $\begin{cases} -2x + 2y = -4 \\ 4x - 9y = 28 \end{cases}$ $x =$ $y =$

40) $\begin{cases} x + 8y = -5 \\ 2x + 6y = 0 \end{cases}$ $x =$ $y =$

41) $\begin{cases} 4x - 3y = -2 \\ x - y = 3 \end{cases}$ $x =$ $y =$

42) $\begin{cases} 2x + 9y = 17 \\ -3x + 8y = 39 \end{cases}$ $x =$ $y =$

How many solutions does the following equation have?

43) $4n = 8 + 5n$

44) $5 - 9f = -9f$

45) $0 = 3z - 3z$

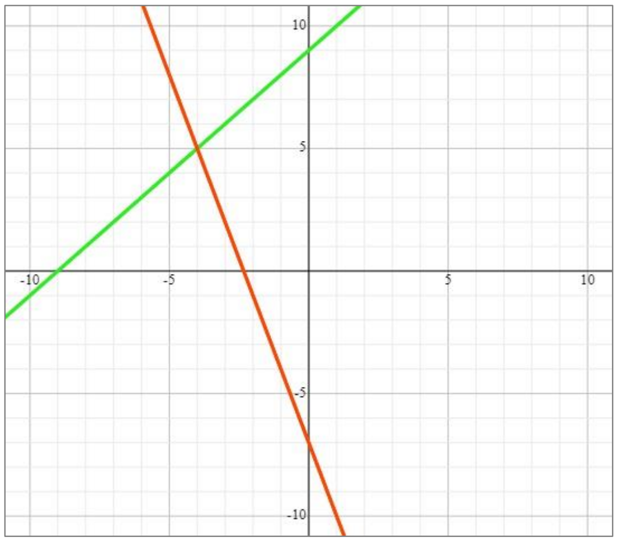
46) $-9x + 2 = -9x$

47) $20 + 12y = 11y$

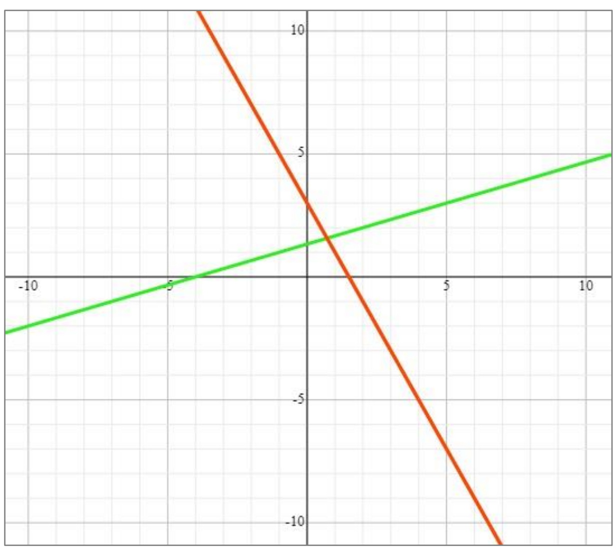
48) $10h - 2 = -4h$

Write a system of equations for the following graph.

49)



50)



Solve each word problem.

51) The equations of two lines are $3x - y = 7$ and $2x + 3y = 1$. What is the value of x in the solution for this system of equations?

52) The perimeter of a rectangle is 100 feet. The rectangle's length is 10 feet less than 5 times its width. What are the length and width of the rectangle?

Solve each system of inequalities and graph them.

53) $\begin{cases} x + 2y \le 3 \\ y - x \ge 0 \\ y \ge -2 \end{cases}$

54) $\begin{cases} x < 3 \\ x + y > -2 \\ y - 1 \le x \end{cases}$

Solve each word problem.

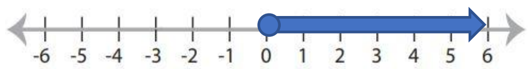
55) James used his first 2 tokens in Glimmer Arcade to play a Roll-and-Score game. Then he played his favorite game, Balloon Bouncer, over and over until he ran out of tickets. Balloon Bouncer costs 4 tokens per game and James started the game with a bucket of 38 tokens. Write an equation James can use to find how many games of Balloon Bouncer, g, he played.

56) Sara buys juice and soda for the party and wants to spend no more than \$46. The price of each bottle of soda is 3 dollars and each bottle of fruit juice is 1 dollar. Write the inequality in a standard form that describes this situation. Use the given numbers and variables below.

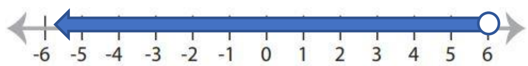
x = the number of bottles of soda
y = the number of bottles of juice

Answers

1) $x \geq 0$



2) $x < 6$



3) $x > -9$

4) $x > -1$

5) $x \leq 19$

6) $x \leq 2$

7) $x \leq 3$

8) $x \geq -6$

9) $x \geq -12$

10) $x < 3$

11) $x < -5$

12) $x \geq -9$

13) $x \geq -2$

14) $x < -6$

15) $x \geq -8$

16) $x > -6$

17) $x > -16$

18) $x < 2$

19) $-10 < x \leq 9$

20) $2 < x < 17$

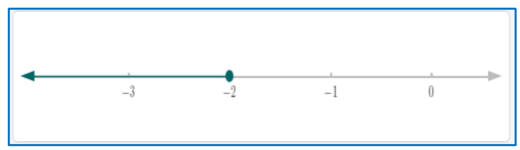
21) $y > 2x + 1$

22) $y \leq \frac{3}{5}x + 2$

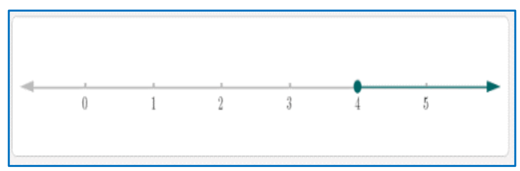
23) $y < -\frac{1}{4}x - 1$

24) $y \geq -2x - 2$

25) $p \leq -2$

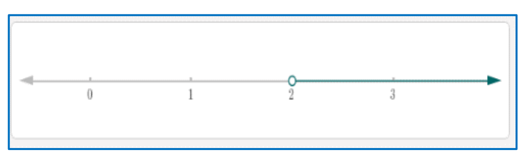


26) $r \geq 4$



27) $f \leq 2$

28) $p > 2$



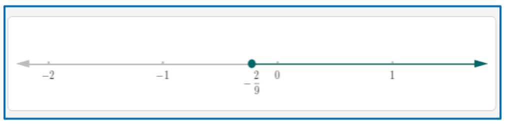
29) $\{(x,y)|y \in R, x \geq \frac{4y+5}{8}\}$

30) $\{(x,y)|y \in R, x \geq \frac{5y+3}{3}\}$

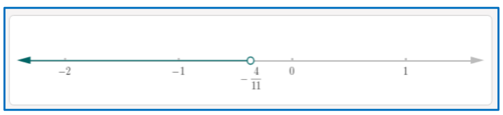
31) $\{(x,y)|y \in R, x \geq \frac{2y-8}{3}\}$

32) $\{(x,y)|y \in R, x \leq \frac{2y+10}{5}\}$

33) $x \geq -\frac{2}{9}$



34) $x < -\frac{4}{11}$



35) $-21 < x < 21$

36) $x > 17$ or $x < -1$

37) $x > 6$ or $x < -18$

38) $-33 < x < 23$

39) $x = -2,\ y = -4$

40) $x = 3,\ y = -1$

41) $x = -11,\ y = -14$

42) $x = -5,\ y = 3$

43) One solution

44) No solution

45) Infinitely solutions

46) No solution

47) One solution

48) One solution

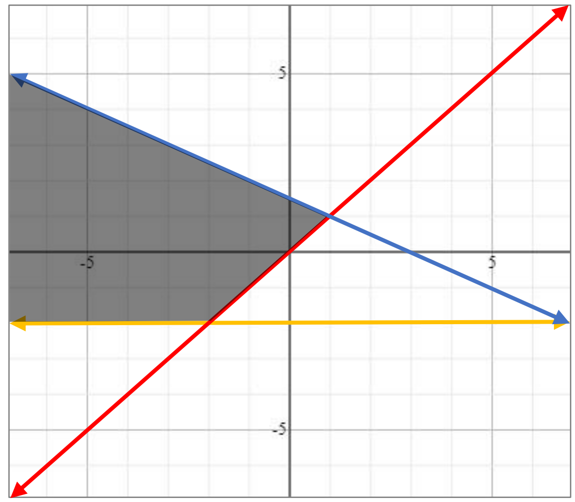
49) $\begin{cases} y = -3x - 7 \\ y = x + 9 \end{cases}$

50) $\begin{cases} 2x + y = 3 \\ -x + 3y = 4 \end{cases}$

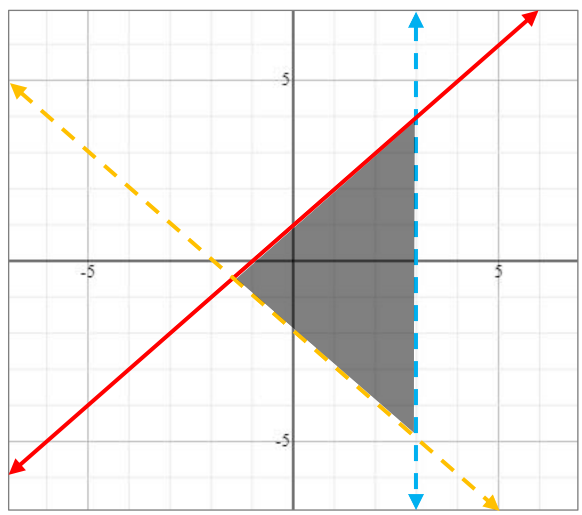
51) $x = 2$

52) 10, 40

53) $\begin{cases} y \le -\frac{1}{2}x + \frac{3}{2} \\ y \ge x \\ y \ge -2 \end{cases}$



54) $\begin{cases} x < 3 \\ y > -x - 2 \\ y \le x + 1 \end{cases}$

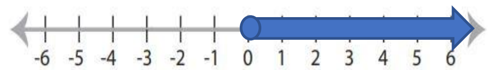


55) $4g + 2 = 38$

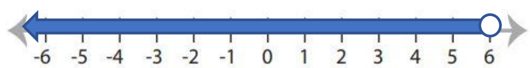
56) $3x + y \le 46$

Answers and Explanations

1. To solve this inequality, we want to isolate x on one side. Here are the steps: Add 2 to both sides of the inequality: $x - 2 + 2 \geq -2 + 2$. This simplifies to $x \geq 0$, because $-2 + 2 = 0$. To graph this inequality, you draw a number line, make a solid circle on 0 to show that 0 is included in the solution (because it's "greater than or equal to"), and shade all the numbers to the right of 0 to indicate that all those numbers are part of the solution.



2. Solving this inequality is about finding the range of x values that make the inequality true: first, add 3 to both sides to cancel out the -3: $2x - 3 + 3 < 9 + 3$. This gives us $2x < 12$. Now, divide both sides by 2 to find the value of x:$\frac{2x}{2} < \frac{12}{2}$. We end up with $x < 6$. For graphing this inequality, you'd draw a number line and place an open circle on 6 (since 6 is not included in the solution) and shade all the numbers to the left of 6, indicating that the solution includes all numbers less than 6.



3. To find the value of x, we reduce the inequality. Subtract 13 from both sides: $x > 4 - 13$, resulting in $x > -9$.

4. Here, we again simplify by moving 6 to the other side. So, $x > 5 - 6$, giving $x > -1$.

5. Bring -12 to the other side, then divide the result by 2: $2x \leq 38$, so $x \leq 19$.

6. Shift -2 to the right side of the inequality and then divide by 8: $x \leq \frac{16}{8}$, simplifying to $x \leq 2$.

7. Move 6 across and divide by 4 to isolate $x \leq \frac{12}{4}$, which simplifies to $x \leq 3$.

8. Start by expanding the left side: $4x + 12 \geq -12$. Next, subtract 12 from both sides to isolate the term with $4x \geq -24$. Finally, divide everything by 4 to solve for $x \geq -6$.

9. First, expand the left side: $12 + 2x \geq -12$. Then, move 12 to the right side by subtracting it: $2x \geq -24$. Divide by 2 to find $x \geq -12$.

10. Expand the multiplication: $3x - 15 < -6$. Add 15 to both sides to bring x terms on one side: $3x < 9$. Divide by 3 to isolate $x < 3$.

11. First, move 10 to the other side by subtracting it: $5x < -25$. Then divide everything by 5 to get $x < -5$.

12. Expand the left side: $36 + 6x \geq -18$. Subtract 36 from both sides: $6x \geq -54$. Divide by 6 to solve for $x \geq -9$.

13. Expand the multiplication: $2x - 10 \geq -14$. Add 10 to both sides: $2x \geq -4$. Divide by 2 to find x: $x \geq -2$.

14. Expand the left side: $6x + 24 < -12$. Subtract 24 from both sides: $6x < -36$. Divide by 6 to isolate $x < -6$.

15. First, expand the left side: $3x - 24 \geq -48$. Add 24 to both sides: $3x \geq -24$. Finally, divide by 3: $x \geq -8$.

16. Simplify the left side: $-6 + 4x > -30$. Add 6 to both sides: $4x > -24$. Divide by 4 to solve for x: $x > -6$.

17. First, expand the multiplication: $4 + 4x > -60$. Subtract 4 from both sides: $4x > -64$. Divide everything by 4 to find $x > -16$.

18. Expand the multiplication: $-12 - 6x > -24$. Add 12 to both sides: $-6x > -12$. Divide by -6 and reverse the inequality sign: $x < 2$.

19. For the first part, $5x \leq 45$, divide both sides by 5 to isolate x: $x \leq 9$. For the second part, $x - 11 > -21$, add 11 to both sides to solve for x: $x > -10$. Combining these two, the solution is x values that satisfy both conditions, which is $-10 < x \leq 9$.

20. This inequality can be split into two parts: $x - 9 > -7$ and $x - 9 < 8$. For the first part, $x - 9 > -7$, add 9 to both sides: $x > 2$. For the second part, $x - 9 < 8$, again add 9 to both sides: $x < 17$. The solution is the range of x that satisfies both parts: $2 < x < 17$.

21. The inequality is likely in the form of $y > mx + b$ or $y \geq mx + b$, where m is the slope of the line and b is the y-intercept. Given two points from the graph, $(0, 1)$ and $(-0.5, 0)$, we can first determine the slope of the line (m) using the slope formula: $m = \frac{0-1}{-0.5-0} = 2$. The slope of the line (m) is 2. Therefore, the equation of the line using the slope-intercept form $y = mx + b$ with the given y-intercept at point $(0, 1)$ is: $y = 2x + 1$. The inequality will be $y > 2x + 1$.

22. To create the inequality from the graph, we first need to determine the slope of the line, which tells us how steep the line is. Given two points from the graph, $(0,2)$ and $(-\frac{10}{3}, 0)$, we can first determine the slope of the line (m) using the slope formula: $m = \frac{0-2}{-\frac{10}{3}-0} = \frac{3}{5}$. Once we have the slope, we use it along with the y-intercept to write the line's equation. The y-intercept is where the line crosses the y-axis, which in this case is at $y = 2$ (the point $(0, 2)$). Putting the slope and

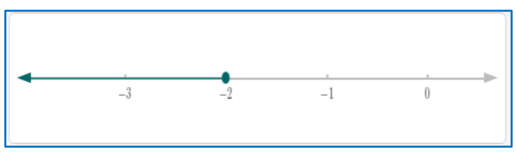
y-intercept together, we get the equation of the line: $y = \frac{3}{5}x + 2$. But for the inequality representing all the points in the shaded area below the line, it will be: $y \leq \frac{3}{5}x + 2$.

23. To find the inequality, we first need to determine the slope of the line. Given the points $(0, -1)$ and $(-4, 0)$, the slope is: $m = \frac{0-(-1)}{-4-0} = -\frac{1}{4}$. The y-intercept is where the line crosses the y-axis. For the point $(0, -1)$, the x-value is 0, so this point is the y-intercept. Therefore, the equation of the line is $y = -\frac{1}{4}x - 1$. The shading below the dashed line tells us that we are looking for values of y that are less than the values on the line, but not equal to them, because the line is not included in the solution set. Thus, the inequality representing the shaded region below the line is: $y < -\frac{1}{4}x - 1$.

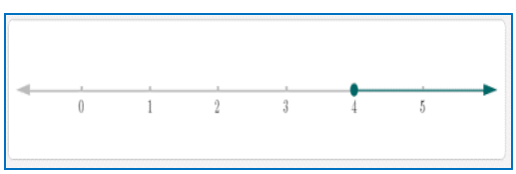
24. First, we find the slope of the line. We have two points: $(0, -2)$ and $(-1, 0)$. The slope is $m = \frac{0-(-2)}{-1-0} = -2$. Next, we use the y-intercept to write the line's equation. The y-intercept is where the line hits the y-axis. Our line crosses at $(0, -2)$, so the y-intercept is -2. In this case, the equation of the line is:

$y = -2x - 2$. The shaded area is above this line, the inequality is: $y \geq -2x - 2$.

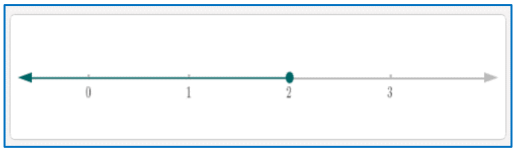
25. To find the value of p, we need to isolate it on one side of the inequality. We start by subtracting 10 from both sides to get $6p \leq -12$. Then, we divide both sides by 6 to find p. This will give us $p \leq -2$. The graph of this solution would show a number line with everything to the left of -2 shaded and a closed circle at -2 because p can also be equal to -2.



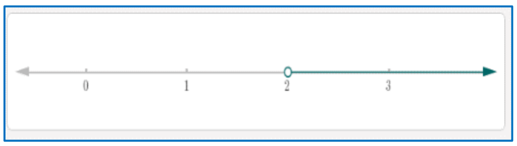
26. Here, we want to find out what r can be. We'll subtract 8 from both sides to get $-r \leq -4$. Multiplying or dividing by a negative number flips the inequality sign, so when we multiply both sides by -1 to get r, the inequality sign flips, giving us $r \geq 4$. On the number line, we'd shade everything to the right of 4 and put a closed circle on 4 since r can be 4 as well.



27. To solve for f, we'll subtract 10 from both sides first, getting $-2f \geq -4$. Next, we divide by -2, which reverses the inequality direction, so we end up with $f \leq 2$. The graph for this would be similar to the first one, with a closed circle at 2 and shading to the left.



28. For this inequality, subtract 1 from both sides to isolate the term with p, giving us $3p > 6$. Dividing by 3 then gives us $p > 2$. Since this is a strict inequality (not including the number 2), we'd draw an open circle at 2 on the number line and shade everything to the right of 2.



29. To solve for x, we need to get x by itself on one side. First, add 3 to both sides to eliminate the -3: $8x \geq 4y + 5$. Next, divide everything by 8 to find x: $x \geq \frac{4y+5}{8}$. The final answer is: $\left\{(x, y) \middle| y \in R, x \geq \frac{4y+5}{8}\right\}$.

30. Here, we first want to get all the x-terms on one side, so subtract x from both sides: $3x - 3 \geq 5y$. Now, add 3 to both sides: $3x \geq 5y + 3$. Finally, divide by 3 to solve for x: $x \geq \frac{5y+3}{3}$. The final answer is: $\left\{(x, y) \middle| y \in R, x \geq \frac{5y+3}{3}\right\}$.

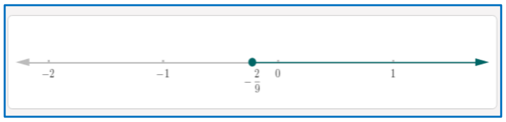
31. To solve for x, we need to reverse the operations affecting x. First, subtract 4 from both sides to get rid of the 4 next to x: $y - 4 \leq \frac{3}{2}x$. Now, multiply everything by $\frac{2}{3}$ to isolate x: $x \geq \frac{2y-8}{3}$. The final answer is: $\left\{(x, y) \middle| y \in R, x \geq \frac{2y-8}{3}\right\}$.

32. Begin by adding $2y$ to both sides to move the y-term to the other side:

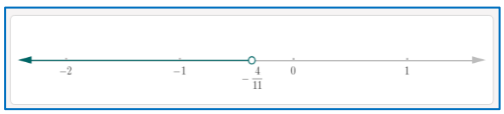
$5x \leq 10 + 2y$. Then, divide by 5 to solve for x: $x \leq \frac{2y+10}{5}$. The final answer is: $\left\{(x, y) \middle| y \in R, x \leq \frac{2y+10}{5}\right\}$.

33. First, we'll collect all the x-terms on one side by adding $2x$ to both sides:

$9x + 3 \geq 1$. Next, we'll move the constant term (3) to the other side by subtracting 3 from both sides: $9x \geq -2$. Then, divide by 9 to isolate x: $x \geq -\frac{2}{9}$. To graph this, we'll draw a number line, place a closed dot on $-\frac{2}{9}$, and shade to the right, indicating that x can be any number greater than or equal to $-\frac{2}{9}$.



34. We want to isolate x, so multiply both sides by -4, which will reverse the inequality because we're multiplying by a negative: $x+4<-32x-8$. Now, get all x-terms to one side by adding $32x$ to both sides: $33x+4<-8$. Subtract 4 from both sides to have only x-terms on one side: $33x<-12$. Finally, divide by 33 to solve for x: $x<-\frac{12}{33}$, which simplifies to $x<-\frac{4}{11}$. For the graph, draw a number line, place an open dot on $-\frac{4}{11}$, and shade to the left, showing that x can be any number less than $-\frac{4}{11}$, but not equal to it.



35. First, add 4 to both sides to isolate the absolute value: $|x|<21$. The inequality now says the distance of x from 0 is less than 21. This means x can be less than 21 and greater than -21. So, we have two inequalities: $x<21$ and $x>-21$, which we can write together as $-21<x<21$.

36. Subtract 6 from both sides first: $|x-8|>9$. This means the distance from x to 8 is more than 9. This leads to two scenarios: $x-8>9$, which simplifies to $x>17$, or $x-8<-9$, which simplifies to $x<-1$.

37. The absolute value being greater than 6 means the expression inside is either more than 6 units away from zero on the positive side or less than -6 on the negative side. Splitting into two cases, we get $\frac{x}{2}+3>6$, which simplifies to $x>6$, or $\frac{x}{2}+3<-6$, which simplifies to $x<-18$.

38. This tells us that the quantity $\frac{(x+5)}{4}$ is less than 7 units away from zero in both the positive and negative directions. We consider two inequalities:

$\frac{(x+5)}{4}<7$, which simplifies to $x<23$, and $\frac{(x+5)}{4}>-7$, which simplifies to $x>-33$. So, the solution to this inequality is $-33<x<23$.

39. We can multiply the first equation by 2 to align the coefficients of x for elimination. This gives us $-4x+4y=-8$ and $4x-9y=28$. Adding these equations eliminates x, resulting in $-5y=20$, and solving for y gives $y=-4$.

Substituting $y = -4$ into one of the original equations and solving for x gives $x = -2$. So, the solution is $x = -2, y = -4$.

40. We first multiply the first equation by -2, which gives $-2x - 16y = 10$, and then add it to the second equation $2x + 6y = 0$. This eliminates x, resulting in $-10y = 10$, and solving for y gives $y = -1$. Substituting $y = -1$ back into one of the original equations, we find $x = 3$. Therefore, the solution is $x = 3, y = -1$.

41. We first multiply the second equation by 3, getting $3x - 3y = 9$. Next, we subtract the second equation from the first one: $(4x - 3y) - (3x - 3y) = -2 - 9$. This simplifies to $x = -11$. Finally, we substitute $x = -11$ into one of the original equations, like $x - y = 3$. Solving for y gives us $y = -14$.

42. We multiply the first equation by 3 and the second by 2 to align the coefficients of x for elimination. This results in $6x + 27y = 51$ and $-6x + 16y = 78$. Adding these equations eliminates x, leading to $43y = 129$, and solving for y gives $y = 3$. Substituting $y = 3$ into one of the original equations, we find $x = -5$. Thus, the solution is $x = -5, y = 3$.

43. Subtract $4n$ from both sides to get $0 = 8 + n$. This simplifies to $-8 = n$, indicating a unique solution, $n = -8$.

44. Add $9f$ to both sides, resulting in $5 = 0$. This is a contradiction, as there's no value of f that can satisfy this equation. Hence, there are no solutions.

45. Simplifying the right side, $3z - 3z$ becomes 0, leading to $0 = 0$. This is a true statement for all values of z, meaning there are infinitely many solutions.

46. Subtract $-9x$ from both sides, yielding $2 = 0$, which is a contradiction. Thus, this equation has no solutions.

47. Subtract $12y$ from both sides, resulting in $20 = -y$. This simplifies to $y = -20$, indicating a unique solution, $y = -20$.

48. Add 2 to both sides, getting $10h = -2$. Dividing both sides by 10 gives $h = -0.2$, a unique solution, $h = -0.2$.

49. We need to find two points for each line to determine its slope and equation. For the green line: Point 1: $(-9,0)$ and point 2: $(0,9)$. For the red line: Point 1: $(-4,5)$ and point 2: $(0,-7)$. The slope of a line is calculated with the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$. For the green line: $m = \frac{9-0}{0-(-9)} = 1$. For the red line: $m = \frac{-7-5}{0-(-4)} = -3$. The equation of a line can be found using the point-slope formula $y - y_1 = m(x - x_1)$, where x_1 and y_1 is a point on the line. For the green line using point $(0,9)$:

$y - 9 = 1(x - 0) \rightarrow y = x + 9$. For the red line using point $(0,-7)$:

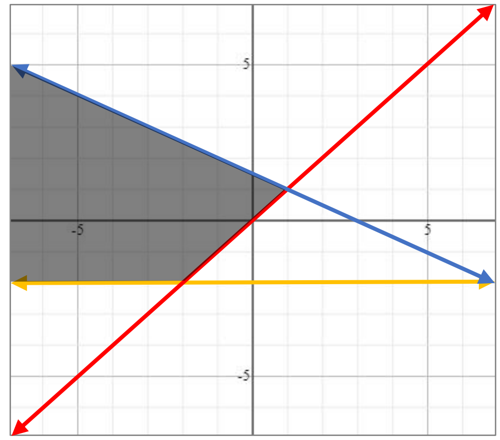
$y + 7 = -3(x - 0) \rightarrow\ y = 3x - 7$. Thus, the system of equations representing the lines in the graph is: $\begin{cases} y = -3x - 7 \\ y = x + 9 \end{cases}$.

50. To write a system of equations for the lines shown in the graph, we need to determine the slope-intercept form of each line, which is $y = mx + b$, where m is the slope and b is the y-intercept of the line. For the red line, we use the two points given: $(0, 3)$ and $(1.5, 0)$. The slope is: $m = \frac{0-3}{1.5-0} = -2$. The y-intercept (b) is the y-value where the line crosses the y-axis, which is 3 for the red line. So, the equation for the red line is $y = -2x + 3$. For the green line, the points are $(-4, 0)$ and $(0, \frac{4}{3})$. Following the same method, the slope m is the change in y divided by the change in x. Here, the change in y is $\frac{4}{3} - 0 = \frac{4}{3}$, and the change in x is $0 - (-4) = 4$. So, the slope $m = \frac{\frac{4}{3}}{4} = \frac{1}{3}$. Since one of the points is on the y-axis $(0, \frac{4}{3})$, this is also the y-intercept (b) for the green line. Thus, the equation for the green line is $y = \frac{1}{3}x + \frac{4}{3}$. Putting it all together, the system of equations for the two lines is: $\begin{cases} 2x + y = 3 \\ -x + 3y = 4 \end{cases}$.

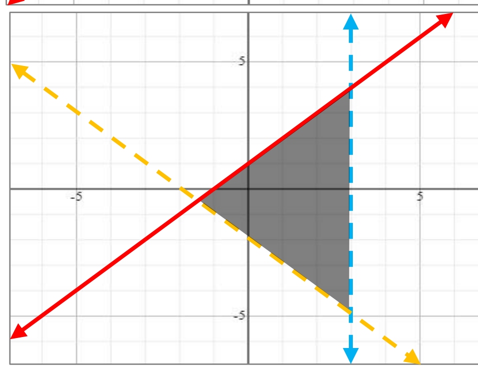
51. We manipulate the equations to align either the x or y coefficients. In this case, we can multiply the first equation by 3 and the second equation by 1, which aligns the y coefficients. This manipulation gives us $9x - 3y = 21$ and $2x + 3y = 1$. Adding these equations together, $3y$ is eliminated, leaving us with $11x = 22$. Solving for x by dividing both sides of the equation by 11, we find that $x = 2$.

52. The perimeter of a rectangle is the total distance around it, which is twice its length plus twice its width $(2L + 2W)$. We're also told that the length (L) is 10 feet less than 5 times the width (W). So, $L = 5W - 10$. Now, we use the perimeter formula: $2L + 2W = 100$. Replace L with $5W - 10$ in this formula, getting $2(5W - 10) + 2W = 100$. Simplifying this equation, we get $10W - 20 + 2W = 100$. Combine like terms to get $12W - 20 = 100$. Add 20 to both sides to isolate the variable term, resulting in $12W = 120$. Finally, divide both sides by 12 to find W, which is 10 feet. Knowing W, we can find L using $L = 5W - 10$, which gives $L = 40$ feet. So, the width is 10 feet, and the length is 40 feet.

53. We rearrange each inequality to solve for y. For $x + 2y \leq 3$, the rearranged form is $y \leq -\frac{1}{2}x + \frac{3}{2}$. For $y - x \geq 0$, it becomes $y \geq x$. The third inequality, $y \geq -2$, remains the same. We then plot these inequalities on a graph. The shaded region where all these conditions overlap is the solution.



54. Again, we rearrange each inequality for y. For $x + y > -2$, it becomes $y > -x - 2$. For $y - 1 \leq x$, the rearranged form is $y \leq x + 1$. We plot these on the graph along with $x < 3$. The solution is where the shaded regions of these inequalities intersect.



55. To write an equation for how many games of Balloon Bouncer James played at Glimmer Arcade, we need to account for all his tokens. James started with 38 tokens. He used 2 tokens for the Roll-and-Score game and the rest on Balloon Bouncer, which costs 4 tokens per game. If g represents the number of Balloon Bouncer games he played, the total tokens used for these games would be $4g$ tokens. The total number of tokens used for both games is $4g$ for Balloon Bouncer plus 2 for Roll-and-Score. Therefore, the equation that represents this situation is $4g + 2 = 38$.

56. Sara's budget for buying soda and juice is \$46. Each bottle of soda costs \$3, and each bottle of juice costs \$1. If x represents the number of soda bottles and y represents the number of juice bottles, the total cost can be represented by $3x + y$. Since she wants to spend no more than \$46, the inequality to describe this situation is $3x + y \leq 46$. Here, $3x$ accounts for the total cost of the soda, and y for the juice, and the inequality ensures that the combined cost doesn't exceed her budget.

CHAPTER

5 Quadratic

Math topics in this chapter:

- ☑ Solving a Quadratic Equations
- ☑ Graphing Quadratic Functions
- ☑ Solve a Quadratic Equation by Factoring
- ☑ Transformations of Quadratic Functions
- ☑ Quadratic Formula and the Discriminant
- ☑ Characteristics of Quadratic Functions: Equations
- ☑ Characteristics of Quadratic Functions: Graphs
- ☑ Complete a Function Table: Quadratic Functions
- ☑ Domain and Range of Quadratic Functions: Equations
- ☑ Factor Quadratics: Special Cases
- ☑ Factor Quadratics Using Algebra Tiles
- ☑ Write a Quadratic Function from Its Vertex and Another Point

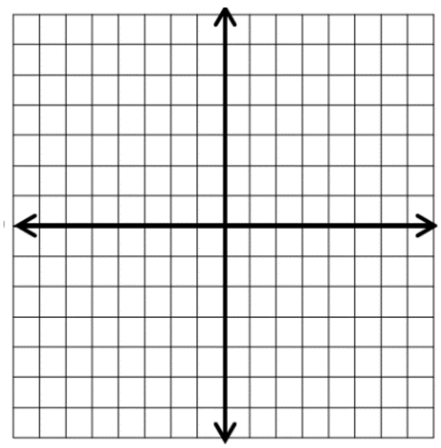
Practices

Solve each equation by factoring or using the quadratic formula.

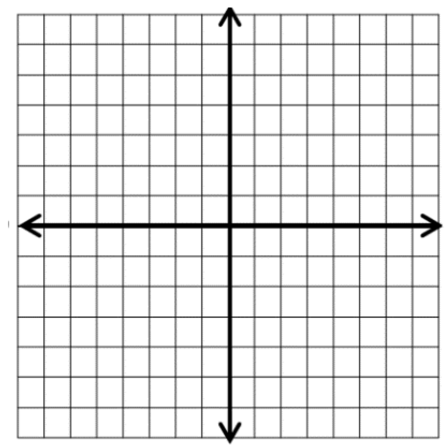
1) $x^2 - x - 2 = 0$
2) $x^2 - 6x + 8 = 0$
3) $x^2 - 4x + 3 = 0$
4) $x^2 + x - 12 = 0$
5) $x^2 + 7x - 18 = 0$
6) $x^2 - 2x - 15 = 0$
7) $x^2 + 6x - 40 = 0$
8) $x^2 - 9x - 36 = 0$

Sketch the graph of each function.

9) $y = (x - 4)^2 - 2$



10) $y = 2(x + 2)^2 - 3$

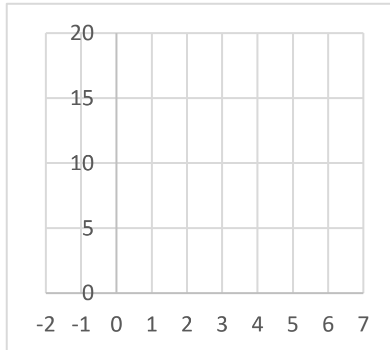


Solve each equation by factoring or using the quadratic formula.

11) $x^2 - 2x - 3 = 0$
12) $x^2 + 9x + 20 = 0$

State the transformations and sketch the graph of the following function.

13) $y = 2(x-3)^2 + 1$



Find the answer to the equation.

14) $2x^2 - 7x + 3 = 0$

15) $x^2 + 8x - 9 = 0$

16) $2x^2 + 5x - 3 = 0$

17) $x^2 + 6x + 9 = 0$

Solve.

18) Find the equation of the axis of symmetry for the parabola $y = x^2 + 7x + 3$.

19) Find the y-intercept of the parabola $x^2 + 25x + 7$.

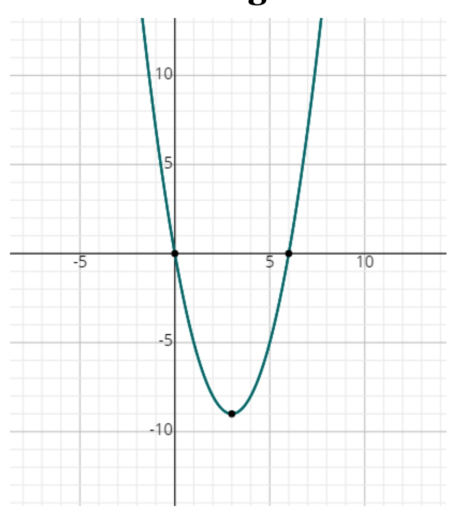
20) Find the vertex of the parabola $y = x^2 - 4x + 3$.

Considering the following graph, determine the following:

21) vertex

22) axis of symmetry

23) y −intercepts



Complete the table.

24)

$g(t) = t^2 + 7$	
t	$g(t)$
-1	
0	
1	

25)

$f(p) = 4p^2$	
p	$f(p)$
-2	
0	
2	

Determine the domain and range of each function.

26) $y = x^2 + 5x + 6$

27) $y = x^2 + 3$

28) $y = -x^2 + 4$

Factor.

29) $25x^2 + 20x + 4$

30) $9x^2 - 1$

31) $3 + 6x + 3x^2$

32) $b^4 - 36$

Use algebra tiles to factor.

33) $x^2 - 3x + 2$

34) $x^2 + 5x + 6$

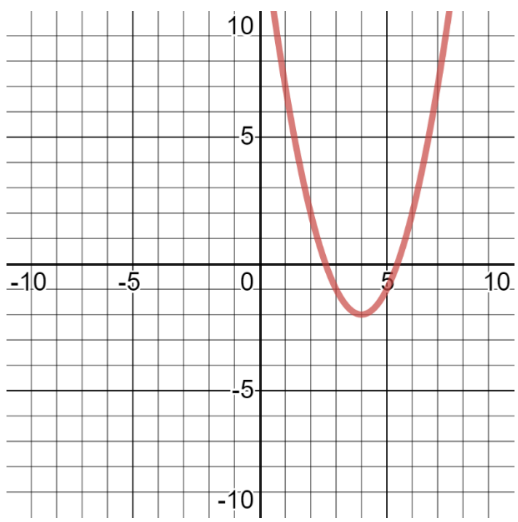
Write each quadratic function as a vertex form.

35) A parabola opening or down has vertex $(0,0)$ and passes through $(8, -16)$.

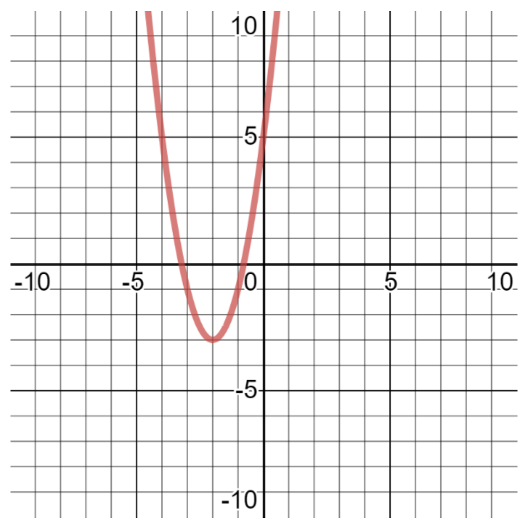
36) A parabola opening up or down has vertex $(0, 2)$ and passes through $(-2,5)$.

Answers

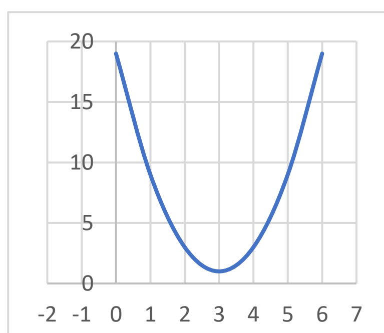
1) $x = 2, x = -1$
2) $x = 2, x = 4$
3) $x = 3, x = 1$
4) $x = 3, x = -4$
5) $x = 2, x = -9$
6) $x = 5, x = -3$
7) $x = 4, x = -10$
8) $x = 12, x = -3$
9) $y = (x - 4)^2 - 2$



10) $y = 2(x + 2)^2 - 3$



11) $\{3, -1\}$
12) $\{-4, -5\}$
13) The graph stretches vertically by a factor of 2.
Move 3 units to the right and 1 unit up.



14) $x_1 = 3, x_2 = \frac{1}{2}$
15) $x_1 = -9, x_2 = 1$
16) $x_1 = -3, x_2 = \frac{1}{2}$
17) $x_1 = x_2 = -3$
18) $x = -\frac{7}{2}$
19) 7

20) $(2, -1)$

21) $(3, -9)$

22) 3

23) 0

24)

$g(t) = t^2 + 7$	
t	$g(t)$
-1	8
0	7
1	8

25)

$f(p) = 4p^2$	
p	$f(p)$
-2	16
0	0
2	16

26) $D = \{x|x \in R\}, R = \{y \in R|y \geq -0.25\}$

27) $D = \{x|x \in R\}, R = \{R|y \geq 3\}$

28) $D = \{x|x \in R\}, R = \{y \in R|y \leq 4\}$

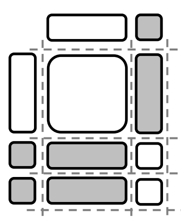
29) $(5x + 2)^2$

30) $(3x - 1)(3x + 1)$

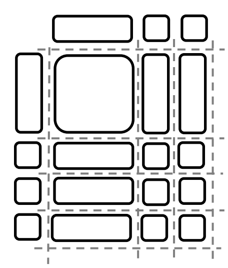
31) $3(1 + x)^2$

32) $(b^2 + 6)(b^2 - 6)$

33) $(x - 1)(x - 2)$



34) $(x + 2)(x + 3)$



35) $y = -\frac{1}{4}x^2$

36) $y = \frac{3}{4}x^2 + 2$

Answers and Explanations

1. The factors of -2 that add up to -1 (the coefficient of x) are -2 and 1. So, we rewrite the equation as $x^2 - 2x + x - 2 = 0$. Factoring by grouping, we get $x(x-2) + 1(x-2) = 0$, which simplifies to $(x-2)(x+1) = 0$. Setting each factor to zero gives the solutions: $x = 2$ and $x = -1$.

2. We look for factors of 8 that add up to -6. These are -2 and -4. Rewriting the equation as $x^2 - 4x - 2x + 8 = 0$, and factoring by grouping, we get $x(x-4) - 2(x-4) = 0$. This simplifies to $(x-4)(x-2) = 0$, giving the solutions: $x = 4$ and $x = 2$.

3. Here, we need factors of 3 that sum up to -4. These factors are -1 and -3. Rewriting the equation as $x^2 - 3x - x + 3 = 0$ and factoring by grouping, we get $x(x-3) - 1(x-3) = 0$. This factors into $(x-3)(x-1) = 0$, yielding solutions: $x = 3$ and $x = 1$.

4. We need factors of -12 that add to 1. These are 4 and -3. Rewrite the equation as $x^2 + 4x - 3x - 12 = 0$. Factoring by grouping, we get $x(x+4) - 3(x+4) = 0$. This factors into $(x+4)(x-3) = 0$, resulting in solutions: $x = -4$ and $x = 3$.

5. Looking for factors of -18 that sum to 7, we find 9 and -2.

Rewrite as $x^2 + 9x - 2x - 18 = 0$. Factoring by grouping, we have $x(x+9) - 2(x+9) = 0$. This factors into $(x+9)(x-2) = 0$, giving solutions: $x = -9$ and $x = 2$.

6. Factors of -15 that add up to -2 are -5 and 3.

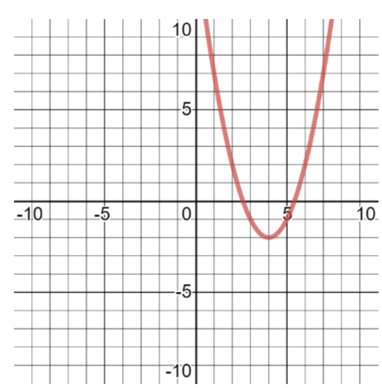
Rewrite as $x^2 - 5x + 3x - 15 = 0$. Factoring by grouping, we get $x(x-5) + 3(x-5) = 0$. This factors into $(x-5)(x+3) = 0$, resulting in solutions: $x = 5$ and $x = -3$.

7. We need factors of -40 that sum to 6. These are 10 and -4.

Rewrite as $x^2 + 10x - 4x - 40 = 0$. Factoring by grouping, we have $x(x+10) - 4(x+10) = 0$. This simplifies to $(x+10)(x-4) = 0$, giving solutions: $x = -10$ and $x = 4$.

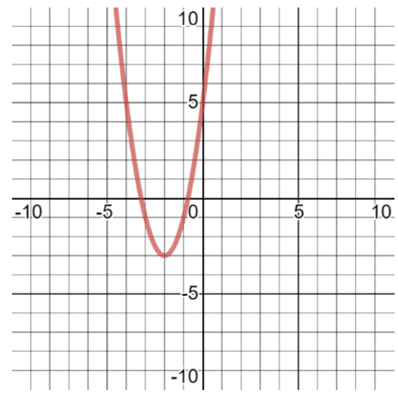
8. Seek numbers that multiply to -36 and add to -9. These are -12 and 3.

Factor as $(x-12)(x+3) = 0$. Solving $x - 12 = 0$ and $x + 3 = 0$ gives $x = 12$ and $x = -3$.



9. This graph represents a parabola. The basic shape of the parabola is $y = x^2$, which opens upwards. The term $(x-4)$

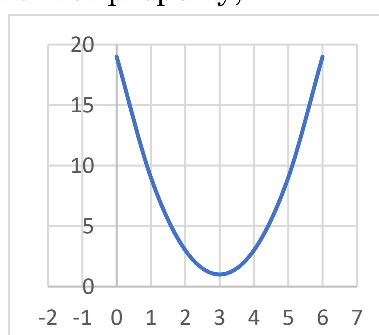
shifts this parabola 4 units to the right, as it changes the x-coordinate of the vertex. The "-2" at the end lowers the parabola by 2 units, moving the vertex down. The vertex of this parabola is at $(4, -2)$, which is the lowest point since the parabola opens upwards.



10. This graph also represents a parabola. The "2" multiplying the square term causes the parabola to be narrower than the standard $y = x^2$. The term $(x + 2)$ shifts the parabola 2 units to the left (opposite direction of the sign). The "-3" moves the parabola down by 3 units, altering the y-coordinate of the vertex. The vertex of this parabola is at $(-2, -3)$, and it opens upwards, making the vertex the lowest point of the parabola.

11. For factoring, we need to find two numbers that multiply to -3 (the constant term) and add to -2 (the coefficient of x). These numbers are -3 and 1. So, we can rewrite the equation as $(x - 3)(x + 1) = 0$. Now, apply the zero-product property, which states that if a product equals zero, then at least one of the factors must be zero. Setting each factor equal to zero gives us $x - 3 = 0$ and $x + 1 = 0$, leading to solutions $x = 3$ and $x = -1$.

12. To factor this, look for two numbers that multiply to 20 (the constant term) and add up to 9 (the coefficient of x). These numbers are 4 and 5. So, we can rewrite the equation as $(x + 4)(x + 5) = 0$. Applying the zero-product property, we set each factor to zero: $x + 4 = 0$ and $x + 5 = 0$. This gives us the solutions $x = -4$ and $x = -5$.



13. The term $(x - 3)$ indicates a horizontal shift of the basic parabola. It moves the parabola 3 units to the right. The coefficient 2 in front of the square term indicates a vertical stretch. This makes the parabola narrower than the standard parabola $y = x^2$. There's an upward shift of 1 unit, as indicated by the " $+ 1$" in the function. This moves the entire graph up.

14. Here, $a = 2$, $b = -7$, and $c = 3$. Substituting these into the formula gives $x_{1,2} = \frac{-(-7) \pm \sqrt{(-7)^2 - 4 \times 2 \times 3}}{2 \times 2}$. Solving this, we get two solutions: $x_1 = \frac{1}{2}$ and $x_2 = 3$.

15. Coefficients are $a = 1$, $b = 8$, and $c = -9$. Plugging these into the quadratic formula: $x_{1,2} = \frac{-8 \pm \sqrt{8^2 - 4 \times 1 \times (-9)}}{2 \times 1}$. This results in the roots $x_1 = -9$ and $x_2 = 1$.

16. Coefficients: $a = 2,\ b = 5,\ c = -3$. Apply the formula: $x_{1,2} = \frac{-5 \pm \sqrt{5^2 - 4 \times 2 \times (-3)}}{2 \times 2}$. The solutions are $x_1 = -3$ and $x_2 = \frac{1}{2}$.

17. Coefficients: $a = 1, b = 6, c = 9$. Using the formula $x_{1,2} = \frac{-6 \pm \sqrt{6^2 - 4 \times 1 \times 9}}{2 \times 1}$. Here, the discriminant (part under the square root) is zero, indicating one real root, $x_{1,2} = -3$.

18. The axis of symmetry of a parabola in the form $ax^2 + bx + c$ is given by $x = -\frac{b}{2a}$. Here, $a = 1$ and $b = 7$, so the axis of symmetry is $x = -\frac{7}{2 \times 1} = -\frac{7}{2}$. This means the line $x = -\frac{7}{2}$ is the axis of symmetry for the parabola.

19. The y-intercept of a parabola is found when $x = 0$. Substituting $x = 0$ in the equation gives the y-intercept. For $y = x^2 + 25x + 7$ substituting $x = 0$ yields $y = 0^2 + (25 \times 0) + 7 = 7$. Therefore, the y-intercept of this parabola is $y = 7$.

20. The vertex of a parabola $y = ax^2 + bx + c$ is at $(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right))$, where $f(x)$ is the parabola's equation. Here, $a = 1$ and $b = -4$, so the x −coordinate of the vertex is $x = -\frac{-4}{2 \times 1} = 2$. Substituting $x = 2$ into the parabola's equation gives the y-coordinate: $y = 2^2 - 4 \times 2 + 3 = -1$. Therefore, the vertex of this parabola is at the point $(2, -1)$.

21. The vertex is the point at the bottom of the parabola since it opens upwards. It's at the "tip" or the lowest point on the graph. You can locate the vertex by finding the point where the parabola turns around, which, on this graph, appears to be at the $(3, -9)$.

22. The axis of symmetry is the vertical line that goes through the vertex and splits the parabola into two symmetrical halves. In this graph, the axis of symmetry would be $x = 3$.

23. The y-intercept is where the parabola crosses the y-axis. From the graph, it's clear that this parabola crosses the y-axis at the point (0,0). There are no other points where the parabola crosses the y-axis.

$g(t) = t^2 + 7$	
t	$g(t)$
-1	8
0	7
1	8

24. To complete the table for each function, you substitute the given values of t into the functions and calculate the corresponding output. When $t = -1, g(t) = (-1)^2 + 7 = 1 + 7 = 8$. When $t = 0, g(t) = (0)^2 + 7 = 0 + 7 = 7$. When $t = 1, g(t) = (1)^2 + 7 = 1 + 7 = 8$.

25. When $p = -2$, $f(p) = 4(-2)^2 = 4 \times 4 = 16$. When $p = 0$, $f(p) = 4(0)^2 = 4 \times 0 = 0$. When $p = 2$, $f(p) = 4(2)^2 = 4 \times 4 = 16$.

$f(p) = 4p^2$	
p	$f(p)$
-2	16
0	0
2	16

26. The domain of any quadratic function, like this one, is all real numbers because there's no restriction on the values that x can take. Let's calculate the vertex to determine the minimum value of y, which will help us in determining the range. The vertex of function is at $(-2.5, -0.25)$. This means the lowest point on the graph of the function is at $y = -0.25$. Therefore, the range is $y \geq -0.25$.

27. Similar to the first function, this is also a quadratic function, where x can be any real number. There's no number that doesn't work in the equation. So, the domain is all real numbers. This equation also makes a U-shaped curve. The lowest point is the vertex. Here, the vertex is simpler because there's no x term to shift it left or right. So, the lowest point is just when $x = 0$, giving $y = 3$. Therefore, the smallest value y can be is 3, and it increases from there. The range is $y \geq 3$.

28. As with the other quadratic functions, there are no limitations on x here. You can choose any number for x. Hence, the domain is all real numbers. This function is also a parabola, but it opens downwards (because of the negative sign before x^2). The highest point on the curve is the vertex. Since there's no x term, the vertex is at $x = 0$, giving $y = 4$. That's the highest point, and the curve goes down from there. So, the range is all values of y that are less than or equal to 4, written as $y \leq 4$.

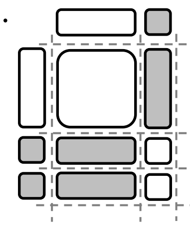
29. This equation can be factored by looking for a perfect square trinomial. A perfect square trinomial is formed when a binomial is squared. We notice that $(5x)^2 = 25x^2$, $2 \times 5x \times 2 = 20x$, and $2^2 = 4$. Therefore, $25x^2 + 20x + 4$ can be factored as $(5x + 2)^2$.

30. This equation is a difference of squares. Here, $9x^2$ is $(3x)^2$ and 1 is 1^2. So, $9x^2 - 1$ factors to $(3x + 1)(3x - 1)$.

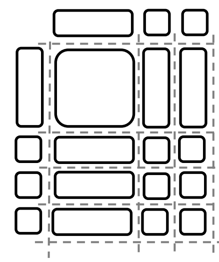
31. It's often easier to factor when the terms are in descending order of their exponents. Rearranging the equation gives us $3x^2 + 6x + 3$. Next, we look for common factors. In this case, each term is divisible by 3. Factoring out 3, we get: $3(x^2 + 2x + 1)$. Now, we need to factor the quadratic expression inside the parentheses. The expression $x^2 + 2x + 1$ is a perfect square trinomial, which factors into the square of a binomial. The pattern for a perfect square trinomial is $a^2 + 2ab + b^2 = (a+b)^2$. Here, $x^2 = (x)^2$, $2 \times x \times 1 = 2x$, and $1^2 = 1$. Putting it all together, the factored form of $3 + 6x + 3x^2$ is: $3(x+1)^2$.

32. In $b^4 - 36$, b^4 is a perfect square because it can be written as $(b^2)^2$and 36 is also a perfect square because it can be written as 6^2. Applying the difference of squares formula, we rewrite $b^4 - 36$ as $(b^2 + 6)(b^2 - 6)$.

33. For $x^2 - 3x + 2$, imagine a set of tiles representing x^2, $-x$ and $+1$. You need to arrange these tiles into a rectangle. The tiles for x^2 form the large square, the $-x$ tiles are rectangles, and $+1$ tiles are small squares. The goal is to create a rectangle where one side is made up of x tiles and the other side with constant number tiles. For this expression, you'll use one x^2 tile, three $-x$ tiles, and two $+1$ tiles. The arrangement that forms a rectangle is one that makes two groups: $(x-1)$ and $(x-2)$. So, $x^2 - 3x + 2$ factors into $(x-1)(x-2)$.



34. For $x^2 + 5x + 6$ use the same method. This time, you have one x^2 tile, five $+x$ tiles, and six $+1$ tiles. Arrange these to form a rectangle. The setup that works is creating two groups: $(x+2)$ and $(x+3)$. So, $x^2 + 5x + 6$ factors into $(x+2)(x+3)$.



35. The vertex form of a quadratic function is $y = a(x-h)^2 + k$, where (h, k) is the vertex. For a parabola with vertex $(0,0)$, the equation simplifies to $y = ax^2$. We need to find the value of a. We know the parabola passes through $(8, -16)$, so we substitute these values into the equation to find a. Substituting $x = 8$ and $y = -16$ gives $-16 = 64a$.

Solving for a give $a = -\frac{16}{64} = -\frac{1}{4}$. So, the equation in vertex form is $y = -\frac{1}{4}x^2$.

36. Starting with the vertex form $y = a(x-h)^2 + k$, we substitute the vertex $(0,2)$, yielding $y = a(x-0)^2 + 2$ or $y = ax^2 + 2$. To find a, we use the point $(-2,5)$. Substituting $x = -2$ and $y = 5$ gives $5 = a(-2)^2 + 2$, which simplifies to $5 = 4a + 2$. Solving for a result in $a = \frac{3}{4}$. Therefore, the quadratic function in vertex form is $y = \frac{3}{4}x^2 + 2$.

CHAPTER

6 Polynomials

Math topics in this chapter:

- ☑ Simplifying Polynomials
- ☑ Adding and Subtracting Polynomials
- ☑ Add and Subtract Polynomials Using Algebra Tiles
- ☑ Multiplying Monomials
- ☑ Multiplying and Dividing Monomials
- ☑ Multiplying a Polynomial and a Monomial
- ☑ Multiply Polynomials Using Area Models
- ☑ Multiplying Binomials
- ☑ Multiply two Binomials Using Algebra Tiles
- ☑ Factoring Trinomials
- ☑ Factoring Polynomials
- ☑ Use a Graph to Factor Polynomials
- ☑ Factoring Special Case Polynomials
- ☑ Add Polynomials to Find Perimeter

Practices

Simplify each polynomial.

1) $3(6x+4)=$
2) $5(3x-8)=$
3) $x(7x+2)+9x=$
4) $6x(x+3)+5x=$
5) $6x(3x+1)-5x=$
6) $x(3x-4)+3x^2-6=$
7) $x^2-5-3x(x+8)=$
8) $2x^2+7-6x(2x+5)=$

Add or subtract polynomials.

9) $(x^2+3)+(2x^2-4)=$
10) $(3x^2-6x)-(x^2+8x)=$
11) $(4x^3-3x^2)+(2x^3-5x^2)=$
12) $(6x^3-7x)-(5x^3-3x)=$
13) $(10x^3+4x^2)+(14x^2-8)=$
14) $(4x^3-9)-(3x^3-7x^2)=$
15) $(9x^3+3x)-(6x^3-4x)=$
16) $(7x^3-5x)-(3x^3+5x)=$

Use algebra tiles to simplify polynomials.

17) $(2x^2-3x+3)-(x^2-x-1)$
18) $(2x^2+2x+5)+(x^2+2x+1)$

Find the products.

19) $3x^2\times 8x^3=$
20) $2x^4\times 9x^3=$
21) $-4a^4b\times 2ab^3=$
22) $(-7x^3yz)\times(3xy^2z^4)=$
23) $-2a^5bc\times 6a^2b^4=$
24) $9u^3t^2\times(-2ut)=$
25) $12x^2z\times 3xy^3=$
26) $11x^3z\times 5xy^5=$
27) $-6a^3bc\times 5a^4b^3=$
28) $-4x^6y^2\times(-12xy)=$

Simplify each expression.

29) $(7x^2y^3)(3x^4y^2) =$

30) $(6x^3y^2)(4x^4y^3) =$

31) $(10x^8y^5)(3x^5y^7) =$

32) $(15a^3b^2)(2a^3b^8) =$

33) $\frac{42x^4y^2}{6x^3y} =$

34) $\frac{49x^5y^6}{7x^2y} =$

35) $\frac{63x^{15}y^{10}}{9x^8y^6} =$

36) $\frac{35x^8y^{12}}{5x^4y^8} =$

Find each product.

37) $3x(5x - y) =$

38) $2x(4x + y) =$

39) $7x(x - 3y) =$

40) $x(2x^2 + 2x - 4) =$

41) $5x(3x^2 + 8x + 2) =$

42) $7x(2x^2 - 9x - 5) =$

Use the area model to find each product.

43) $3x(x + 2)$

44) $(a - 3)(2a + 2)$

Find each product.

45) $(x - 3)(x + 3) =$

46) $(x - 6)(x + 6) =$

47) $(x + 10)(x + 4) =$

48) $(x - 6)(x + 7) =$

49) $(x + 2)(x - 5) =$

50) $(x - 10)(x + 3) =$

Use algebra tiles to simplify.

51) $(x + 1)(x + 6)$

52) $(2x + 1)(x - 4)$

Factor each trinomial.

53) $x^2 + 6x + 8 =$

54) $x^2 + 3x - 10 =$

55) $x^2 + 2x - 48 =$

56) $x^2 - 10x + 16 =$

57) $2x^2 - 10x + 12 =$

58) $3x^2 - 10x + 3 =$

Factor each expression.

59) $4x^2 - 4x - 8$

60) $6x^2 + 37x + 6$

61) $16x^2 + 60x - 100$

62) $4x^2 - 17x + 4$

Use a graph to factor the following polynomial.

63) $x^2 - 4$

64) $-(x + 2)^2$

Factor each completely.

65) $36x^2 - 121$

66) $-36x^4 + 4x^2$

67) $-36x^2 + 400 =$

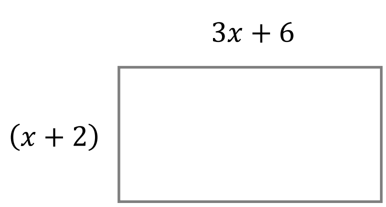
68) $49x^2 - 56x + 16 =$

69) $1 - x^2 =$

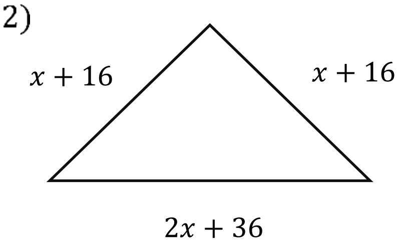
70) $81x^4 - 900x^2 =$

Find the perimeter.

71)



72)



Answers

1) $18x + 12$

2) $15x - 40$

3) $7x^2 + 11x$

4) $6x^2 + 23x$

5) $18x^2 + x$

6) $6x^2 - 4x - 6$

7) $-2x^2 - 24x - 5$

8) $-10x^2 - 30x + 7$

9) $3x^2 - 1$

10) $2x^2 - 14x$

11) $6x^3 - 8x^2$

12) $x^3 - 4x$

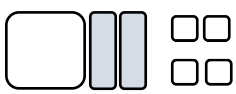
13) $10x^3 + 18x^2 - 8$

14) $x^3 + 7x^2 - 9$

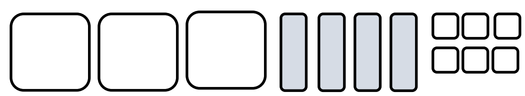
15) $3x^3 + 7x$

16) $4x^3 - 10x$

17) $x^2 - 2x + 4$



18) $3x^2 + 4x + 6$



19) $24x^5$

20) $18x^7$

21) $-8a^5b^4$

22) $-21x^4y^3z^5$

23) $-12a^7b^5c$

24) $-18u^4t^3$

25) $36x^3y^3z$

26) $55x^4y^5z$

27) $-30a^7b^4c$

28) $48x^7y^3$

29) $21x^6y^5$

30) $24x^7y^5$

31) $30x^{13}y^{12}$

32) $30a^6b^{10}$

33) $7xy$

34) $7x^3y^5$

35) $7x^7y^4$

36) $7x^4y^4$

37) $15x^2 - 3xy$

38) $8x^2 + 2xy$

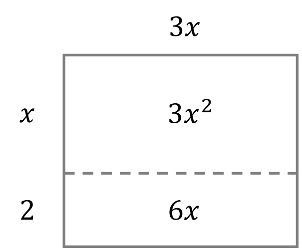
39) $7x^2 - 21xy$

40) $2x^3 + 2x^2 - 4x$

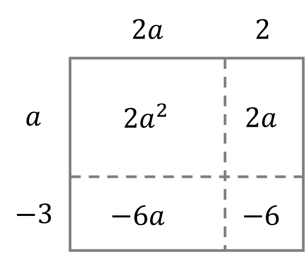
41) $15x^3 + 40x^2 + 10x$

42) $14x^3 - 63x^2 - 35x$

43) $3x^2 + 6x$



44) $2a^2 - 4a - 6$



45) $x^2 - 9$

46) $x^2 - 36$

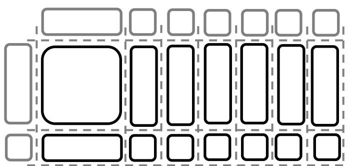
47) $x^2 + 14x + 40$

48) $x^2 + x - 42$

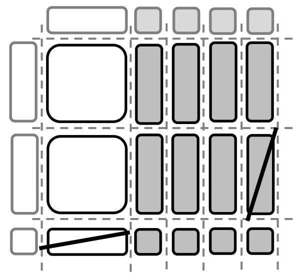
49) $x^2 - 3x - 10$

50) $x^2 - 7x - 30$

51) $x^2 + 7x + 6$



52) $2x^2 - 7x - 4$



53) $(x + 4)(x + 2)$

54) $(x + 5)(x - 2)$

55) $(x - 6)(x + 8)$

56) $(x - 8)(x - 2)$

57) $2(x - 3)(x - 2)$

58) $(3x - 1)(x - 3)$

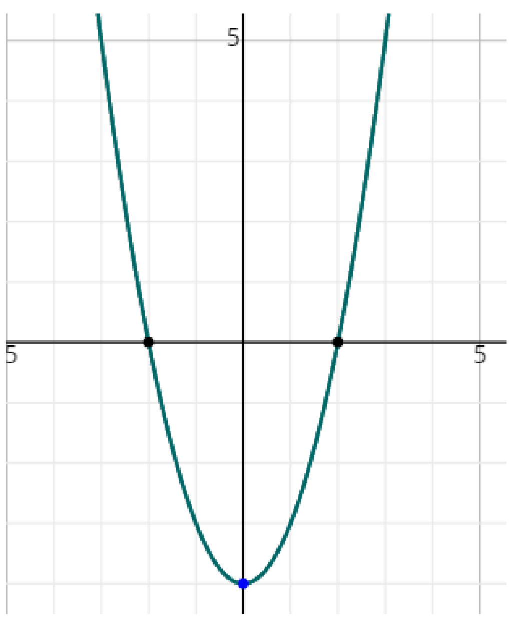
59) $4(x + 1)(x - 2)$

60) $(x + 6)(6x + 1)$

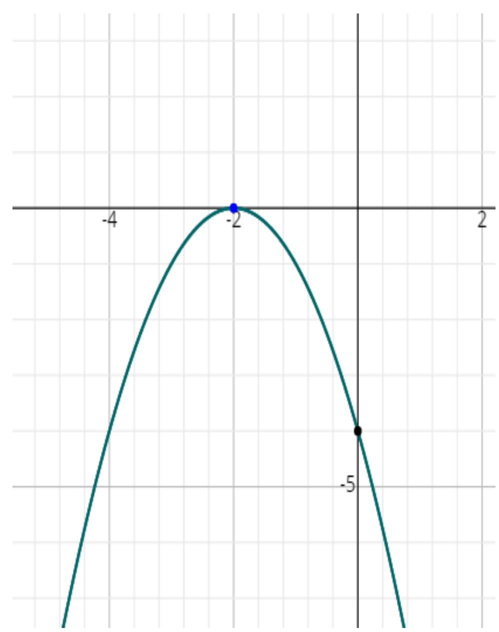
61) $4(x + 5)(4x - 5)$

62) $(x - 4)(4x - 1)$

63) $x = \pm 2$



64) $x = -2$



65) $(6x - 11)(6x + 11)$

66) $4x^2(1 - 3x)(1 + 3x)$

67) $4(10 + 3x)(10 - 3x)$

68) $(7x - 4)^2$

69) $(1 + x)(1 - x)$

70) $9x^2(3x + 10)(3x - 10)$

71) $8x + 16$

72) $4x + 68$

Answers and Explanations

1. First, distribute the 3 to both $6x$ and 4. This means multiplying 3 by each term inside the parentheses. $3 \times 6x$ gives $18x$, and 3×4 gives 12. So, the simplified form is $18x + 12$.

2. Here, you'll distribute 5 to $3x$ and -8. Multiplying 5 by $3x$ gives $15x$, and 5 times -8 gives -40. Thus, the polynomial simplifies to $15x - 40$.

3. Start by expanding $x(7x + 2)$. Multiply x with $7x$ to get $7x^2$, and x with 2 to get $2x$. Now, add $9x$ to the expanded form. So, it becomes $7x^2 + 11x$.

4. Multiply $6x$ by each term inside the parentheses (x and 3). This gives $6x^2$ and $18x$. Then, add $5x$ to these. The result is $6x^2 + 23x$.

5. Distribute $6x$ to $3x$ and 1. You get $18x^2$ and $6x$. Subtracting $5x$ from $6x$ results in $18x^2 + x$.

6. Multiply x by $3x$ and -4, getting $3x^2$ and $-4x$. Add these to $3x^2 - 6$. The simplified form is $6x^2 - 4x - 6$.

7. Start by expanding $3x(x + 8)$, which gives $3x^2$ and $24x$. Now, subtract these from $x^2 - 5$. The result is $-2x^2 - 24x - 5$.

8. First, expand $6x(2x + 5)$ to get $12x^2$ and $30x$. Then, subtract these from $2x^2 + 7$. You end up with $-10x^2 - 30x + 7$.

9. Add the like terms. Here, x^2 and $2x^2$ are like terms, so they combine to $3x^2$. The constants 3 and -4 also combine to give -1. Thus, the simplified expression is $3x^2 - 1$.

10. Subtract each term in the second polynomial from the corresponding term in the first. $3x^2 - x^2$ equals $2x^2$, and $-6x - 8x$ equals $-14x$. So, the answer is $2x^2 - 14x$.

11. Combine like terms. $4x^3$ and $2x^3$ add up to $6x^3$, and $-3x^2$ and $-5x^2$ add up to $-8x^2$. The result is $6x^3 - 8x^2$.

12. Subtract the terms in the second polynomial from the first. $6x^3 - 5x^3$ is x^3, and $-7x - (-3x)$ simplifies to $-4x$. The final expression is $x^3 - 4x$.

13. Add like terms. $10x^3$ has no like term, so it remains as is. $4x^2$ and $14x^2$ make $18x^2$. The constant -8 stands alone. So, the simplified form is $10x^3 + 18x^2 - 8$.

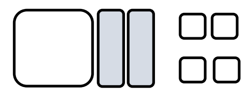
14. Here, $4x^3 - 3x^3$ equals x^3. There's no like term for $-7x^2$, so it's just subtracted, becoming $+7x^2$. Finally, -9 has no like term, so it remains. The answer is $x^3+7x^2 - 9$.

15. Subtract the second polynomial from the first. $9x^3 - 6x^3$ is $3x^3$, and $3x - (-4x)$ simplifies to $7x$. Thus, the result is $3x^3 + 7x$.

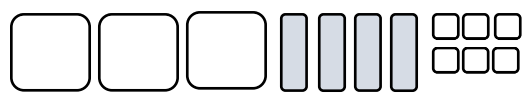
16. Subtract the terms in the second polynomial from the first. $7x^3 - 3x^3$ results in $4x^3$, and

$-5x - 5x$ equals $-10x$. The final expression is $4x^3 - 10x$.

17. First, represent $2x^2$ with two large squares, $-3x$ with three negative long rectangles, and $+3$ with three small squares. Next, for $-(x^2 - x - 1)$, use one negative large square for $-x^2$, one positive long rectangle for $+x$, and one positive small square for $+1$. Now, combine these tiles. The two large squares and one negative large square leave one large square (x^2). The three negative rectangles and one positive rectangle leave two negative rectangles ($-2x$). The three small squares and one positive small square leave four small squares ($+4$). So, the simplified form is $x^2 - 2x + 4$.



18. Represent $2x^2$ with two large squares, $2x$ with two long rectangles, and $+5$ with five small squares. For $x^2 + 2x + 1$, use one large square, two long rectangles, and one small square. Combine these tiles. The two large squares and one large square make three large squares ($3x^2$). The four long rectangles represent $4x$. The five small squares and one small square make six small squares ($+6$). The simplified expression is $3x^2 + 4x + 6$.



19. Multiply the coefficients 3 and 8 to get 24. For the variables, add the exponents of x (2 and 3), resulting in x^{2+3}. So, the product is $24x^5$.

20. First, multiply the coefficients 2 and 9, which equals 18. Then, combine x^4 and x^3 by adding their exponents $(4 + 3)$, leading to x^7. Thus, the product is $18x^7$.

21. Multiply the coefficients -4 and 2 to get -8. Combine a^4 and a (which is a^1) by adding exponents $(4 + 1 = 5)$ to get a^5. Similarly, combine b and b^3 to get b^4. So, the result is $-8a^5b^4$.

22. Here, multiply -7 and 3 to get -21. For x^3 and x, add their exponents $(3 + 1 = 4)$ to get x^4. For y, combine y and y^2 to y^3. Lastly, add the exponents of z and z^4 $(1 + 4 = 5)$ to get z^5. The final product is $-21x^4y^3z^5$.

23. Multiply -2 and 6 to get -12. Combine a^5 and a^2 to get a^7, and b and b^4 to get b^5. The c term remains as is. The result is $-12a^7b^5c$.

24. Multiply 9 and -2 to get -18. Add the exponents of u^3 and u to get u^4, and the exponents of t^2 and t to get t^3. So, the product is $-18u^4t^3$.

25. Multiply the coefficients 12 and 3 to get 36. Combine x^2 and x to x^3. The y^3 and z terms don't have like terms, so they are just appended. The result is $36x^3\,y^3z$.

26. Here, multiply 11 and 5 to get 55. Add the exponents of x^3 and x to get x^4. Since z and y^5 don't have like terms, they remain as is. The final product is $55x^4\,y^5z$.

27. First, multiply the coefficients (numerical values) together: $-6 \times 5 = -30$. Next, multiply the 'a' terms: $a^3 \times a^4$. When multiplying with the same base, add the exponents: $3 + 4 = 7$. So, this becomes a^7. Then, multiply the 'b' terms: $b \times b^3$. Similarly, add the exponents for 'b': $1 + 3 = 4$ (Note: 'b' is b^1). This results in b^4. Lastly, the 'c' term remains as is since there's no other 'c' term to multiply with. Combine all these: $-30a^7b^4c$.

28. Multiply the coefficients: $-4 \times (-12) = 48$. Multiply the 'x' terms: $x^6 \times x$. Add their exponents: $6 + 1 = 7$. This gives x^7. Multiply the 'y' terms: $y^2 \times y$. The exponents add up to $2 + 1 = 3$, resulting in y^3. Combine these results: $48x^7y^3$.

29. We multiply the coefficients (numbers) and add the exponents of like bases (x and y). So, $7 \times 3 = 21$ and $x^{2+4} = x^6$, $y^{3+2} = y^5$. The simplified expression is $21x^6y^5$.

30. Multiply the coefficients (6 and 4) and add the exponents of x and y separately. $6 \times 4 = 24$, $x^{3+4} = x^7$, and $y^{2+3} = y^5$. This gives $24x^7y^5$.

31. Here, multiply 10 and 3, and add the exponents of x and y. $10 \times 3 = 30$, $x^{8+5} = x^{13}$, $y^{5+7} = y^{12}$. The result is $30x^{13}y^{12}$.

32. Multiply 15 and 2, and add the exponents of a and b. $15 \times 2 = 30$, $a^{3+3} = a^6$, $b^{2+8} = b^{10}$. So, we get $30a^6b^{10}$.

33. Divide the coefficients and subtract the exponents of x and y in the denominator from those in the numerator. $42 \div 6 = 7$, $x^{4-3} = x$, and $y^{2-1} = y$. The simplified form is $7xy$.

34. Again, divide the coefficients and subtract the exponents.

$49 \div 7 = 7$, $x^{5-2} = x^3$, and $y^{6-1} = y^5$. This simplifies to $7x^3y^5$.

35. Divide 63 by 9 and subtract the exponents of x and y in the denominator from the numerator. $63 \div 9 = 7$, $x^{15-8} = x^7$, $y^{10-6} = y^4$. Thus, the simplified expression is $7x^7y^4$.

36. Finally, divide 35 by 5 and subtract the exponents. $35 \div 5 = 7$, $x^{8-4} = x^4$, and $y^{12-8} = y^4$. The result is $7x^4y^4$.

37. Multiply $3x$ by each term inside the parentheses. First, $3x \times 5x = 15x^2$. (Multiplying the coefficients and adding the exponents of x). Next, $3x \times (-y) = -3xy$ (multiplying the coefficient of x by y and keeping the sign). The product is $15x^2 - 3xy$.

38. Here, multiply $2x$ by $4x$ to get $8x^2$ and $2x$ by y to get $2xy$. The result is $8x^2 + 2xy$.

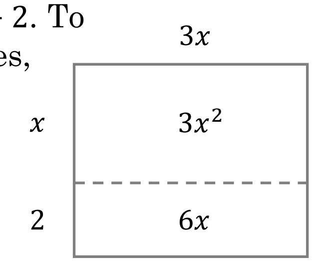
39. Distribute $7x$ across x and $-3y$. $7x \times x = 7x^2$and $7x \times (-3y) = -21xy$. This results in $7x^2 - 21xy$.

40. Multiply x with each term inside the parentheses. $x \times 2x^2 = 2x^3, x \times 2x = 2x^2$, and $x \times (-4) = -4x$. Combine these for $2x^3 + 2x^2 - 4x$.

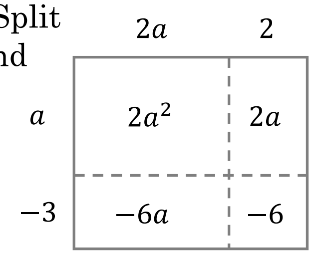
41. Apply distribution: $5x \times 3x^2 = 15x^3$, $5x \times 8x = 40x^2$, and $5x \times 2 = 10x$. The product is $15x^3 + 40x^2 + 10x$.

42. Here, multiply $7x$ by $2x$ to get $14x^3$, $7x$ by $-9x$ to get $-63x^2$ and $7x$ by -5 to get $-35x$. This results in $14x^3 - 63x^2 - 35x$.

43. Picture a rectangle with one side as $3x$ and another as $x + 2$. To find the area, divide the rectangle into two smaller rectangles, one with side $3x$ and the other side x, and the second with side $3x$ and the other side 2. The area of the first smaller rectangle is $3x \times x = 3x^2$. (multiplying the lengths of its sides). The area of the second smaller rectangle is $3x \times 2 = 6x$. The total area, or the product, is the sum of these areas: $3x^2 + 6x$.



44. Visualize this as a rectangle with sides $a - 3$ and $2a + 2$. Split it into two smaller rectangles: one with sides $a - 3$ and $2a$, and the other with sides $a - 3$ and 2. The area of the first is $2a \times (a - 3) = 2a^2 - 6a$ (multiplying $2a$ with each term in $a - 3$). The area of the second is $2 \times (a - 3) = 2a - 6$ (multiplying 2 with each term in $a - 3$). The total area, or product, is $2a^2 - 6a + 2a - 6,$, which simplifies to $2a^2 - 4a - 6$.



45. Multiply each term in the first binomial by each term in the second. This gives $x \times x = x^2$, $x \times 3 = 3x$, $-3 \times x = -3x$, and $-3 \times 3 = -9$. Combining these, we get $x^2 + 3x - 3x - 9$. Notice that $3x$ and $-3x$ cancel out, so the final answer is $x^2 - 9$.

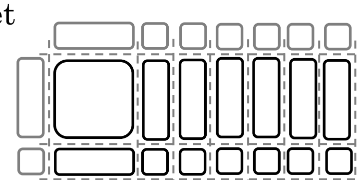
46. Following the same method, we multiply each term: $x \times x = x^2$, $x \times 6 = 6x$, $-6 \times x = -6x$, and $-6 \times 6 = -36$. Combining gives $x^2 + 6x - 6x - 36$. Again, $6x$ and $-6x$ cancel out, resulting in $x^2 - 36$.

47. The product is found by multiplying: $x \times x = x^2$, $x \times 4 = 4x$, $10 \times x = 10x$, and $10 \times 4 = 40$. Adding these up gives $x^2 + 4x + 10x + 40$. Combining like terms ($4x$ and $10x$), we get $x^2 + 14x + 40$.

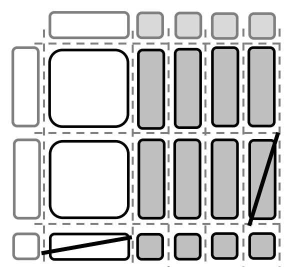
48. Multiply each term to get $x \times x = x^2$, $x \times 7 = 7x$, $-6 \times x = -6x$, and $-6 \times 7 = -42$. Summing these gives $x^2 + 7x - 6x - 42$, which simplifies to $x^2 + x - 42$.

49. Multiply each term: $x \times x = x^2$, $x \times (-5) = -5x$, $2 \times x = 2x$, and $2 \times (-5) = -10$. Adding up these products gives $x^2 - 5x + 2x - 10$, , which simplifies to $x^2 - 3x - 10$.

50. The multiplication of each term yields $x \times x = x^2$, $x \times 3 = 3x$, $-10 \times x = -10x$, and $-10 \times 3 = -30$. Combining these results in $x^2 + 3x - 10x - 30$, which simplifies to $x^2 - 7x - 30$.



51. Imagine a set of tiles representing $x + 1$ and another set representing $x + 6$. To find the product, we arrange these tiles to form a rectangle. The tiles for $x + 1$ form one side of the rectangle, and the tiles for $x + 6$ form the other. The area of the rectangle is the product. In this case, the area consists of $x \times x$ (which is x^2) $x \times 6$ (which is $6x$), $1 \times x$ (which is x), and 1×6 (which is 6). Adding these areas together, we get $x^2 + 6x + x + 6$, which simplifies to $x^2 + 7x + 6$.



52. Here, we use algebra tiles for $2x + 1$ and $x - 4$ to form another rectangle. The area of this rectangle is found by multiplying the lengths of its sides. We have $2x \times x$ (which gives $2x^2$) $2x \times -4$ (which is $-8x$), $1 \times x$ (which is x), and $1 \times (-4)$ (which is -4). When these areas are combined, we get $2x^2 - 8x + x - 4$, which simplifies to $2x^2 - 7x - 4$.

53. We need two numbers that multiply to 8 and add up to 6. The numbers 2 and 4 fit this description, as $2 \times 4 = 8$ and $2 + 4 = 6$. So, the factorization is $(x + 2)(x + 4)$.

54. We look for numbers that multiply to -10 and add to 3. The numbers 5 and -2 work here, since $5 \times (-2) = -10$ and $5 + (-2) = 3$. Thus, the factorization is $(x + 5)(x - 2)$.

55. We need numbers that multiply to -48 and add to 2. The numbers 8 and -6 meet these criteria ($8 \times (-6) = -48$ and $8 + (-6) = 2$). Therefore, it factors to $(x + 8)(x - 6)$.

56. Here, we seek numbers that multiply to 16 and add to -10. The numbers -2 and -8 do the trick ($-2 \times (-8) = 16$ and $-2 + (-8) = -10$). So, the factorization is $(x - 2)(x - 8)$.

57. First, notice all coefficients are even, so we can factor out a 2: $2(x^2 - 5x + 6)$. Now, factor $x^2 - 5x + 6$. We need numbers that add to -5 and multiply to 6. These are -3 and -2. The factored form is $2(x-3)(x-2)$.

58. Again, the coefficient of x^2 is not 1, so we look for numbers that multiply to $3 \times 3 = 9$ and add to -10.

The numbers -9 and -1 fit ($-9 \times (-1) = 9$ and $-9 + (-1) = -10$). Therefore, it factors to $(3x-1)(x-3)$.

59. Initially, notice that each term in the expression is divisible by 4. Therefore, we factor out 4, resulting in $4(x^2 - x - 2)$. Now, focus on factoring $x^2 - x - 2$. We need two numbers that add up to -1 (the coefficient of 'x') and multiply to -2 (the constant term). These numbers are 1 and -2.

Thus, the factored form is $4(x+1)(x-2)$.

60. This requires a bit more thought because the coefficients are larger. We seek two numbers that add up to 37 and multiply to $6 \times 6 = 36$. The numbers 36 and 1 fit this criterion. Place these numbers in the expression, giving $6x^2 + 36x + x + 6$. Then, group and factor by grouping: $6x(x+6) + 1(x+6)$.

The final factored form is $(6x+1)(x+6)$.

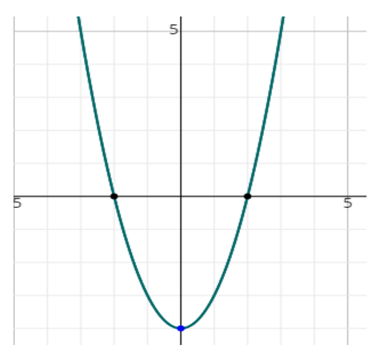
61. Start by noticing all terms are divisible by 4. So, we factor out 4, resulting in $4(4x^2 + 15x - 25)$. Next, factor $4x^2 + 15x - 25$. The numbers we're looking for are those that add to 15 and multiply to $4 \times (-25) = -100$. The suitable numbers are 20 and -5. After rearranging and grouping, the factored form becomes $4(4x^2 + 20x - 5x - 25)$ which simplifies to $4(4x(x+5) - 5(x+5))$, and finally to $4(4x-5)(x+5)$.

62. This one also needs careful consideration. We want two numbers that add up to -17 and multiply to $4 \times 4 = 16$. These numbers are -16 and -1. By splitting the middle term, we get $4x^2 - 16x - x + 4$.

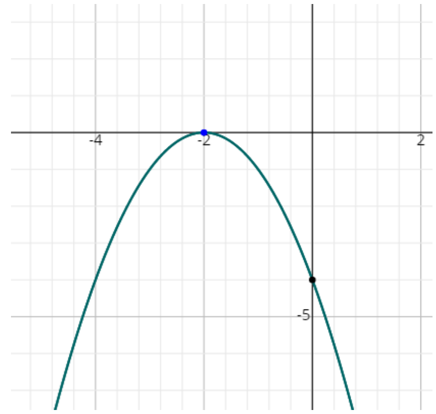
Grouping these gives $4x(x-4) - 1(x-4)$.

The expression then factors to $(4x-1)(x-4)$.

63. The graph of this polynomial is a parabola opening upwards. The equation $x^2 - 4$ can be rewritten as $(x-2)(x+2)$ using the difference of squares formula. Graphically, this parabola will intersect the x-axis at $x = 2$ and $x = -2$. So, the factored form $(x-2)(x+2)$ directly corresponds to these intercepts, indicating the points where the graph touches the x-axis.



64. This is the graph of a downward-opening parabola because of the negative sign in front. The term $(x+2)^2$ means that the parabola is a shifted version of the basic x^2 graph, moved 2 units to the left. For the roots, we set $(x+2)^2 = 0$. The only solution to this is $x = -2$, which means the graph touches the x-axis at this point only. Since it's a perfect square, the graph only touches the axis at one point, reflecting the fact that $x = -2$ is a repeated or double root.



65. This is a difference of squares, as both $36x^2$ and 121 are perfect squares. It can be rewritten as $(6x)^2 - 11^2$. The factored form, using the difference of squares formula $a^2 - b^2 = (a-b)(a+b)$, is $(6x-11)(6x+11)$.

66. First, factor out the greatest common factor, which is $4x^2$, giving $4x^2(-9x^2+1)$. Notice that $1 - 9x^2$ is also a difference of squares. Thus, it factors further into $4x^2(1-3x)(1+3x)$.

67. Firstly, we notice that both terms, $-36x^2$ and 400, are divisible by 4. However, since our first term is negative, we factor out -4. This step changes the expression to $-4(9x^2 - 100)$.

The next step involves recognizing that $9x^2 - 100$ is a difference of two squares. In this case, $9x^2$ is $3x$ squared, and 100 is 10 squared. Therefore, $9x^2 - 100$ factors into $(3x+10)(3x-10)$. Now, we substitute this back into our expression to get $-4(3x+10)(3x-10)$ or $4(10+3x)(10-3x)$.

68. This is a perfect square trinomial. It factors into a binomial squared. The square root of $49x^2$ is $7x$, and the square root of 16 is 4. The factored form is $(7x-4)^2$.

69. This is another difference of squares, where $a = 1$ and $b = x$.

It factors to $(1-x)(1+x)$.

70. Start by factoring out the greatest common factor, which is $9x^2$, yielding $9x^2(9x^2 - 100)$. The expression inside the parentheses is again a difference of squares. So, it factors to $9x^2(3x-10)(3x+10)$.

71. To find the perimeter, you add all four sides: $P = (3x+6) + (3x+6) + (x+2) + (x+2)$. Simplify by combining like terms:

$P = 2(3x+6) + 2(x+2) = 6x + 12 + 2x + 4 = 8x + 16$.

72. Add the three sides: $P = (x+16) + (x+16) + (2x+36)$. Combine like terms to simplify: $P = 2(x+16) + (2x+36) = 2x + 32 + 2x + 36 = 4x + 68$.

CHAPTER

7 Relations and Functions

Math topics in this chapter:

- ☑ Function Notation and Evaluation
- ☑ Adding and Subtracting Functions
- ☑ Multiplying and Dividing Functions
- ☑ Composition of Functions
- ☑ Evaluate an Exponential Function
- ☑ Match Exponential Functions and Graphs
- ☑ Write Exponential Functions: Word Problems
- ☑ Function Inverses
- ☑ Domain and Range of Relations
- ☑ Rate of Change and Slope
- ☑ Complete a Function Table from an Equation

Practices

Evaluate each function.

1) $f(x) = x - 2$, find $f(-1)$
2) $g(x) = 2x + 4$, find $g(3)$
3) $g(n) = 2n - 8$, find $g(-1)$
4) $h(n) = n^2 - 1$, find $h(-2)$
5) $f(x) = x^2 + 12$, find $f(5)$
6) $g(x) = 2x^2 - 9$, find $g(-2)$
7) $w(x) = 2x^2 - 4x$, find $w(2n)$
8) $p(x) = 4x^3 - 10$, find $p(-3a)$

Perform the indicated operation.

9) $g(x) = x - 2$
$h(x) = 2x + 6$
Find: $(h + g)(3)$
_____

10) $f(x) = 3x + 2$
$g(x) = -x - 6$
Find: $(f + g)(2)$
_____

11) $f(x) = 5x + 8$
$g(x) = 3x - 12$
Find: $(f - g)(-2)$
_____

12) $h(x) = 2x^2 - 10$
$g(x) = 3x + 12$
Find: $(h + g)(3)$
_____

13) $g(x) = 12x - 8$
$h(x) = 3x^2 + 14$
Find: $(h - g)(x)$
_____

14) $h(x) = -2x^2 - 18$
$g(x) = 4x^2 + 15$
Find: $(h - g)(a)$
_____

Perform the indicated operation.

15) $g(x) = x - 5$
$h(x) = x + 6$
Find: $(g.h)(-1)$
____

16) $f(x) = 2x + 2$
$g(x) = -x - 6$
Find: $(\frac{f}{g})(-2)$
____

17) $f(x) = 5x + 3$
$g(x) = 2x - 4$
Find: $(\frac{f}{g})(5)$
____

18) $h(x) = x^2 - 2$
$g(x) = x + 4$
Find: $(g.h)(3)$
____

19) $g(x) = 4x - 12$
$h(x) = x^2 + 4$
Find: $(g.h)(-2)$
____

20) $h(x) = 3x^2 - 8$
$g(x) = 4x + 6$
Find: $(\frac{h}{g})(-4)$
____

Solve.

21) $f(x) = 2x$
$g(x) = x + 3$
Find: $(fog)(2)$

______

24) $h(x) = 2x - 2$
$g(x) = x + 4$
Find: $(goh)(2)$

______

22) $f(x) = x + 2$
$g(x) = x - 6$
Find: $(fog)(-1)$

______

25) $f(x) = 2x - 8$
$g(x) = x + 10$
Find: $(fog)(-2)$

______

23) $f(x) = 3x$
$g(x) = x + 4$
Find: $(gof)(4)$

______

26) $f(x) = x^2 - 8$
$g(x) = 2x + 3$
Find: $(gof)(4)$

______

Use the following function to find: $f(x) = 3x(\frac{1}{2})^{2x+2}$

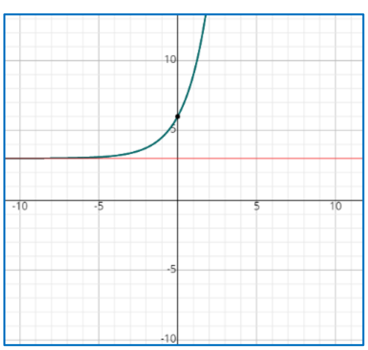
27) $f(2)$

28) $f(4)$

Match each exponential function to its graph.

29) $f(x) = -4(3)^x$, $f(x) = 2^x + 5$, $f(x) = 3(2)^x + 3$

A



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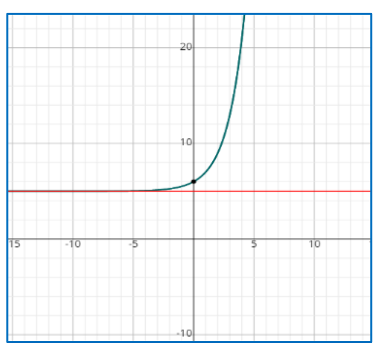
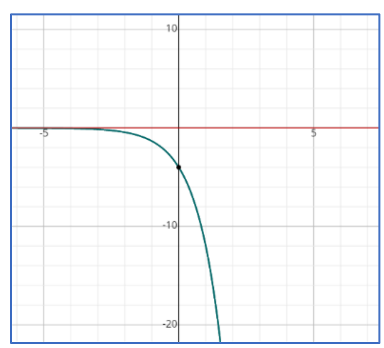


C



Solve.

30) As of 2019, the world population is 8.716 billion and growing at a rate of 1.2% per year. Write an equation to model population growth, where $p(t)$ is the population in billions of people and t is the time in years.

31) You decide to buy a used car that costs $20,000. You have heard that the car may depreciate at a rate of 10% per year. At this rate, how much will the car be worth in 6 years?

Find the inverse of each function.

32) $f(x) = -\frac{1}{x} - 9$

$f^{-1}(x) =$ ________

33) $g(x) = \sqrt{x} - 2$

$g^{-1}(x) =$ ________

34) $h(x) = -\frac{5}{x+3}$

$h^{-1}(x) =$ ________

35) $f(x) = 6x + 6$

$f^{-1}(x) =$ ________

Find the domain and range of each relation.

36) $\{(1,-1),(2,-4),(0,5),(-1,6)\}$

37) $\{(10,-5),(-16,-8),(-4,19),(16,7),(6,-14)\}$

38) $\{(4,7),(-15,6),(-20,9),(13,8),(7,5)\}$

Solve.

39) Average food preparation time in a restaurant was tracked daily as part of an efficiency improvement program.

Day	Food preparation time (minutes)
Tuesday	45
Wednesday	49
Thursday	32
Friday	15
Saturday	25

According to the table, what was the rate of change between Tuesday and Wednesday?

Complete the table.

40)

$f(x) = 3x - 2$	
x	$f(x)$
-3	
0	
2	

41)

$f(x) = 2x$	
x	$f(x)$
1	
2	
3	

Answers

1) -3
2) 10
3) -10
4) 3
5) 37
6) -1
7) $8n^2 - 8n$
8) $-108a^3 - 10$
9) 13
10) 0
11) 16
12) 29
13) $3x^2 - 12x + 22$
14) $-6a^2 - 33$
15) -30
16) $\frac{1}{2}$
17) $\frac{14}{3}$
18) 49
19) -160
20) -4
21) 10
22) -5
23) 16
24) 6
25) 8
26) 19
27) $\frac{3}{32}$
28) $\frac{3}{256}$
29) $A = 3(2)^x + 3,$

    $B = 2^x + 5,$

    $C = -4(3)^x$
30) $p(t) = 8.716(1 + 0.012)^t$
31) $A = 20{,}000(1 - 0.1)^6$
32) $-\frac{1}{x+9}$
33) $x^2 + 4x + 4$
34) $-\frac{5}{x} - 3$
35) $\frac{x-6}{6}$

36) $D = (1,2,0,-1)$,

$R = (-1,-4,5,6)$

37) $D = (10,-16,-4,16,6)$,

$R = (-5,-8,19,7,-14)$

38) $D = (4,-15,-20,13,7)$,

$R = (7,6,9,8,5)$

39) 4

40)

$f(x) = 3x - 2$	
x	$f(x)$
-3	-11
0	-2
2	4

41).

$f(x) = 2x$	
x	$f(x)$
1	2
2	4
3	6

Answers and Explanations

1. Replace x with -1 in the function $f(x) = x - 2$. So, $f(-1) = -1 - 2 = -3$. This means when x is -1, the value of the function is -3.

2. Substitute x with 3 in $g(x) = 2x + 4$. This gives $g(3) = 2 \times 3 + 4 = 10$. Thus, when x is 3, the function evaluates to 10.

3. Here, replace n with -1 in $g(n) = 2n - 8$.

This results in $g(-1) = 2 \times -1 - 8 = -10$. So, the function value is -10 when n is -1.

4. For $h(-2)$, substitute n with -2 in $h(n) = n^2 - 1$.

We get $h(-2) = -2^2 - 1 = 4 - 1 = 3$. Hence, $h(-2)$ is 3.

5. Replace x with 5 in $f(x) = x^2 + 12$. So, $f(5) = 5^2 + 12 = 25 + 12 = 37$. The function value is 37 when x is 5.

6. In $g(x) = 2x^2 - 9$, substitute x with -2, giving $g(-2) = 2 \times (-2)^2 - 9 = 8 - 9 = -1$. Therefore, $g(-2)$ is -1.

7. For $w(2n)$, replace x with $2n$ in $w(x) = 2x^2 - 4x$.

This leads to $w(2n) = 2 \times (2n)^2 - 4 \times 2n = 8n^2 - 8n$. Thus, $w(2n)$ equals $8n^2 - 8n$.

8. Substitute x with $-3a$ in $p(x) = 4x^3 - 10$.

We get $p(-3a) = 4(-3a)^3 - 10 = -108a^3 - 10$. So, $p(-3a)$ is $-108a^3 - 10$.

9. We add g and h and then substitute $x = 3$: $(h + g)(x) = (2x + 6) + (x - 2)$, which simplifies to $3x + 4$. Substituting $x = 3$ gives us $3(3) + 4 = 9 + 4 = 13$.

10. We add f and g and substitute $x = 2$: $(f + g)(x) = (3x + 2) + (-x - 6)$, simplifying to $2x - 4$. Substituting $x = 2$ gives us $2(2) - 4 = 4 - 4 = 0$.

11. We subtract g from f and substitute $x = -2$:

$(f - g)(x) = (5x + 8) - (3x - 12)$, simplifying to $2x + 20$. Substituting $x = -2$ gives us $2(-2) + 20 = -4 + 20 = 16$.

12. We add h and g and substitute $x = 3$: $(h + g)(x) = (2x^2 - 10) + (3x + 12)$, which simplifies to $2x^2 + 3x + 2$.

Substituting $x = 3$ gives us $2(3)^2 + 3(3) + 2 = 18 + 9 + 2 = 29$.

13. We subtract g from h without substituting any value for x:

$(h - g)(x) = (3x^2 + 14) - (12x - 8)$, simplifying to $3x^2 - 12x + 22$.

14. We subtract g from h and substitute x with a:

$(h-g)(x)=(-2x^2-18)-(4x^2+15)$, simplifying to $-6x^2-33$. When we substitute x with a, the expression remains $-6a^2-33$.

15. First, we find $g(-1)$, which is $-1-5=-6$.

Then, we find $h(-1)$, so $h(-1)=-1+6=5$. Therefore, $(g.h)(-1)=-6\times 5=-30$.

16. We calculate $f(-2)$ as $2\times(-2)+2=-4+2=-2$. Similarly, $g(-2)$ is $-(-2)-6=2-6=-4$. We then divide these results: $\frac{-2}{-4}=\frac{1}{2}$.

17. Here, $f(5)=5\times 5+3=25+3=28$. For $g(5)$, we get $(2\times 5)-4=10-4=6$. Dividing, $\frac{28}{6}=\frac{14}{3}$. Thus, $\left(\frac{f}{g}\right)(5)=\frac{14}{3}$.

18. First, calculate $h(3)=3^2-2=9-2=7$. Then, $g(3)=3+4=7$.

So, $(g.h)(3)=7\times 7=49$.

19. We start with $h(-2)=(-2)^2+4=4+4=8$.

Then, $g(-2)=4\times(-2)-12=-8-12=-20$.

Therefore, $(g.h)(-2)=8\times(-20)=-160$.

20. Calculating $h(-4)=3\times(-4)^2-8=(3\times 16)-8=48-8=40$. For $g(-4)$, it's

$4\times(-4)+6=-16+6=-10$. Dividing these gives $\frac{40}{-10}=-4$.

Thus, $(\frac{h}{g})(-4)=-4$.

21. First, we find $g(2)$, which is $2+3=5$. Then, we use this value in $f(x)$, so $f(5)=2\times 5=10$. Therefore, $(fog)(2)=10$.

22. We calculate $g(-1)$ as $-1-6=-7$. Then, we substitute this into $f(x)$, so $f(-7)=-7+2=-5$. Hence, $(fog)(-1)=-5$.

23. First, find $f(4)$, which is $3\times 4=12$. Next, use this value in $g(x)$, resulting in $g(12)=12+4=16$. Thus, $(gof)(4)=16$.

24. Compute $h(2)$ by substituting 2 into $h(x)$, giving $h(2)=(2\times 2)-2=4-2=2$. Then, use this result in $g(x)$, so $g(2)=2+4=6$. Therefore, $(goh)(2)=6$.

25. Calculate $g(-2)$ as $-2+10=8$. Next, use this value in $f(x)$, resulting in $f(8)=2\times 8-8=16-8=8$. So, $(fog)(-2)=8$.

26. Start by finding $f(4)$, which is $4^2 - 8 = 16 - 8 = 8$. Then, substitute this result into $g(x)$, giving $g(8) = (2 \times 8) + 3 = 16 + 3 = 19$. Hence, $(gof)(4) = 19$.

27. To evaluate $f(2)$, we substitute $x = 2$ into the function $f(x) = 3x(\frac{1}{2})^{2x+2}$. This becomes $3 \times 2(\frac{1}{2})^{2\times2+2} = 6 \times (\frac{1}{2})^6$. The expression $(\frac{1}{2})^6$ represents $\frac{1}{2}$ multiplied by itself 6 times, which equals $\frac{1}{64}$. Thus, $f(2) = 6 \times \frac{1}{64} = \frac{6}{64} = \frac{3}{32}$.

28. For $f(4)$, we substitute $x = 4$ into the same function, resulting in $3 \times 4(\frac{1}{2})^{2\times4+2} = 12 \times (\frac{1}{2})^{10}$. The term $(\frac{1}{2})^{10}$ is $\frac{1}{2}$ raised to the 10th power, which equals $\frac{1}{1{,}024}$. Therefore, $f(4) = 12 \times \frac{1}{1{,}024} = \frac{12}{1{,}024} = \frac{3}{256}$.

29. $f(x) = -4(3)^x$: This function has a negative coefficient in front of the base, which means it will reflect across the x-axis, resulting in a graph that decreases as x increases. The base is greater than 1, so the reflection will still show an exponential decay. Therefore, we're looking for a graph that goes downwards as we move from left to right. This matches with graph C.

$f(x) = 2^x + 5$: This function is a standard exponential growth function with a vertical shift upwards by 5 units. Since there's no negative sign or coefficient greater than 1 in front of the base, the graph will increase as x increases and will be above the standard 2^x graph by 5 units. This is represented by graph B.

$f(x) = 3(2)^x + 3$: Similar to the second function, this one also represents exponential growth with a base of 2. However, it has a larger vertical shift than the second function due to the "+3" outside the exponent, moving it further up. The coefficient of 3 in front of the 2^x will make the graph grow faster than the standard 2^xgraph. This is shown by graph A, which grows faster and starts higher due to the vertical shift compared to graph B.

30. To model the world population growth based on the given data, we can use an exponential growth equation. The standard form for exponential growth is $p(t) = p_0(1 + r)^t$, where p_0 is the initial amount (in this case, the initial population), r is the growth rate (as a decimal), and t is the time in years. Here, $p_0 = 8.716$ billion (the population in 2019). The growth rate $r = 1.2\% = 0.012$ (converted from a percentage to a decimal), t will be the number of years since 2019. So, the equation to model the population growth is $p(t) = 8.716(1 + 0.012)^t$.

31. For the depreciation of the car, we'll use an exponential decay model since the car's value decreases over time. The formula for exponential decay is similar to growth: $A_t = A_0(1 - r)^t$, where A_0 is the initial value, r is the rate of decay, and t is the time in years. The initial value of the car, A_0 is \$20,000, the annual depreciation rate $r = 10\% = 0.1$.

For $t = 6$ years, the equation becomes $A(6) = 20{,}000(1 - 0.1)^6$.

32. First, replace $f(x)$ with y: $y = -\frac{1}{x} - 9$. To find the inverse, swap x and y:

$x = -\frac{1}{y} - 9$. Next, isolate y. Start by adding 9 to both sides: $x + 9 = -\frac{1}{y}$. Multiply both sides by $-y$ to get rid of the fraction: $-y(x + 9) = 1$. Finally, solve for y by dividing by $-(x + 9)$: $y = -\frac{1}{x+9}$. Therefore, $f^{-1}(x) = -\frac{1}{x+9}$.

33. Write the function as $y = \sqrt{x} - 2$. To find its inverse, exchange x and y:

$x = \sqrt{y} - 2$. Rearrange to solve for y. First, add 2 to both sides: $x + 2 = \sqrt{y}$. Then square both sides to eliminate the square root: $(x + 2)^2 = y$.

Thus, $g(x)^{-1} = (x + 2)^2 = x^2 + 4x + 4$.

34. Start with $y = -\frac{5}{x+3}$. Swap x and y to get $x = -\frac{5}{y+3}$. Rearrange to find y. Multiply both sides by $y + 3$ and then by $-x(y + 3) = 5$.

Divide by $-x$ to get $y + 3 = -\frac{5}{x}$. Subtract 3 to isolate y: $y = -\frac{5}{x} - 3$. Therefore, $h(x)^{-1} = -\frac{5}{x} - 3$.

35. Represent the function as $y = 6x + 6$. Exchange x and y: $x = 6y + 6$. Now, solve for y. Subtract 6 from both sides: $x - 6 = 6y$. Divide everything by 6:

$y = \frac{x-6}{6}$. Hence, $f^{-1}(x) = \frac{x-6}{6}$.

36. Look at the first numbers in each pair: $1, 2, 0, -1$. These represent the x-values. So, the domain is $\{1, 2, 0, -1\}$. Now look at the second numbers: $-1, -4, 5, 6$. These are the y-values. Therefore, the range is $\{-1, -4, 5, 6\}$.

37. The first numbers are $10, -16, -4, 16, 6$. These form the domain: $\{10, -16, -4, 16, 6\}$. The second numbers are $-5, -8, 19, 7, -14$. These make up the range: $\{-5, -8, 19, 7, -14\}$.

38. The first elements are $4, -15, -20, 13, 7$. So, the domain is $\{4, -15, -20, 13, 7\}$. The second elements are $7, 6, 9, 8, 5$. Thus, the range is $\{7, 6, 9, 8, 5\}$.

40. To find the rate of change, we subtract the value at the starting point from the value at the ending point, and then divide by the time passed. $Rate\ of\ Change = \frac{Wednesday's\ time - Tuesday's\ time}{Time\ passed} = \frac{49-45}{1} = 4$.

41. For the first function, $f(x) = 3x - 2$: When $f(-3) = 3(-3) - 2 = -9 - 2 = -11$.

When $f(0) = 3(0) - 2 = 0 - 2 = -2$. When $f(2) = 3(2) - 2 = 6 - 2 = 4$.

$f(x) = 3x - 2$	
x	$f(x)$
-3	-11
0	-2
2	4

42. For the second function, $f(x) = 2x$: When $f(1) = 2(1) = 2$. When $f(2) = 2(2) = 4$. When $f(3) = 2(3) = 6$.

$f(x) = 2x$	
x	$f(x)$
1	2
2	4
3	6

CHAPTER

8 Radical Expressions

Math topics in this chapter:

- ☑ Finding Slope
- ☑ Simplifying Radical Expressions
- ☑ Adding and Subtracting Radical Expressions
- ☑ Multiplying Radical Expressions
- ☑ Rationalizing Radical Expressions
- ☑ Radical Equations
- ☑ Domain and Range of Radical Functions
- ☑ Simplify Radicals with Fractions

Practices

Evaluate.

1) $\sqrt{49} =$ ________

2) $\sqrt{4} \times \sqrt{81} =$ ________

3) $\sqrt{16} \times \sqrt{4x^2} =$ ________

4) $\sqrt{289} =$ ________

5) $\sqrt{25b^4} =$ ________

6) $\sqrt{9} \times \sqrt{x^2} =$ ________

Simplify.

7) $\sqrt{6} + 6\sqrt{6} =$

8) $9\sqrt{8} - 6\sqrt{2} =$

9) $-\sqrt{7} - 5\sqrt{7} =$

10) $10\sqrt{2} + 3\sqrt{18} =$

11) $\sqrt{12} - 6\sqrt{3} =$

12) $-2\sqrt{x} + 6\sqrt{x} =$

Evaluate.

13) $\sqrt{4} \times 2\sqrt{9} =$

14) $\sqrt{5} \times 3\sqrt{20y} =$

15) $-6\sqrt{4} \times 3\sqrt{4} =$

16) $-9\sqrt{3b^2} \times \left(-\sqrt{6}\right) =$

Simplify.

17) $\frac{1+\sqrt{5}}{1-\sqrt{3}} =$

18) $\frac{2+\sqrt{6}}{\sqrt{2}-\sqrt{5}} =$

19) $\frac{\sqrt{7}}{\sqrt{6}-\sqrt{3}} =$

20) $\frac{\sqrt{8a}}{\sqrt{a^5}} =$

Solve for x in each equation.

21) $2\sqrt{2x-4} = 8$

22) $9 = \sqrt{4x-1}$

23) $\sqrt{x} + 6 = 11$

24) $\sqrt{5x} = \sqrt{x+3}$

Identify the domain and range of each function.

25) $y = \sqrt{x+1}$

26) $y = \sqrt{x-2} + 6$

27) $y = \sqrt{x} - 1$

28) $y = \sqrt{x-4}$

Simplify.

29) $\sqrt{\frac{625}{36}}$

30) $\sqrt{\frac{1,296}{25}}$

31) $\sqrt{\frac{147}{64}}$

32) $\sqrt{\frac{98}{18}}$

Answers

1) 7
2) 18
3) $8x$
4) 17
5) $5b^2$
6) $3x$
7) $7\sqrt{6}$
8) $12\sqrt{2}$
9) $-6\sqrt{7}$
10) $19\sqrt{2}$
11) $-4\sqrt{3}$
12) $4\sqrt{x}$
13) 12
14) $30y$
15) -72
16) $27b\sqrt{2}$
17) $-\frac{(1+\sqrt{5})(1+\sqrt{3})}{2}$
18) $-\frac{2\sqrt{2}+2\sqrt{5}+2\sqrt{3}+\sqrt{30}}{3}$
19) $\frac{\sqrt{7}(\sqrt{6}+\sqrt{3})}{3}$
20) $\frac{2\sqrt{2}}{a^2}$
21) $x = 10$
22) $x = 20.5$
23) $x = 25$
24) $x = \frac{3}{4}$
25) $x \geq -1,\ y \geq 0$
26) $x \geq 2,\ y \geq 6$
27) $x \geq 0,\ y \geq -1$
28) $x \geq 4,\ y \geq 0$
29) $\frac{25}{6}$
30) $\frac{36}{5}$
31) $\frac{7\sqrt{3}}{8}$
32) $\frac{7}{3}$

Answers and Explanations

1. The square root of 49 is the number that when multiplied by itself equals 49. This number is 7 because $7 \times 7 = 49$.

2. The square root of 4 is 2, and the square root of 81 is 9. When you multiply these together, 2×9, you get 18.

3. The square root of 16 is 4, and the square root of $4x^2$ is $2x$. Multiplying these together, $4 \times 2x$, gives $8x$.

4. The square root of 289 is the number that when squared equals 289. This number is 17 because $17 \times 17 = 289$.

5. The square root of $25b^4$ is the product of the square root of 25, which is 5, and the square root of b^4, which is b^2. So, $5 \times\ b^2 = 5b^2$.

6. The square root of 9 is 3, and the square root of x^2 is x. Multiplying these gives $3 \times x$, or $3x$.

7. Both terms have the square root of 6, so they can be combined just like you would combine like terms such as $1x + 6x$. Therefore, $\sqrt{6} + 6\sqrt{6}$ simplifies to $7\sqrt{6}$.

8. Here, we can simplify $\sqrt{8}$ to $2\sqrt{2}$ (since 8 is 4 times 2, and the square root of 4 is 2). This gives us $9 \times 2\sqrt{2} - 6\sqrt{2}$, which simplifies to $18\sqrt{2} - 6\sqrt{2}$, and further to $12\sqrt{2}$.

9. These are like terms since both involve the square root of 7. Combining them like regular numbers, we get $-1\sqrt{7} - 5\sqrt{7}$ which simplifies to $-6\sqrt{7}$.

10. We can simplify $\sqrt{18}$ to $3\sqrt{2}$ (since 18 is 9 times 2, and the square root of 9 is 3). This gives us $10\sqrt{2} + 3 \times 3\sqrt{2}$, which simplifies to $10\sqrt{2} + 9\sqrt{2}$, and further simplifies to $19\sqrt{2}$.

11. We can simplify $\sqrt{12}$ to $2\sqrt{3}$ (since 12 is 4 times 3, and the square root of 4 is 2). This gives us $2\sqrt{3} - 6\sqrt{3}$, which simplifies to $-4\sqrt{3}$.

12. These are like terms since both involve the square root of x. We combine them just like we would with similar variables, $-2\sqrt{x} + 6\sqrt{x}$, which simplifies to $4\sqrt{x}$.

13. First, evaluate the square roots: $\sqrt{4}$ is 2, and $\sqrt{9}$ is 3. Now multiply the numbers outside the square roots: 2 times 2, which equals 4. Then multiply this by the square root we found earlier, which is 3, to get $4 \times 3 = 12$.

14. We can't simplify $\sqrt{5}$, but we can simplify $\sqrt{20}$. Since 20 is 4 times 5 and the square root of 4 is 2, $\sqrt{20}$ is $2\sqrt{5}$. Now multiply the coefficients (numbers outside the square roots): 3 times 2 equals 6. Finally, multiply this by the $\sqrt{5}$ we left untouched earlier to get $6\sqrt{5} \times \sqrt{5}$, which simplifies to $6 \times 5 = 30$ since $\sqrt{5} \times \sqrt{5} = 5$. So, the expression becomes $30y$.

15. The square root of 4 is 2, so $\sqrt{4}$ is 2. Multiply -6 by 2 to get -12, and 3 by 2 to get 6. Then multiply -12 by 6, which equals -72.

16. We'll simplify $\sqrt{(3b^2)}$. Since b^2 is just b times b, and the square root of a squared number is the number itself, $\sqrt{(3b^2)}$ is $b\sqrt{3}$. Now, multiply -9 by $b\sqrt{3}$ to get $-9b\sqrt{3}$. Finally, multiply this by $-\sqrt{6}$ to get $-9b\sqrt{3} \times -\sqrt{6}$, which simplifies to $9b\sqrt{18}$. The square root of 18 can be simplified further to $3\sqrt{2}$ (since 18 is 9 times 2 and the square root of 9 is 3), resulting in $9b \times 3\sqrt{2} = 27b\sqrt{2}$.

17. To simplify, we multiply the numerator and denominator by the conjugate of the denominator. The conjugate of $1 - \sqrt{3}$ is $1 + \sqrt{3}$. This removes the square root from the denominator. $\frac{1+\sqrt{5}}{1-\sqrt{3}} \times \frac{1+\sqrt{3}}{1+\sqrt{3}}$. Multiply out both the numerator and the denominator: $\frac{(1+\sqrt{5})(1+\sqrt{3})}{1-3} = -\frac{(1+\sqrt{5})(1+\sqrt{3})}{2}$.

18. Here again, we use the conjugate to eliminate the square root from the denominator. The conjugate of $\sqrt{2} - \sqrt{5}$ is $\sqrt{2} + \sqrt{5}$. $\frac{2+\sqrt{6}}{\sqrt{2}-\sqrt{5}} \times \frac{\sqrt{2}+\sqrt{5}}{\sqrt{2}+\sqrt{5}}$. Multiply and simplify: $\frac{(2+\sqrt{6})(\sqrt{2}+\sqrt{5})}{2-5}$. Expand and simplify: $-\frac{2\sqrt{2}+2\sqrt{5}+2\sqrt{3}+\sqrt{30}}{3}$.

19. Use the conjugate technique again. Multiply the expression by $\frac{\sqrt{6}+3}{\sqrt{6}+3}$. $\frac{\sqrt{7}}{\sqrt{6}-\sqrt{3}} \times \frac{\sqrt{6}+3}{\sqrt{6}+3}$. Simplify: $\frac{\sqrt{7}(\sqrt{6}+\sqrt{3})}{6-3} = \frac{\sqrt{7}(\sqrt{6}+\sqrt{3})}{3}$.

20. Simplify each square root separately.

$\sqrt{8a} = \sqrt{2 \times 4 \times a} = 2\sqrt{2a}$ and $\sqrt{a^5} = a^2\sqrt{a}$. Now divide: $\frac{2\sqrt{2a}}{a^2\sqrt{a}}$. Simplify the square roots: $\frac{2}{a^2} \times \sqrt{\frac{2a}{a}} = \frac{2\sqrt{2}}{a^2}$.

21. To solve, first divide both sides by 2: $\sqrt{2x-4} = 4$. Now, square both sides to eliminate the square root: $2x - 4 = 16$. Next, add 4 to both sides: $2x = 20$. Finally, divide by 2: $x = 10$.

22. Start by squaring both sides to remove the square root: $81 = 4x - 1$. Then, add 1 to both sides: $82 = 4x$. Divide by 4 to isolate $x = 20.5$.

23. First, isolate the square root by subtracting 6 from both sides: $\sqrt{x} = 5$. Now, square both sides to eliminate the square root: $x = 25$.

24. Square both sides to remove the square roots: $5x = x + 3$. Next, subtract x from both sides: $4x = 3$. Finally, divide by 4: $x = \frac{3}{4}$.

25. The expression inside the square root, $x + 1$, must be greater than or equal to 0. Solving $x + 1 \geq 0$ gives us $x \geq -1$. So, the domain is all real numbers x such that $x \geq -1$. Since square roots always yield non-negative results, y will be 0 or positive. Therefore, the range is all real numbers y such that $y \geq 0$.

26. For $x - 2$ to be non-negative, $x \geq 2$. Thus, the domain is $x \geq 2$. The smallest value of $\sqrt{(x-2)}$ is 0 (when $x = 2$), and since we add 6 to it, the smallest value of y is 6. Hence, the range is $y \geq 6$.

27. The square root of x requires x to be non-negative, so $x \geq 0$. This makes the domain $x \geq 0$. The smallest value $\sqrt{x}$ can take is 0 (when $x = 0$) and subtracting 1 from it gives -1. So, the range starts from -1 and goes upwards, making it $y \geq -1$.

28. For $x - 4$ to be non-negative, x must be at least 4. Therefore, the domain is $x \geq 4$. The square root function yields non-negative outputs. Thus, the smallest value for y is 0 (when $x = 4$), making the range $y \geq 0$.

29. The square root of 625 is 25 (because $25 \times 25 = 625$), and the square root of 36 is 6 (since $6 \times 6 = 36$). So, $\sqrt{\frac{625}{36}}$ becomes $\frac{25}{6}$.

30. Here, the square root of 1,296 is 36 (as $36 \times 36 = 1{,}296$), and the square root of 25 is 5 ($5 \times 5 = 25$). Thus, $\sqrt{\frac{1{,}296}{25}}$ simplifies to $\frac{36}{5}$.

31. For this, the square root of 147 is not a whole number, but we can simplify it. Let's find the largest perfect square that divides into 147. The factors of 147 include $1, 3, 7, 21, 49, 147$. The largest perfect square among these is 49. So, 147 can be written as 49×3. The square root of 49 is 7, so the square root of 147 simplifies to $7\sqrt{3}$. The square root of 64 is 8 ($8 \times 8 = 64$). So, $\sqrt{\frac{147}{64}}$ becomes $\frac{7\sqrt{3}}{8}$.

32. The number 98 doesn't have a perfect square root, but it can be simplified. Let's find the largest perfect square that divides into 98. The factors of 98 include $1, 2, 7, 14, 49, 98$. The largest perfect square among these is 49. So, 98 can be written as 49×2. The square root of 49 is 7, so the square root of 98 simplifies to $7\sqrt{2}$. The number 18 is not a perfect square, so its square root does not simplify to a whole number. The factors of 18 include $1, 2, 3, 6, 9, 18$. The largest perfect square is 9. So, 18 can be written as 9×2, and the square root of 18 simplifies to $3\sqrt{2}$. Combining these, $\sqrt{\frac{98}{18}}$ simplifies to $\frac{7\sqrt{2}}{3\sqrt{2}}$. When you simplify this fraction, the $\sqrt{2}$ in the numerator and denominator cancel out, leaving $\frac{7}{3}$.

CHAPTER

9 Statistics and Probabilities

Math topics in this chapter:

- ☑ Mean, Median, Mode, and Range of the Given Data
- ☑ Pie Graph
- ☑ Scatter Plots
- ☑ Probability Problems
- ☑ Permutations and Combinations
- ☑ Calculate and Interpret Correlation Coefficients
- ☑ Equation of a Regression Line and Interpret Regression Lines
- ☑ Correlation and Causation

Practices

Find the values of the given data.

1) 6, 11, 5, 3, 6
Mode: _____ Range: _____
Mean: _____ Median: _____

2) 4, 9, 1, 9, 6, 7
Mode: _____ Range: _____
Mean: _____ Median: _____

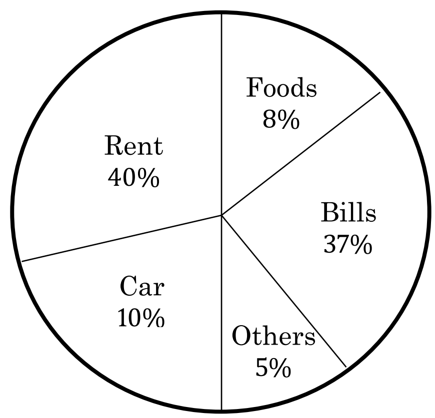
3) 10, 3, 6, 10, 4, 15
Mode: _____ Range: _____
Mean: _____ Median: _____

4) 12, 4, 8, 9, 3, 12, 15
Mode: _____ Range: _____
Mean: _____ Median: _____

The circle graph below shows all of Bob's expenses for last month. Bob spent $790 on his Rent last month.

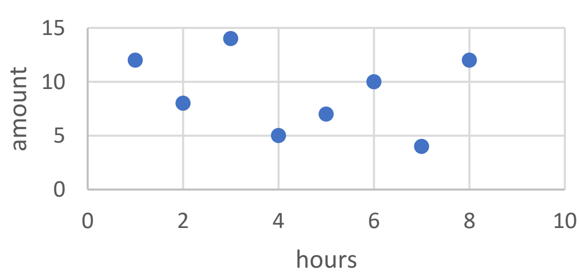
5) How much did Bob's total expenses last month? ________
6) How much did Bob spend for foods last month? ________
7) How much did Bob spend on his bills last month? ________
8) How much did Bob spend on his car last month? ________

Bob's last month expenses



Make a scatter plot of the data.

9) Does this scatter plot show a positive trend, a negative trend, or no trend?



hours	amount
1	$12
2	$8
3	$14
4	$5
5	$7
6	$10
7	$4
8	$12

Solve.

10) Bag A contains 8 red marbles and 6 green marbles. Bag B contains 5 black marbles and 7 orange marbles. What is the probability of selecting a green marble at random from bag A? What is the probability of selecting a black marble at random from Bag B?

________________ ______________

Solve.

11) Susan is baking cookies. She uses sugar, flour, butter, and eggs. How many different orders of ingredients can she try? ______________
12) Jason is planning for his vacation. He wants to go to a museum, go to the beach, and play volleyball. How many different ways of ordering are there for him? ______________
13) In how many ways can a team of 6 basketball players choose a captain and co-captain? ______________
14) How many ways can the first and second place be awarded to 11 people? _____
15) A professor is going to arrange her 5 students in a straight line. In how many ways can she do this? ______________
16) In how many ways can a teacher choose 12 out of 15 students? ______________

Find the correlation coefficient of the following data.

17)

x	12	14	18	21	28
y	2	4	6	8	12

18)

x	50	51	52	53	54
y	4.1	4.2	4.3	4.4	4.5

Determine the linear regression equation from the given set of data.

19)

x	2	3	5	8
y	3	6	4	13

20)

x	2	4	6	8
y	4	7	10	12

Determine whether the following relationships reflect both correlation and causation or not.

21) The number of cold and snowy days and the amount of coffee at the ski resort.

22) The number of miles traveled, and the gas used.

Answers

1) Mode: 6, Range: 8,
Mean: 6.2, Median: 6
2) Mode: 9, Range: 8,
Mean: 6, Median: 6.5
3) Mode: 10, Range: 12,
Mean: 8, Median: 8
4) Mode: 12, Range: 12,
Mean: 9, Median: 9
5) $1,975
6) $158
7) $730.75
8) $197.50
9) No trend
10) $\frac{3}{7}, \frac{5}{12}$
11) 24
12) 6
13) 30
14) 110
15) 120
16) 455
17) 0.997
18) 1
19) $y = 1.47x - 0.14$
20) $y = 1.35x + 1.5$
21) Correlation no causation
22) Correlation and causation

Answers and Explanations

1. The mode is the most frequently occurring number, which is 6 in this case. The range is the difference between the highest and lowest values.

Here, it's $11 - 3 = 8$. The mean is the average, calculated by adding all the numbers and dividing by the count. Here, it's $\frac{6+11+5+3+6}{5} = 6.2$. The median is the middle value when the numbers are in order. For this set, when ordered, the middle value is 6.

2. The most frequent number is 9. The highest number is 9 and the lowest is 1, so the range is $9 - 1 = 8$. The average is $\frac{4+9+1+9+6+7}{6} = 6$. Sorting these numbers, the middle values are 6 and 7, so the median (average of these two) is 6.5.

3. The number that appears most frequently is 10. The range is $15 - 3 = 12$. The mean is $\frac{10+3+6+10+4+15}{6} = 8$. When sorted, the middle numbers are 6 and 10, and their average is 8.

4. The mode is 12, as it appears most frequently. The difference between the highest (15) and lowest (3) values is 12. The mean is calculated as $\frac{12+4+8+9+3+12+15}{7} = 9$. In the sorted list, the middle number is 9.

5. First, we know Bob's rent, which is 40% of his total expenses, was \$790. To find the total expenses, we use the formula: $Total\ expenses = \frac{Expense\ Amount}{Percentage\ of\ Total}$. Here, it's $Total\ expenses = \frac{\$790}{40\%}$, which is $Total\ expenses = \frac{\$790}{0.4}$. When we calculate this, we find out that Bob's total expenses were \$1,975.

6. To calculate spending on food, which is 8% of the total, we multiply the total expenses by the percentage for food. That is, $\$1{,}975 \times 8\%$ (or 0.08). This calculation gives us the food expense, which is \$158.

7. Similarly, for bills, which are 37% of total expenses, we multiply \$1,975 by 37% (or 0.37). This gives us the expense for bills, amounting to \$730.75.

8. Finally, for the car expenses, 10% of the total, we do the same: multiply \$1,975 by 10% (or 0.1). This results in car expenses of \$197.50.

9. Looking at your data: $1 \rightarrow \$12, 2 \rightarrow \$8, 3 \rightarrow \$14, 4 \rightarrow \$5, 5 \rightarrow \$7, 6 \rightarrow \$10, 7 \rightarrow \$4, 8 \rightarrow \$12$, it doesn't consistently go up or down as the sequence progresses. The values fluctuate up and down without a clear direction. This indicates that the scatter plot does not show a positive or negative trend; instead, it shows no clear trend.

10. In Bag A, there are 6 green marbles, which are our favorable outcomes. The total number of marbles in Bag A is 8 red $+ 6$ green $= 14$ marbles. So, the probability of picking a green marble is 6 (green marbles) divided by 14 (total marbles), which gives us $\frac{6}{14}$. Simplifying this, we get $\frac{3}{7}$.

In Bag B, there are 5 black marbles, which are our favorable outcomes. The total number of marbles in Bag B is 5 black $+ 7$ orange $= 12$ marbles. Therefore, the probability of picking a black marble is 5 (black marbles) divided by 12 (total marbles), which is $\frac{5}{12}$.

11. Susan has 4 ingredients (sugar, flour, butter, eggs). The number of ways to arrange 4 items is calculated by multiplying the number of choices for each position: 4 choices for the first, then 3, then 2, then 1. This is $4 \times 3 \times 2 \times 1 = 24$ different orders.

12. Jason has 3 activities (museum, beach, volleyball). The number of ways to arrange 3 items is $3 \times 2 \times 1 = 6$ different orders.

13. Out of 6 players, one is chosen as captain and another as co-captain. The first choice has 6 options, and the second choice has 5 (as one player is already chosen as captain). This is $6 \times 5 = 30$ ways.

14. For 11 people, first place can be awarded in 11 ways, and second place in 10 ways (since one person is already chosen for first place). This is $11 \times 10 = 110$ ways.

15. Arranging 5 students is like choosing positions for each one: 5 choices for the first, 4 for the second, and so on. This is $5 \times 4 \times 3 \times 2 \times 1 = 120$ ways.

16. This is a combination problem, as the order doesn't matter. The formula for combinations is $nC_r = \frac{n!}{r!(n-r)!}$, where n is the total number and r is the number chosen. For 12 out of 15, it's $\binom{15}{12}$ which is $\frac{15!}{12!\times(15-12)!}$.

This simplifies to $\frac{15\times14\times13}{3\times2\times1} = 455$ ways.

17. The sum of x is 93 and the sum of y is 32. The sum of x squared is 1,889 and the sum of y squared is 264. The sum of the products of x and y is 692. Apply the correlation formula: $r = \frac{1}{n-1} . \sum_{i=1}^{n} \frac{(x_i-\bar{x})(y_i-\bar{y})}{s_x s_y}$, where n is 5 (the number of data points). The calculation yields a correlation coefficient of approximately 0.997, indicating a very strong positive linear relationship between x and y.

18. The sum of x is 260 and the sum of y is 21.5. The sum of x squared is 13,530 and the sum of y squared is 92.55. The sum of the products of x and y is 1,119.

Using the same formula as above, the correlation coefficient is calculated as 1.0, indicating a perfect positive linear relationship between x and y.

19. To determine the linear regression equation from a set of data, we use the formula $y = mx + b$, where m is the slope and b is the y-intercept of the line. The slope represents how much y increases for a one-unit increase in x, and the y-intercept is the value of y when x is 0. For the first dataset, the linear regression equation is calculated as follows: the slope (m) is approximately 1.476, meaning for each unit increase in x, y increases by about 1.476 units. The y-intercept (b) is approximately -0.143, which is the value of y when x is 0. Therefore, the linear regression equation for the first dataset is approximately $y = 1.47x - 0.14$.

20. For the second dataset, the linear regression equation is: the slope (m) is approximately 1.35. The y-intercept (b) is approximately 1.5. Thus, the linear regression equation for the second dataset is approximately $y = 1.35x + 1.5$.

21. The relationship between the number of cold and snowy days and the amount of coffee consumed at the ski resort likely shows correlation but not necessarily causation. Correlation here means that as the number of cold and snowy days increases, the consumption of coffee at the ski resort may also increase. This correlation could be due to people seeking warmth and comfort from hot beverages like coffee during cold conditions. However, this doesn't imply causation, which would mean that cold and snowy days directly cause an increase in coffee consumption. There could be other factors influencing coffee consumption, such as personal preferences or the presence of other warm beverages.

22. The relationship between the number of miles traveled and the amount of gas used typically reflects both correlation and causation. This is because as you travel more miles, you logically use more gas – this is a direct cause-and-effect relationship. The correlation is evident as the increase in one variable (miles traveled) is associated with an increase in the other (gas used). The causation is clear as the physical act of traveling miles in a gas-powered vehicle consumes fuel. This relationship is a straightforward example of causation where one variable directly affects the other.

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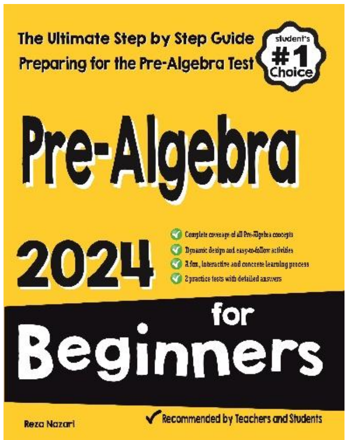

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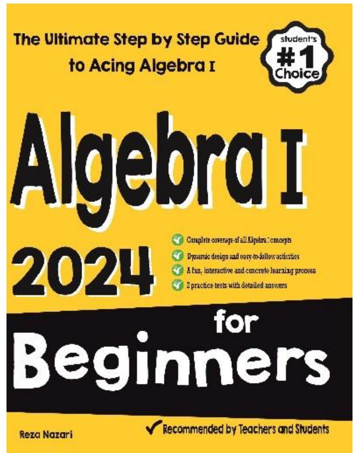

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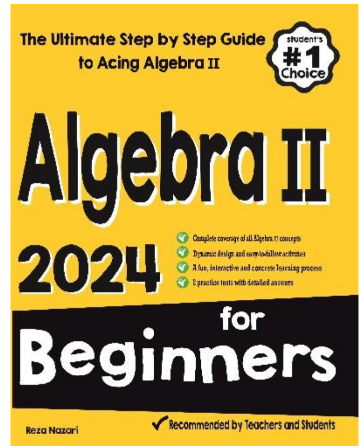

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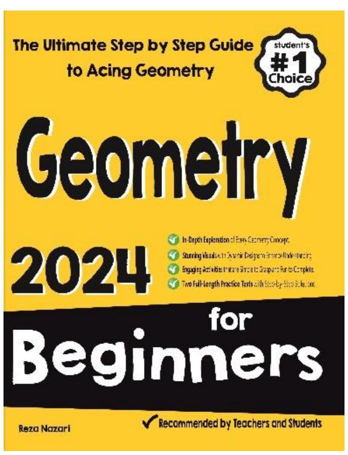

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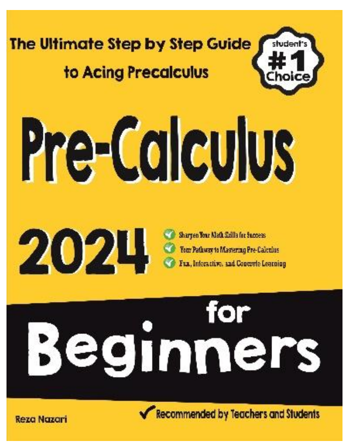

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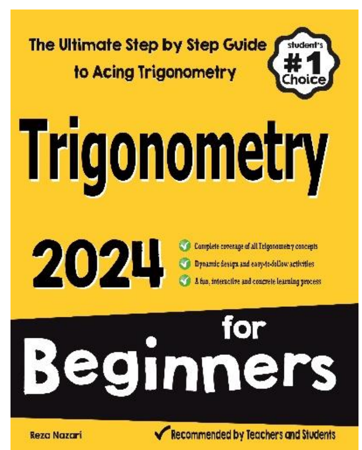

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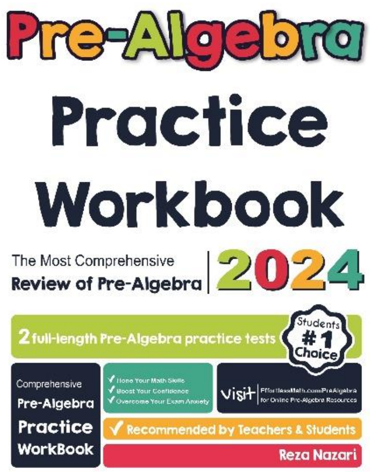

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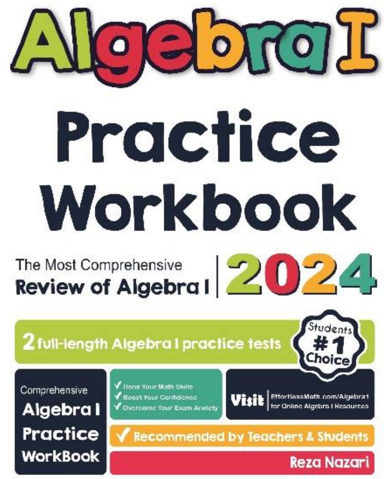

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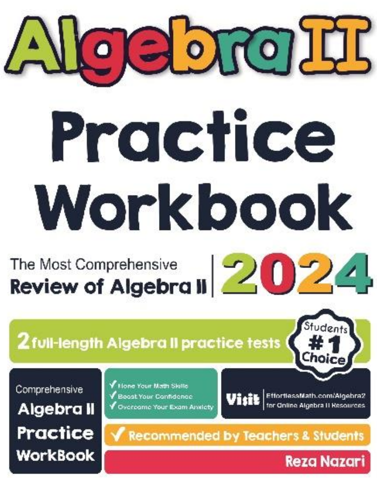

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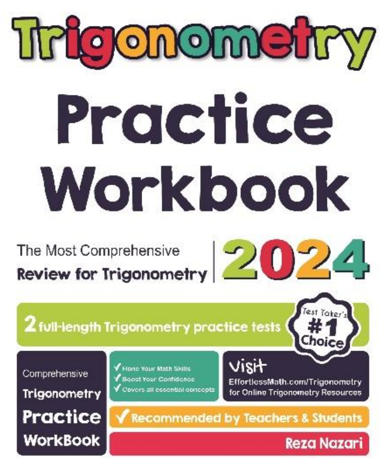

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