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TSI Math for Beginners

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TSI MATH FOR BEGINNERS

The Ultimate Step by Step Guide to Preparing for the TSI Math Test

Answers and Solutions

By

Reza Nazari

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CHAPTER

1 Fractions and Mixed Numbers

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- ☑ Adding and Subtracting Fractions
- ☑ Multiplying and Dividing Fractions
- ☑ Adding Mixed Numbers
- ☑ Subtracting Mixed Numbers
- ☑ Multiplying Mixed Numbers
- ☑ Dividing Mixed Numbers

Practices

Simplify each fraction.

1) $\frac{2}{8} =$

2) $\frac{5}{15} =$

3) $\frac{10}{90} =$

4) $\frac{12}{16} =$

5) $\frac{25}{45} =$

6) $\frac{42}{54} =$

7) $\frac{48}{60} =$

8) $\frac{52}{169} =$

Find the sum or difference.

9) $\frac{3}{10} + \frac{2}{10} =$

10) $\frac{4}{9} - \frac{1}{9} =$

11) $\frac{2}{3} + \frac{6}{15} =$

12) $\frac{17}{24} - \frac{5}{8} =$

13) $\frac{7}{54} - \frac{1}{9} =$

14) $\frac{4}{5} - \frac{1}{6} =$

15) $\frac{6}{7} - \frac{3}{8} =$

16) $\frac{2}{13} + \frac{1}{4} =$

Find the products or quotients.

17) $\frac{2}{9} \div \frac{4}{3} =$

18) $\frac{14}{5} \div \frac{28}{35} =$

19) $\frac{9}{25} \times \frac{5}{27} =$

20) $\frac{65}{72} \times \frac{12}{15} =$

Find the sum.

21) $2\frac{1}{5} + 1\frac{2}{5} =$

22) $5\frac{1}{9} + 2\frac{7}{9} =$

23) $2\frac{3}{4} + 1\frac{1}{8} =$

24) $2\frac{2}{7} + 4\frac{1}{21} =$

25) $5\frac{3}{5} + 1\frac{4}{9} =$

26) $3\frac{3}{11} + 4\frac{6}{7} =$

Find the difference.

27) $5\frac{1}{3} - 4\frac{2}{3} =$
28) $4\frac{7}{10} - 1\frac{3}{10} =$
29) $3\frac{1}{3} - 2\frac{2}{9} =$
30) $6\frac{1}{2} - 3\frac{1}{3} =$
31) $4\frac{3}{4} - 2\frac{1}{28} =$
32) $4\frac{2}{7} - 3\frac{1}{6} =$
33) $5\frac{3}{10} - 3\frac{3}{4} =$
34) $6\frac{9}{20} - 2\frac{1}{3} =$

Find the products.

35) $1\frac{1}{2} \times 2\frac{3}{7} =$
36) $1\frac{3}{4} \times 1\frac{3}{5} =$
37) $4\frac{1}{2} \times 1\frac{5}{6} =$
38) $1\frac{2}{7} \times 3\frac{1}{5} =$
39) $2\frac{1}{5} \times 5\frac{1}{2} =$
40) $2\frac{1}{2} \times 4\frac{4}{5} =$
41) $3\frac{1}{5} \times 4\frac{1}{2} =$
42) $4\frac{9}{10} \times 4\frac{1}{2} =$

Solve.

43) $1\frac{1}{3} \div 1\frac{2}{3} =$
44) $2\frac{1}{4} \div 1\frac{1}{2} =$
45) $5\frac{1}{3} \div 3\frac{1}{2} =$
46) $3\frac{2}{7} \div 1\frac{1}{8} =$
47) $4\frac{1}{5} \div 2\frac{2}{3} =$
48) $1\frac{2}{3} \div 1\frac{3}{8} =$
49) $4\frac{1}{2} \div 2\frac{2}{3} =$
50) $1\frac{2}{11} \div 1\frac{1}{8} =$

Answers

1) $\frac{1}{4}$
2) $\frac{1}{3}$
3) $\frac{1}{9}$
4) $\frac{3}{4}$
5) $\frac{5}{9}$
6) $\frac{7}{9}$
7) $\frac{4}{5}$
8) $\frac{4}{13}$
9) $\frac{1}{2}$
10) $\frac{1}{3}$
11) $\frac{16}{15} = 1\frac{1}{15}$
12) $\frac{1}{12}$
13) $\frac{1}{54}$
14) $\frac{19}{30}$
15) $\frac{27}{56}$
16) $\frac{21}{52}$
17) $\frac{1}{6}$
18) $\frac{7}{2} = 3\frac{1}{2}$
19) $\frac{1}{15}$
20) $\frac{13}{18}$
21) $3\frac{3}{5}$
22) $7\frac{8}{9}$
23) $3\frac{7}{8}$
24) $6\frac{1}{3}$
25) $7\frac{2}{45}$
26) $8\frac{10}{77}$
27) $\frac{2}{3}$
28) $3\frac{2}{5}$
29) $1\frac{1}{9}$
30) $3\frac{1}{6}$
31) $2\frac{5}{7}$
32) $1\frac{5}{42}$
33) $1\frac{11}{20}$
34) $4\frac{7}{60}$
35) $3\frac{9}{14}$
36) $2\frac{4}{5}$
37) $8\frac{1}{4}$
38) $4\frac{4}{35}$
39) $12\frac{1}{10}$
40) 12
41) $14\frac{2}{5}$
42) $22\frac{1}{20}$
43) $\frac{4}{5}$
44) $1\frac{1}{2}$
45) $1\frac{11}{21}$
46) $2\frac{58}{63}$
47) $1\frac{23}{40}$
48) $1\frac{7}{33}$
49) $1\frac{11}{16}$
50) $1\frac{5}{99}$

Answers and Explanations

1. We can simplify the fraction $\frac{2}{8}$ by finding a common factor of both numbers and dividing them by it. For 2 and 8, the common factor is 2. $2 \div 2 = 1$, and $8 \div 2 = 4$. So, $\frac{2}{8}$ simplifies to $\frac{1}{4}$.

2. Here, we can observe that both numbers are multiples of 5: $5 \div 5 = 1$, and $15 \div 5 = 3$. Thus, $\frac{5}{15}$ is equivalent to $\frac{1}{3}$.

3. For 10 and 90, they share a factor of 10. Then, we have: $10 \div 10 = 1$, and $90 \div 10 = 9$. Therefore, $\frac{10}{90}$ reduces to $\frac{1}{9}$.

4. To simplify $\frac{12}{16}$, we can notice that both numbers are even. The highest even number that can divide both is 4. So, $12 \div 4 = 3$, and $16 \div 4 = 4$. This means $\frac{12}{16}$ can be simplified to $\frac{3}{4}$.

5. 25 and 45 have a common factor of 5. We get: $25 \div 5 = 5$, and $45 \div 5 = 9$. Hence, $\frac{25}{45}$ becomes $\frac{5}{9}$.

6. Both 42 and 54 can be divided by 6. We have: $42 \div 6 = 7$, and $54 \div 6 = 9$. So, $\frac{42}{54}$ simplifies to $\frac{7}{9}$.

7. The biggest number that can evenly divide both 48 and 60 is 12. So, $48 \div 12 = 4$, and $60 \div 12 = 5$. Thus, $\frac{48}{60}$ can be represented as $\frac{4}{5}$.

8. Recognizing that 169 is 13 squared, we can see that 52 is divisible by 13. $52 \div 13 = 4$, and also $169 \div 13 = 13$. So, $\frac{52}{169}$ is simplified to $\frac{4}{13}$.

9. When the denominators (the bottom numbers) are the same, you can simply add the numerators (the top numbers). Here, both fractions have a denominator of 10: $3+2=5$. So, $\frac{3}{10}+\frac{2}{10}=\frac{5}{10}$. Simplifying $\frac{5}{10}$ gives $\frac{1}{2}$ because both 5 and 10 are divisible by 5.

10. Similarly, with the same denominators, just subtract the numerators. $4-1=3$. So, $\frac{4}{9}-\frac{1}{9}=\frac{3}{9}$. When simplified, $\frac{3}{9}$ becomes $\frac{1}{3}$ as both 3 and 9 are divisible by 3.

11. To add fractions with different denominators, find a common denominator. The smallest common multiple of 3 and 15 is 15. Convert $\frac{2}{3}$ to have a denominator of 15 by multiplying top and bottom by 5. So, $\frac{2}{3}\times\frac{5}{5}=\frac{10}{15}$. Now, add the numerators: $10+6=16$.

Therefore, $\frac{2}{3}+\frac{6}{15}=\frac{10}{15}+\frac{6}{15}=\frac{16}{15}$ or $1\frac{1}{15}$.

12. Convert both fractions to have a common denominator, which is 24. $\frac{5}{8}$ can be changed to $\frac{15}{24}$ by multiplying by $\frac{3}{3}$. Now, subtract the numerators: $17-15=2$. So, $\frac{17}{24}-\frac{5}{8}=\frac{17}{24}-\frac{15}{24}=\frac{2}{24}$, which simplifies to $\frac{1}{12}$.

13. To make the denominators match, convert $\frac{1}{9}$ into a fraction with a 54 denominator. Multiply $\frac{1}{9}$ by $\frac{6}{6}$ to get $\frac{6}{54}$. Now, $7-6=1$. So, $\frac{7}{54}-\frac{1}{9}=\frac{7}{54}-\frac{6}{54}=\frac{1}{54}$.

14. The least common multiple of 5 and 6 is 30. Convert $\frac{4}{5}$ to $\frac{24}{30}$ and $\frac{1}{6}$ to $\frac{5}{30}$. So, $\frac{4}{5}-\frac{1}{6}=\frac{24}{30}-\frac{5}{30}=\frac{24-5}{30}=\frac{19}{30}$.

15. The least common multiple of 7 and 8 is 56. Convert $\frac{6}{7}$ to $\frac{48}{56}$ and $\frac{3}{8}$ to $\frac{21}{56}$. Thus, $\frac{6}{7}-\frac{3}{8}=\frac{48}{56}-\frac{21}{56}=\frac{48-21}{56}=\frac{27}{56}$.

16. For denominators of 13 and 4, the smallest common multiple is 52. Convert $\frac{2}{13}$ to $\frac{8}{52}$ and $\frac{1}{4}$ to $\frac{13}{52}$. Then, $8 + 13 = 21$, resulting in $\frac{2}{13} + \frac{1}{4} = \frac{21}{52}$.

17. For dividing fractions, you can multiply the first fraction by the reciprocal of the second fraction. The reciprocal is simply flipping the numerator and the denominator. $\frac{2}{9} \times \frac{3}{4}$. Now, multiply the numerators together and the denominators together: $\frac{2}{9} \times \frac{3}{4} = \frac{6}{36}$. Simplify: $\frac{6}{36} = \frac{1}{6}$ (since 6 can be divided from both the numerator and the denominator).

18. For this division, multiply the first fraction by the reciprocal of the second fraction. $\frac{14}{5} \times \frac{35}{28} = \frac{490}{140}$. But this fraction can be made simpler! Both 490 and 140 can be divided by 70. Doing that gives us $\frac{7}{2} = 3\frac{1}{2}$.

19. For multiplying fractions, simply multiply the numerators together and the denominators together. $\frac{9}{25} \times \frac{5}{27} = \frac{9}{5\times5} \times \frac{5}{3\times9} = \frac{1}{5} \times \frac{1}{3} = \frac{1}{15}$.

20. Before multiplying, let's simplify the fractions to make our multiplication easier. $\frac{12}{15}$ can be simplified to $\frac{4}{5}$ by dividing both the numerator and the denominator by 3. Multiply the simplified fractions: $\frac{65}{72} \times \frac{4}{5} = \frac{13\times5}{4\times18} \times \frac{4}{5} = \frac{13}{18}$.

21. When adding mixed numbers, we add whole numbers together and then fractions separately. $2 + 1 = 3$ (Whole numbers), $\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$ (fractions). So, the result is $3\frac{3}{5}$.

22. For the whole number part, $5 + 2 = 7$. When you add the fractions $\frac{1}{9}$ and $\frac{7}{9}$, you get $\frac{8}{9}$. So, the result is $7\frac{8}{9}$.

23. Adding the whole numbers first: $2 + 1 = 3$. Then, for the fractions: $\frac{3}{4}$ is the same as $\frac{6}{8}$. So, $\frac{6}{8} + \frac{1}{8} = \frac{7}{8}$. Combining, we get $3\frac{7}{8}$.

24. Combine the whole numbers: $2+4=6$. For the fractions, $\frac{2}{7}$ can be expressed as $\frac{6}{21}$. So, $\frac{6}{21}+\frac{1}{21}=\frac{7}{21}$, which simplifies to $\frac{1}{3}$. Thus, the sum is $6\frac{1}{3}$.

25. Begin with the whole numbers: $5+1=6$. Now, for the fractions: convert $\frac{3}{5}$ to $\frac{27}{45}$ and $\frac{4}{9}$ to $\frac{20}{45}$. Adding these gives $\frac{47}{45}$, which is 1 and $\frac{2}{45}$ when simplified. Adding the 1 to the 6, we get $7\frac{2}{45}$.

26. First, total the whole numbers: $3+4=7$. Next, convert the fractions to have common denominators. $\frac{3}{11}$ is equivalent to $\frac{21}{77}$, and $\frac{6}{7}$ equals $\frac{66}{77}$. Adding these fractions results in $\frac{87}{77}$, which is 1 and $\frac{10}{77}$. Including this to the whole number gives $8\frac{10}{77}$.

27. Whole numbers: $5-4=1$. Fractions: $\frac{1}{3}-\frac{2}{3}=-\frac{1}{3}$. Since this is negative, we'll borrow 1 from our whole number result. This makes our whole number 0 and our fraction $\frac{4}{3}$. Now, $\frac{4}{3}-\frac{2}{3}=\frac{2}{3}$.

28. Start by subtracting the fractions. $\frac{7}{10}-\frac{3}{10}=\frac{4}{10}$, which can be simplified to $\frac{2}{5}$. Then subtract the whole numbers: $4-1=3$. So, the answer is $3\frac{2}{5}$.

29. To subtract the fractions, find a common denominator, which is 9. $\frac{1}{3}$ is the same as $\frac{3}{9}$. Now, $\frac{1}{3}-\frac{2}{9}=\frac{3}{9}-\frac{2}{9}=\frac{1}{9}$. Subtract the whole numbers: $3-2=1$. So, the result is $1\frac{1}{9}$.

30. Using the knowledge from the previous steps, first transform $\frac{1}{2}$ to $\frac{3}{6}$ and $\frac{1}{2}$ to $\frac{2}{6}$. Now, $\frac{3}{6}-\frac{2}{6}=\frac{1}{6}$. Subtracting whole numbers, $6-3=3$. So, the answer is $3\frac{1}{6}$.

31. The denominators 4 and 28 aren't as straightforward. Convert $\frac{3}{4}$ to $\frac{21}{28}$. Now, $\frac{21}{28}-\frac{1}{28}=\frac{20}{28}$, which simplifies to $\frac{5}{7}$. Subtract the whole numbers: $4-2=2$. Answer: $2\frac{5}{7}$.

32. With different denominators, convert $\frac{2}{7}$ to $\frac{12}{42}$ and $\frac{1}{6}$ to $\frac{7}{42}$. Now, $\frac{12}{42}-\frac{7}{42}=\frac{5}{42}$. For the whole numbers: $4-3=1$. So, the answer is $1\frac{5}{42}$.

33. First, find a common denominator. In this case, it´s 20. $\frac{3}{10}$ is equivalent to $\frac{6}{20}$ and $\frac{3}{4}$ is equivalent to $\frac{15}{20}$. Now, $\frac{6}{20}-\frac{15}{20}=-\frac{9}{20}$, which is negative, so we borrow 1 from the whole number. $5-1=4$ (After borrowing). And $1+\frac{6}{20}=\frac{20+6}{20}=\frac{26}{20}$. Next, $\frac{26}{20}-\frac{15}{20}=\frac{11}{20}$. Now, subtract the whole numbers: $4-3=1$. So, the final answer is $1\frac{11}{20}$.

34. Convert $\frac{1}{3}$ to $\frac{20}{60}$. Convert $\frac{9}{20}$ to $\frac{27}{60}$. Subtracting, $\frac{27}{60}-\frac{20}{60}=\frac{7}{60}$. For whole numbers: $6-2=4$. So, the result is $4\frac{7}{60}$.

35. First, convert the mixed numbers to improper fractions. $1\frac{1}{2}=\frac{3}{2}$ and $2\frac{3}{7}=\frac{17}{7}$. Multiply the numerators $(3\times 17=51)$ and denominators $(2\times 7=14)$. This gives $\frac{51}{14}$, which simplifies to $3\frac{9}{14}$.

36. Convert: $1\frac{3}{4}=\frac{7}{4}$ and $1\frac{3}{5}=\frac{8}{5}$. Multiplying the numerators $(7\times 8=56)$ and denominators $(4\times 5=20)$ yields $\frac{56}{20}$, which is $2\frac{16}{20}$ or $2\frac{4}{5}$ when simplified.

37. For this, convert and then multiply. $4\frac{1}{2}$ becomes $\frac{9}{2}$ and $1\frac{5}{6}$ becomes $\frac{11}{6}$. Our result is $\frac{99}{12}$. Breaking that down, we get $8\frac{3}{12}$, which is $8\frac{1}{4}$ in its simplest form.

38. Make them improper: $1\frac{2}{7}=\frac{9}{7}$ and $3\frac{1}{5}=\frac{16}{5}$. After multiplication, we get $\frac{144}{35}$. This equates to $4\frac{4}{35}$.

39. Transforming: $2\frac{1}{5}=\frac{11}{5}$ and $5\frac{1}{2}=\frac{11}{2}$. Multiplying yields $\frac{121}{10}$, which is the same as $12\frac{1}{10}$.

40. Upon conversion, $2\frac{1}{2}$ is $\frac{5}{2}$ and $4\frac{4}{5}$ is $\frac{24}{5}$. The product is $\frac{120}{10}$, which simplifies to 12.

41. In improper form: $3\frac{1}{5} = \frac{16}{5}$ and $4\frac{1}{2} = \frac{9}{2}$. Multiplying, we get $\frac{144}{10}$, which is $14\frac{4}{10}$ or $14\frac{2}{5}$.

42. This turns into $\frac{49}{10}$ and $\frac{9}{2}$. The multiplication results in $\frac{441}{20}$, which breaks down to $22\frac{1}{20}$.

43. First, convert both mixed numbers to improper fractions. $1\frac{1}{3} = \frac{4}{3}$ and $1\frac{2}{3} = \frac{5}{3}$. Now, divide by multiplying with the reciprocal of the second fraction: $\frac{4}{3} \times \frac{3}{5} = \frac{12}{15}$. Simplifying gives: $\frac{4}{5}$.

44. Transform to improper fractions: $2\frac{1}{4} = \frac{9}{4}$, $1\frac{1}{2} = \frac{3}{2}$. Perform division: $\frac{9}{4} \times \frac{2}{3} = \frac{18}{12}$. On simplification: $\frac{3}{2}$ or $1\frac{1}{2}$.

45. Change to improper fractions: $5\frac{1}{3} = \frac{16}{3}$, $3\frac{1}{2} = \frac{7}{2}$. Divide the two: $\frac{16}{3} \times \frac{2}{7} = \frac{32}{21}$.

Simplified form: $1\frac{11}{21}$.

46. Convert: $3\frac{2}{7} = \frac{23}{7}$, $1\frac{1}{8} = \frac{9}{8}$. Perform division: $\frac{23}{7} \times \frac{8}{9} = \frac{184}{63}$. This is equal to $2\frac{58}{63}$.

47. Change numbers: $4\frac{1}{5} = \frac{21}{5}$, $2\frac{2}{3} = \frac{8}{3}$. Execute division: $\frac{21}{5} \times \frac{3}{8} = \frac{63}{40}$. Final result is $1\frac{23}{40}$.

48. Adjust to improper form: $1\frac{2}{3} = \frac{5}{3}$, $1\frac{3}{8} = \frac{11}{8}$. Carry out division: $\frac{5}{3} \times \frac{8}{11} = \frac{40}{33}$. The result is $1\frac{7}{33}$.

49. Modify to improper fractions: $4\frac{1}{2} = \frac{9}{2}$, $2\frac{2}{3} = \frac{8}{3}$. Implement division: $\frac{9}{2} \times \frac{3}{8} = \frac{27}{16}$. That's $1\frac{11}{16}$.

50. Turn to improper fractions: $1\frac{2}{11} = \frac{13}{11}$, $1\frac{1}{8} = \frac{9}{8}$. Execute division: $\frac{13}{11} \times \frac{8}{9} = \frac{104}{99}$. Concluding result: $1\frac{5}{99}$.

CHAPTER

2 Decimals

Math topics in this chapter:

- ☑ Comparing Decimals
- ☑ Rounding Decimals
- ☑ Adding and Subtracting Decimals
- ☑ Multiplying and Dividing Decimals

Practices

Compare. Use >, =, and <

1) 0.5 □ 0.6
2) 0.9 □ 0.8
3) 0.1 □ 0.2
4) 0.02 □ 0.06
5) 0.05 □ 0.08
6) 0.12 □ 0.09
7) 3.2 □ 2.5
8) 4.8 □ 8.4
9) 0.005 □ 0.05
10) 2.02 □ 20.020
11) 55.100 □ 55.10
12) 0.44 □ 0.440
13) 6.01 □ 6.0100
14) 0.77 □ 77.0

Round each decimal to the nearest whole number.

15) 5.8
16) 6.4
17) 12.3
18) 9.2
19) 7.6
20) 22.4
21) 6.8
22) 15.9
23) 13.41
24) 16.78
25) 67.58
26) 42.67
27) 55.89
28) 14.32
29) 78.88
30) 98.29

Find the sum or difference.

31) $12.1 + 36.2 =$
32) $56.3 - 22.2 =$
33) $45.1 + 12.8 =$
34) $27.9 - 16.4 =$
35) $98.8 - 56.6 =$
36) $28.45 + 13.22 =$
37) $16.78 + 45.11 =$
38) $86.16 - 72.12 =$
39) $96.23 - 28.32 =$
40) $57.33 + 67.46 =$
41) $46.26 - 39.49 =$
42) $44.95 + 76.53 =$
43) $79.37 - 52.89 =$
44) $19.99 + 28.7 =$
45) $83.48 - 49.3 =$
46) $19.6 + 42.98 =$

Find the product or quotient.

47) $3.3 \times 0.2 =$
48) $2.4 \div 0.3 =$
49) $8.1 \times 1.4 =$
50) $4.8 \div 0.2 =$
51) $4.1 \times 0.3 =$
52) $8.6 \div 0.2 =$
53) $9.9 \times 0.8 =$
54) $1.84 \div 0.2 =$
55) $2.1 \times 8.4 =$
56) $1.6 \times 4.5 =$
57) $9.2 \times 3.1 =$
58) $36.6 \div 1.6 =$
59) $1.91 \times 5.2 =$
60) $3.65 \times 1.4 =$
61) $24.82 \div 0.4 =$
62) $12.4 \times 4.20 =$

Answers

1) <
2) >
3) <
4) <
5) <
6) >
7) >
8) <
9) <
10) <
11) =
12) =
13) =
14) <
15) 6
16) 6
17) 12
18) 9
19) 8
20) 22
21) 7
22) 16
23) 13
24) 17
25) 68
26) 43
27) 56
28) 14
29) 79
30) 98
31) 48.3
32) 34.1
33) 57.9
34) 11.5
35) 42.2
36) 41.67
37) 61.89
38) 14.04
39) 67.91
40) 124.79
41) 6.77
42) 121.48
43) 26.48
44) 48.69
45) 34.18
46) 62.58
47) 0.66
48) 8
49) 11.34
50) 24
51) 1.23
52) 43
53) 7.92
54) 9.2
55) 17.64
56) 7.2
57) 28.52
58) 22.875
59) 9.932
60) 5.11
61) 62.05
62) 52.08

Answers and Explanations

1. Look at the first number after the decimal point: 0.5 has 5 while 0.6 has 6. Since 5 is smaller than 6, 0.5 is less than 0.6. So, $0.5 < 0.6$.

2. Examine the first digit after the point: 0.9 has 9 and 0.8 has 8. As 9 is greater than 8, 0.9 is bigger than 0.8. So, $0.9 > 0.8$.

3. Focus on the initial digit post the dot: for 0.1 it's 1 and for 0.2 it's 2. Clearly, 1 is not as big as 2, so 0.1 is smaller and the answer is: $0.1 < 0.2$.

4. Moving to the second place after the decimal: 0.02 has 2, while 0.06 has 6. Given that 2 is not as high as 6, 0.02 is less. So, $0.02 < 0.06$.

5. Again, observe the second digit after the point: in 0.05, it's 5, whereas in 0.08, it's 8. Since 5 doesn't exceed 8, 0.05 is the smaller number. Answer: $0.05 < 0.08$.

6. Initially, focus on the first digit after the point: 0.12 has 1 and 0.09 has 0. Since $1 > 0$, you might think 0.12 is greater, but to be thorough, check the next digit. For 0.12, it's 2 and for 0.09, it's 9. But since the first digit already determined the larger number, no need to compare further. So, $0.12 > 0.09$.

7. First, compare the whole numbers: 3 in 3.2 and 2 in 2.5. As 3 surpasses 2, 3.2 is certainly bigger than 2.5 without needing to look at the decimals. Answer: $3.2 > 2.5$.

8. Start by looking at the whole numbers: 4 in 4.8 and 8 in 8.4. Given that 4 doesn't come close to 8, 4.8 is evidently smaller than 8.4. Answer: $4.8 < 8.4$.

9. When comparing these two decimals, we look at each digit's position. The first number has its highest value in the thousandths place, while the second has its highest value in the hundredths place. Any non-zero digit in the hundredths place is larger than any digit in the thousandths place. Thus, $0.005 < 0.05$.

10. Starting from the left, the first number starts with a 2 in the ones place, while the second number starts with a 20 in the tens place. This immediately makes the second number ten times larger than the first. The decimal portions only reinforce this difference. So: $2.02 < 20.020$.

11. Decimal numbers can have trailing zeros after the decimal point without changing their value. In this case, 55.100 and 55.10 represent the same value. Thus, $55.100 = 55.10$.

12. Both numbers have a 4 in the tenths place. Moving to the hundredths place, they both have a 4 again. Even though 0.440 has an extra 0 in the thousandths place, this does not change its value, so the two numbers are equivalent. $0.44 = 0.440$

13. Considering the place values, both numbers have a 6 in the ones place. Moving to the right, both have a 0 in the tenths and a 1 in the hundredths place. The additional zeros in 6.0100 at the thousandths and ten-thousandths places do not alter its value. Hence, $6.01 = 6.0100$.

14. In the number 0.77, the 7 is in the tenths place, meaning it's less than one. In contrast, for 77.0, the first 7 is in the tens place, representing a value of seventy. Clearly, seventy is much greater than a fraction that's less than one. So, $0.77 < 77.0$.

15. Rounding involves looking at the number immediately to the right of the decimal point. If it's 5 or greater, we increase the whole number by 1.

Here, 8 is greater than 5, so 5.8 rounds up to 6.

16. For rounding, a key number to remember is 5. Numbers less than 5 cause the whole number to remain the same. Since 4 is less than 5, 6.4 rounds down to 6.

17. We're focusing on the first digit after the decimal point, which is 3. As it's below 5, the number stays as 12 when rounded.

18. The number immediately after the decimal is 2. Given that 2 is smaller than 5, we don't add anything to the whole number. So, 9.2 remains 9.

19. By observing the digit right after the decimal (6 in this case), and noting that it's 5 or more, we bump the whole number up. So, 7 becomes 8.

20. When the number after the decimal is smaller than 5, like the 4 here, we keep the whole number as is. Thus, 22.4 rounds to 22.

21. Here, the number 8 is greater than 5. This means we raise the whole number part up by one, so 6 turns into 7.

22. We determine the rounding based on the first number after the decimal. With 9 being greater than 5, 15 increases to 16.

23. Although there are more numbers after the decimal, we only need to consider the first one for rounding. Here, 4 causes 13 to remain the same.

24. The first digit after the point is 7. As it's over 5, we elevate the 16 to 17.

25. We're concerned with the 5 immediately following the decimal. Given that it's exactly 5, we raise 67 by one, resulting in 68.

26. Checking the first digit after the decimal, we see it's 6. This pushes 42 up to 43.

27. The number 8 is greater than 5. Therefore, the whole part, 55, gets a boost of one, turning it into 56.

28. With 3 being our key number (as it's directly after the decimal), and it being less than 5, 14 remains unchanged.

29. Observing the first digit after the point, 8 tells us to increment 78 by one, so it becomes 79.

30. By focusing on the 2, which is less than 5, 98 remains the same when rounded.

31. When you add numbers with decimals, line up the decimal points. Start by adding the numbers to the right of the decimal ($0.1 + 0.2 = 0.3$). Next, add the whole numbers ($12 + 36 = 48$). Put them together, and you get 48.3.

32. For subtraction, also line up the decimal points. Subtract the numbers to the right of the decimal ($0.3 - 0.2 = 0.1$). Subtract the whole numbers ($56 - 22 = 34$). Combine to get 34.1.

33. Adding from the right, $0.1 + 0.8 = 0.9$. Add the whole numbers: $45 + 12 = 57$. So, 57.9 is the answer.

34. Begin from the decimals: $0.9 - 0.4 = 0.5$. For whole numbers: $27 - 16 = 11$. Together, the answer is 11.5.

35. Subtracting the decimals: $0.8 - 0.6 = 0.2$. Subtracting the whole numbers: $98 - 56 = 42$. In total, the answer is 42.2.

36. Starting from the rightmost decimal: $0.05 + 0.02 = 0.07$. For the next position, $0.4 + 0.2 = 0.6$. Now, add the whole numbers: $28 + 13 = 41$. Summing them up, the result is 41.67.

37. Rightmost decimal addition: $0.08 + 0.01 = 0.09$. Next, $0.7 + 0.1 = 0.8$. For the whole numbers: $16 + 45 = 61$. Combined, the result is 61.89.

38. Starting from the rightmost decimal: $0.06 - 0.02 = 0.04$. Then, $0.1 - 0.1 = 0$. For the whole numbers: $86 - 72 = 14$. Altogether, the result is 14.04.

39. Beginning with the decimals: $0.03 - 0.02 = 0.01$. Next, $0.2 - 0.3 = -0.1$. Since you're borrowing, this makes the 96 become 95 and the 0.2 becomes 1.2. Now, $1.2 - 0.3 = 0.9$. For the whole numbers: $95 - 28 = 67$. So, 67.91 is the answer.

40. Starting rightmost: $0.03 + 0.06 = 0.09$. Then, $0.3 + 0.4 = 0.7$. For whole numbers: $57 + 67 = 124$. The result is 124.79.

41. Starting with decimals: $0.06 - 0.09$ needs borrowing. $0.16 - 0.09 = 0.07$. Now, $0.2 - 0.4 = -0.2$, requiring another borrow. $1.2 - 0.4 = 0.8$. Whole numbers: $45 - 39 = 6$. The result is 6.77.

42. Rightmost addition: $0.05 + 0.03 = 0.08$. Then, $0.9 + 0.5 = 1.4$ (carry the 1). Whole numbers with carry: $45 + 76 = 121$. So, 121.48 is the answer.

43. Starting with decimals: $0.07 - 0.09$ needs borrowing. $0.17 - 0.09 = 0.08$. Next, $0.2 - 0.8 = -0.5$, requiring a borrow. $1.2 - 0.8 = 0.5$. Whole numbers: $78 - 52 = 26$. Result is 26.48.

44. Rightmost addition: $0.09 + 0 = 0.09$. Then, $0.9 + 0.7 = 1.6$ (carry the 1). Whole numbers with carry: $20 + 28 = 48$. The result is 48.69.

45. Starting from the right, $0.08 - 0 = 0.08$. Then, $0.4 - 0.3 = 0.1$. Whole numbers: $83 - 49 = 34$. Result is 34.18.

46. Beginning with rightmost: $0 + 0.08 = 0.08$. Then, $0.6 + 0.9 = 1.5$ (carry the 1). Whole numbers with carry: $20 + 42 = 62$. The result is 62.58.

47. Multiplying whole numbers is straightforward. When multiplying decimals, ignore the decimal points initially, and then place the decimal back in the final answer. For this problem, treat it as $33 \times 2 = 66$. Since there are 2 total decimal places in the original numbers (1 in 3.3 and 1 in 0.2), the answer will also have 2 decimal places. Thus, 0.66 is the answer.

48. Think of division as "how many times does 0.3 fit into 2.4? "or" distribute 2.4 into 0.3-sized groups. "Dividing without decimals, it's $24 \div 3$ which is 8. Since the decimals do not affect this specific division, the answer remains 8.

49. This is similar to multiplying 81 by 14. That's $1{,}134$. There are two decimal places across the original numbers (1 in 8.1 and 1 in 1.4). So, your answer should have two decimal places: 11.34.

50. Here, think of distributing 4.8 into 0.2-sized portions. Without decimals, $48 \div 2$ is 24. So, the answer is 24.

51. For 41×3, you get 123. With 2 decimal places in total (1 in each number), the answer is 1.23.

52. You're asking "How many times does 0.2 fit into 8.6?". Without decimals, $86 \div 2$ is 43. Thus, the answer is 43.

53. This resembles 99×8, which gives 792. Two decimal places in total mean the answer is 7.92.

54. Here, distribute 1.84 into 0.2-sized groups. For $184 \div 2$, you get 92. So, the answer is 9.2.

55. Similar to 21×84 which is $1{,}764$. Two decimal places total means the answer is 17.64.

56. Like multiplying 16 and 45, you get 720. With two decimal places in total, it's 7.2.

57. This is akin to 92×31, which is $2{,}852$. Again, two decimals mean it's 28.52.

58. You're checking how many times 1.6 fits into 36.6. This is trickier; division gives you about 22.875.

59. View it as 191×52 which is $9{,}932$. Counting 3 decimals in total gives 9.932.

60. Think 365×14, yielding $5{,}110$. With three decimals, the answer is 5.11.

61. Dividing 2482 by 4, you get 620.5. With two decimals, the answer is 62.05.

62. This is like 1240×42, which is $52{,}080$. Counting 4 decimals

yield 52.08.

CHAPTER

3 Integers and Order of Operations

Math topics in this chapter:

- ☑ Adding and Subtracting Integers
- ☑ Multiplying and Dividing Integers
- ☑ Order of Operations
- ☑ Integers and Absolute Value

Practices

Find each sum or difference.

1) $-9 + 16 =$
2) $-18 - 6 =$
3) $-24 + 10 =$
4) $30 + (-5) =$
5) $15 + (-3) =$
6) $(-13) + (-4) =$
7) $25 + (3 - 10) =$
8) $12 - (-6 + 9) =$
9) $5 - (-2 + 7) =$
10) $(-11) + (-5 + 6) =$
11) $(-3) + (9 - 16) =$
12) $(-8) - (13 + 4) =$
13) $(-7 + 9) - 39 =$
14) $(-30 + 6) - 14 =$
15) $(-5 + 9) + (-3 + 7) =$
16) $(8 - 19) - (-4 + 12) =$
17) $(-9 + 2) - (6 - 7) =$
18) $(-12 - 5) - (-4 - 14) =$

Solve.

19) $3 \times (-6) =$
20) $(-32) \div 4 =$
21) $(-5) \times 4 =$
22) $(25) \div (-5) =$
23) $(-72) \div 8 =$
24) $(-2) \times (-6) \times 5 =$
25) $(-2) \times 3 \times (-7) =$
26) $(-1) \times (-3) \times (-5) =$
27) $(-2) \times (-3) \times (-6) =$
28) $(-12 + 3) \times (-5) =$
29) $(-3 + 4) \times (-11) =$
30) $(-9) \times (6 - 5) =$
31) $(-3 - 7) \times (-6) =$
32) $(-7 + 3) \times (-9 + 6) =$
33) $(-15) \div (-17 + 12) =$
34) $(-3 - 2) \times (-9 + 7) =$
35) $(-15 + 31) \div (-2) =$
36) $(-64) \div (-16 + 8) =$

Evaluate each expression.

37) $3 + (2 \times 5) =$

38) $(5 \times 4) - 7 =$

39) $(-9 \times 2) + 6 =$

40) $(7 \times 3) - (-5) =$

41) $(-8) + (2 \times 7) =$

42) $(9 - 6) + (3 \times 4) =$

43) $(-19 + 5) + (6 \times 2) =$

44) $(32 \div 4) + (1 - 13) =$

45) $(-36 \div 6) - (12 + 3) =$

46) $(-16 + 5) - (54 \div 9) =$

47) $(-20 + 4) - (35 \div 5) =$

48) $(42 \div 7) + (2 \times 3) =$

49) $(28 \div 4) + (2 \times 6) =$

50) $2[(3 \times 3) - (4 \times 5)] =$

51) $3[(2 \times 8) + (4 \times 3)] =$

52) $2[(9 \times 3) - (6 \times 4)] =$

53) $4[(4 \times 8) \div (4 \times 4)] =$

54) $-5[(10 \times 8) \div (5 \times 8)] =$

Find the answers.

55) $|-5| + |7 - 10| =$

56) $|-4 + 6| + |-2| =$

57) $|-9| + |1 - 9| =$

58) $|-7| - |8 - 12| =$

59) $|9 - 11| + |8 - 15| =$

60) $|-7 + 10| - |-8 + 3| =$

61) $|-12 + 6| - |3 - 9| =$

62) $5 + |2 - 6| + |3 - 4| =$

63) $-4 + |2 - 6| + |1 - 9| =$

64) $\frac{|-42|}{7} \times \frac{|-64|}{8} =$

65) $\frac{|-100|}{10} \times \frac{|-36|}{6} =$

66) $|4 \times (-2)| \times \frac{|-27|}{3} =$

67) $|-3 \times 2| \times \frac{|-40|}{8} =$

68) $\frac{|-54|}{6} - |-3 \times 7| =$

69) $\frac{|-72|}{8} + |-7 \times 5| =$

70) $\frac{|-121|}{11} + |-6 \times 4| =$

71) $\frac{|(-6)\times(-3)|}{9} \times \frac{|2\times(-20)|}{5} =$

72) $\frac{|(-3)\times(-8)|}{6} \times \frac{|9\times(-4)|}{12} =$

Answers

1) 7
2) -24
3) -14
4) 25
5) 12
6) -17
7) 18
8) 9
9) 0
10) -10
11) -10
12) -25
13) -37
14) -38
15) 8
16) -19
17) -6
18) 1
19) -18
20) -8
21) -20
22) -5
23) -9
24) 60
25) 42
26) -15
27) -36
28) 45
29) -11
30) -9
31) 60
32) 12
33) 3
34) 10
35) -8
36) 8
37) 13
38) 13
39) -12
40) 26
41) 6
42) 15
43) -2
44) -4
45) -21
46) -17
47) -23
48) 12
49) 19
50) -22
51) 84
52) 6
53) 8
54) -10
55) 8
56) 4
57) 17
58) 3
59) 9
60) -2
61) 0
62) 10
63) 8
64) 48
65) 60
66) 72
67) 30
68) -12
69) 44
70) 35
71) 16
72) 12

Answers and Explanations

1. Start with -9. Moving to the right on the number line by 16 units will take you to $+7$. So, $-9+16=7$.

2. From -18, moving 6 units further to the left on the number line will take you to -24. Thus, $-18-6=-24$.

3. Picture -24 as being 24 units to the left of zero. If you then move 10 units to the right, you would land on -14. So, $-24+10=-14$.

4. Think of 30 as a foundation. Adding a negative is like taking away, so you're removing 5 from 30. This results in 25. Hence, $30+(-5)=25$.

5. You're at 15 and you're moving backwards (or subtracting) by 3. This means you decrease the 15 by 3 to get 12. Therefore, $15+(-3)=12$.

6. Combine two debts: If you owe 13 dollars and then owe another 4 dollars, you end up owing 17 dollars in total. So, $(-13)+(-4)=-17$.

7. Inside the parentheses, we subtract 10 from 3, which is -7. Adding 25 to -7, you're essentially taking 7 away from 25. The result is 18. So, $25+(3-10)=25-7=18$.

8. Within the parentheses, $-6+9$ equals 3. Then, 12 minus this 3 is 9. Thus, $12-(-6+9)=12-3=9$.

9. Resolving inside the parentheses first, $-2+7$ equals 5. Now, subtracting 5 from 5 gives 0. Therefore, $5-(-2+7)=5-5=0$.

10. First, calculate what is inside the parentheses. $-5+6=1$. Next, add this result to -11: $-11+1=-10$. So, the final answer is: -10.

11. Tackle the equation inside the parentheses $(9-16)$ first. You are taking away 16 from 9. This gives you -7. Then, you're adding this result to -3. Combining -3 and -7 gives -10.

12. Firstly, resolve the addition inside the parentheses $(13 + 4)$. That's simple addition and gives 17. Now, subtract this number from -8. Instead of subtracting 17, you can think of it as adding its negative, which means $(-8) + (-17) = -25$.

13. Starting with $(-7 + 9)$, you are adding a smaller negative number to a positive one, so it will be closer to the positive number. The result is 2. Then, subtract 39 from 2 to get -37.

14. Break it down: $(-30 + 6)$ gives -24 because you're reducing a large negative number by a smaller positive one. Then, subtracting 14 from -24 takes it further negative to -38.

15. Evaluate each pair inside the parentheses: $(-5 + 9) = 4$ and $(-3 + 7) = 4$. Summing those two results gives 8.

16. Working with the first set, $(8 - 19)$ is -11 since you're taking a larger number from a smaller one. For the second set, $(-4 + 12)$ is 8. Subtract 8 from -11, and you get $-11 - 8 = -19$.

17. Initially, $(-9 + 2)$ results in -7. For the second set, $(6 - 7)$ equals -1. Subtracting -1 is the same as adding 1, so $(-7) + 1 = -6$.

18. Starting with the first parentheses, $-12 - 5 = -17$. For the second, $-4 - 14 = -18$. Subtracting -18 is like adding 18 to -17, resulting in $-17 - (-18) = -17 + 18 = 1$.

19. When you multiply a positive number with a negative number, the result is always negative. Here, 3 (positive) times 6 is 18. Because one of the numbers is negative, the result is -18.

20. Division distributes the negative sign as you'd expect: $-32 \div 4 = -8$.

21. Imagine owing 5 dollars (which is -5) and you owe it to 4 people. You'd owe 20 dollars in total. So, -5 multiplied by 4 is -20.

22. Here, we have a positive number divided by a negative number. If you had 25 apples and needed to split them into groups of -5 (like owing 5 apples), you would have -5 groups.

23. Consider having a debt of 72 dollars. If you split or distribute this debt among 8 people, each person would share an equal debt. 72 divided by 8 is 9. Since it's a debt (negative), each person would have a debt of 9 dollars. $(-72) \div 8 = -9$.

24. When you multiply two negative numbers, they become positive. So, -2 times -6 is 12. Now, take this positive 12 and multiply it by 5. This gives you 60.

25. Starting with the first two numbers: -2 multiplied by 3 gives -6 (a negative times a positive is negative). Now, multiply -6 by -7. Two negatives multiplied give a positive: 6 times 7 is 42.

26. First, multiplying -1 and -3 yields 3, because the product of two negatives is positive. However, when you then multiply this positive 3 by -5, you return to a negative value. 3 times 5 is 15, and since one number is negative, the result is -15.

27. When multiplying two negative numbers, the result is positive. So, $(-2) \times (-3) = 6$. Now, when we multiply this result (6) with another negative number (-6), the result becomes negative. Hence, $6 \times (-6) = -36$.

28. Begin by solving the expression inside the parentheses. $-12 + 3 = -9$. Now, we have to multiply this result with -5. When you multiply a negative number by a negative number, the result is positive. Therefore, $-9 \times (-5) = 45$.

29. Start with the arithmetic inside the parentheses. $-3 + 4 = 1$. Then, multiply this number by -11. Multiplying a positive number with a negative number gives a negative result. Thus, $1 \times (-11) = -11$.

30. Address the equation within the parentheses first. $6 - 5 = 1$. Now, multiply this value with -9. When a negative number is multiplied with a positive number, the result is negative. As such, $-9 \times 1 = -9$.

31. Compute the sum within the parentheses first: $-3 - 7 = -10$. Then, multiply this result by -6. As established, the product of two negative numbers is positive. Hence, $-10 \times (-6) = 60$.

32. Calculate the sums inside both sets of parentheses: $-7 + 3 = -4$ and $-9 + 6 = -3$. Now, multiplying these results, $-4 \times (-3) = 12$, as a negative time a negative produces a positive outcome.

33. First, figure out the value inside the parentheses: $-17 + 12 = -5$. Now, dividing -15 by this result gives 3, because when you divide a negative number by another negative number, the outcome is positive. Therefore, $-15 \div (-5) = 3$.

34. Start with the arithmetic inside the parentheses: $-3 - 2 = -5$ and $-9 + 7 = -2$. Multiply these two values together. As you already know, a negative multiplied by a negative result in a positive. This gives $-5 \times (-2) = 10$.

35. First, simplify the addition: $-15 + 31 = 16$. Next, divide this value by -2. When a positive number is divided by a negative number, the outcome is negative. Thus, $16 \div (-2) = -8$.

36. Work out the sum inside the parentheses: $-16 + 8 = -8$. Now, when dividing -64 by this result, you get a positive 8, because dividing one negative number by another negative number yields a positive outcome.

37. First, perform the multiplication. 2 multiplied by 5 is 10. Then, add 10 to 3. The answer is 13.

38. You start by multiplying 5 and 4, which gives you 20. Afterward, you subtract 7 from the result. Hence, 20 minus 7 is 13.

39. Multiplication is the first operation to consider here. -9 times 2 gives -18. Now, if you add 6 to -18, you will get -12.

40. Begin by multiplying 7 by 3 to get 21. Subtracting a negative is equivalent to adding its positive counterpart. So, 21 added to 5 is 26.

41. Firstly, 2 multiplied by 7 yields 14. Combine this result with −8. It's like having 14 and owing 8, leaving you with 6.

42. Deduct 6 from 9 first, which equals 3. Then multiply 3 by 4 to get 12. Add the two results, 3 and 12, together to obtain 15.

43. Simplify the addition inside the parentheses to get −14. Separately, 6 times 2 is 12. Combining −14 and 12 gives −2.

44. Divide 32 by 4 to receive 8. In the second set of parentheses, subtract 13 from 1 to get −12. Summing 8 and −12 results in −4.

45. The division of −36 by 6 results in −6. Add 12 and 3 together to get 15. Now, subtract 15 from −6, leading to −21.

46. Begin with the addition inside the parentheses to get −11. Separately, divide 54 by 9 to find 6. Now, subtract 6 from −11 to get −17.

47. Add 4 to −20, resulting in −16. Next, divide 35 by 5 to obtain 7. Taking 7 away from −16 gives −23.

48. Start by dividing 42 by 7, which is 6. Multiply 2 by 3 to get 6. Summing both results, 6 and 6, you get 12.

49. Divide 28 by 4 to yield 7. Multiplying 2 and 6 results in 12. Combine 7 and 12 for a total of 19.

50. Inside the parentheses, 3 times 3 is 9, and 4 times 5 is 20. Subtracting the latter from the former gives −11. Multiplying −11 by 2 results in −22.

51. 2 times 8 is 16, and 4 times 3 is 12. Combining 16 and 12 results in 28. When you multiply 28 by 3, you get 84.

52. In the parentheses, 9 multiplied by 3 equals 27. Then, 6 times 4 is 24. The difference between 27 and 24 is 3. When 3 is doubled, it becomes 6.

53. Within the brackets, 4 times 8 is 32, and 4 times 4 is 16. Now, 32 divided by 16 is 2. Quadrupling 2 gives 8.

54. 10 multiplied by 8 yields 80. On the other hand, 5 times 8 is 40.Now, 80 divided by 40 is 2. Multiplying 2 by -5 results in -10.

55. Absolute value is the distance of a number from zero. Thus, $|-5|$ is 5 because the distance of -5 from zero is 5 units. For $|7-10|$, perform the subtraction first: $7-10=-3$. $|-3|$ is 3. So, the answer is $5+3=8$.

56. $|-4+6|$ is the absolute value of 2, which is 2. $|-2|$ is 2. Summing them gives $2+2=4$.

57. $|-9|$ is 9. For $|1-9|$, perform subtraction: $1-9=-8$. $|-8|$ is 8. So, $9+8=17$.

58. $|-7|$ is 7. $|8-12|=|-4|$ is 4. Subtracting gives $7-4=3$.

59. $|9-11|$ is $|-2|$ which is 2. $|8-15|$ is $|-7|$ which is 7. Summing them gives $2+7=9$.

60. $|-7+10|=|3|$ is 3. $|-8+3|=|-5|$ is 5. Subtracting gives $3-5=-2$.

61. $|-12+6|=|-6|$ which is 6. $|3-9|=|-6|$ which is 6. The result is $6-6=0$.

62. $|2-6|=|-4|$ which is 4. $|3-4|=|-1|$ which is 1. Adding gives $5+4+1=10$.

63. Following the previous methodology, $|2-6|$ is 4 and $|1-9|$ is 8. Thus, $-4+4+8=8$.

64. $|-42|$ is 42 and $|-64|$ is 64. So, $\left(\frac{42}{7}\right)\times\left(\frac{64}{8}\right)=6\times 8=48$.

65. $|-100|$ is 100 and $|-36|$ is 36. Then, $\left(\frac{100}{10}\right)\times\left(\frac{36}{6}\right)=10\times 6=60$.

66. $|4\times(-2)|=|-8|$ which is 8. $|-27|$ is 27. So, $8\times\left(\frac{27}{3}\right)=8\times 9=72$.

67. $|-3\times 2|$ is $|-6|$ which is 6. $|-40|$ is 40. Thus, $6\times\left(\frac{40}{8}\right)=6\times 5=30$.

68. $|-54|$ is 54 and $|-3\times 7|$ is $|-21|$ which is 21. So, $\left(\frac{54}{6}\right)-21=9-21=-12$.

69. $|-72|$ is 72 and $|-7\times 5|$ is $|-35|$ which is 35. Thus, $\left(\frac{72}{8}\right)+35=9+35=44$.

70. $|-121|$ is 121 and $|-6\times 4|$ is $|-24|$ which is 24. So, $\left(\frac{121}{11}\right)+24=11+24=35$.

71. $|(-6) \times (-3)|$ is $|18|$ which is 18. $|2 \times (-20)|$ is $|-40|$ which is 40. Thus, $\left(\frac{18}{9}\right) \times \left(\frac{40}{5}\right) = 2 \times 8 = 16$.

72. $|(-3) \times (-8)|$ is $|24|$ which is 24. $|9 \times (-4)|$ is $|-36|$ which is 36. So, $\left(\frac{24}{6}\right) \times \left(\frac{36}{12}\right) = 4 \times 3 = 12$.

CHAPTER

4 Ratios and Proportions

Math topics in this chapter:

- ☑ Simplifying Ratios
- ☑ Proportional Ratios
- ☑ Similarity and Ratios

Practices

Reduce each ratio.

1) $2:18 =$ ___:___
2) $5:35 =$ ___:___
3) $8:72 =$ ___:___
4) $24:36 =$ ___:___
5) $25:40 =$ ___:___
6) $40:72 =$ ___:___
7) $28:63 =$ ___:___
8) $18:81 =$ ___:___
9) $13:52 =$ ___:___
10) $56:72 =$ ___:___
11) $42:63 =$ ___:___
12) $32:96 =$ ___:___

Solve.

13) Bob has 16 red cards and 20 green cards. What is the ratio of Bob's red cards to his green cards? ________

14) In a party, 34 soft drinks are required for every 20 guests. If there are 260 guests, how many soft drinks are required? ________

15) Sara has 56 blue pens and 28 black pens. What is the ratio of Sara's black pens to her blue pens? ________

16) In Jack's class, 48 of the students are tall and 20 are short. In Michael's class 28 students are tall and 12 students are short. Which class has a higher ratio of tall to short students? ________

17) The price of 6 apples at the Quick Market is $1.52. The price of 5 of the same apples at Walmart is $1.32. Which place is the better buy? _______

18) The bakers at a Bakery can make 180 bagels in 6 hours. How many bagels can they bake in 16 hours? What is that rate per hour? ________

19) You can buy 6 cans of green beans at a supermarket for $3.48. How much does it cost to buy 38 cans of green beans? ________

Solve each proportion.

20) $\frac{3}{2} = \frac{9}{x} \Rightarrow x =$ ____

21) $\frac{7}{2} = \frac{x}{4} \Rightarrow x =$ ____

22) $\frac{1}{3} = \frac{2}{x} \Rightarrow x =$ ____

23) $\frac{1}{4} = \frac{5}{x} \Rightarrow x =$ ____

24) $\frac{9}{6} = \frac{x}{2} \Rightarrow x =$ ____

25) $\frac{3}{6} = \frac{5}{x} \Rightarrow x =$ ____

26) $\frac{7}{x} = \frac{2}{6} \Rightarrow x =$ ____

27) $\frac{2}{x} = \frac{4}{10} \Rightarrow x =$ ____

28) $\frac{3}{2} = \frac{x}{8} \Rightarrow x =$ ____

29) $\frac{x}{6} = \frac{5}{3} \Rightarrow x =$ ____

30) $\frac{3}{9} = \frac{5}{x} \Rightarrow x =$ ____

31) $\frac{4}{18} = \frac{2}{x} \Rightarrow x =$ ____

32) $\frac{6}{16} = \frac{3}{x} \Rightarrow x =$ ____

33) $\frac{2}{5} = \frac{x}{20} \Rightarrow x =$ ____

34) $\frac{28}{8} = \frac{x}{2} \Rightarrow x =$ ____

35) $\frac{3}{5} = \frac{x}{15} \Rightarrow x =$ ____

36) $\frac{2}{7} = \frac{x}{14} \Rightarrow x =$ ____

37) $\frac{x}{18} = \frac{3}{2} \Rightarrow x =$ ____

38) $\frac{x}{24} = \frac{2}{6} \Rightarrow x =$ ____

39) $\frac{5}{x} = \frac{4}{20} \Rightarrow x =$ ____

40) $\frac{10}{x} = \frac{20}{80} \Rightarrow x =$ ____

41) $\frac{90}{6} = \frac{x}{2} \Rightarrow x =$ ____

Solve each problem.

42) Two rectangles are similar. The first is 8 $feet$ wide and 32 $feet$ long. The second is 12 $feet$ wide. What is the length of the second rectangle? ________________

43) Two rectangles are similar. One is 4.6 $meters$ by 7 $meters$. The longer side of the second rectangle is 28 $meters$. What is the other side of the second rectangle? __________________

Answers

1) 1: 9
2) 1: 7
3) 1: 9
4) 2: 3
5) 5: 8
6) 5: 9
7) 4: 9
8) 2: 9
9) 1: 4
10) 7: 9
11) 2: 3
12) 1: 3
13) 4: 5
14) 442
15) 1: 2
16) $Jack's\ class: \frac{48}{20} = \frac{12}{5}$ $Michael's\ class: \frac{28}{12} = \frac{7}{3}$ Jack's class has a higher ratio of tall to short student: $\frac{12}{5} > \frac{7}{3}$
17) Quick market
18) 480, 30 bagels per hour
19) $22.04
20) 6
21) 14
22) 6
23) 20
24) 3
25) 10
26) 21
27) 5
28) 12
29) 10
30) 15
31) 9
32) 8
33) 8
34) 7
35) 9
36) 4
37) 27
38) 8
39) 25
40) 40
41) 30
42) 48 $feet$
43) 18.4 $meters$

Answers and Explanations

1. The common factor of 2 and 18 is 2. Divide both numbers by 2: $2 \div 2 = 1$, and $18 \div 2 = 9$. So, $2:18$ is reduced to $1:9$.

2. Looking for the largest number that both 5 and 35 are divisible by, it's 5. Dividing both sides of the ratio: $5 \div 5 = 1$, and $35 \div 5 = 7$. Therefore, $5:35$ is $1:7$.

3. The common factor between 8 and 72 is 8: $8 \div 8 = 1$, and $72 \div 8 = 9$. The reduced ratio is $1:9$.

4. Recognizing both numbers are divisible by 12, we get: $24 \div 12 = 2$, $36 \div 12 = 3$. The simplified ratio is $2:3$.

5. These numbers can both be divided by 5: $25 \div 5 = 5$, $40 \div 5 = 8$. So, $25:40$ becomes $5:8$.

6. Both numbers share 8 as a common factor: $40 \div 8 = 5$, $72 \div 8 = 9$. Resulting in a ratio of $5:9$.

7. Considering 7 as their common divisor: $28 \div 7 = 4$, $63 \div 7 = 9$. We get a ratio of $4:9$.

8. By noting that both are divisible by 9: $18 \div 9 = 2$, $81 \div 9 = 9$. This simplifies to $2:9$.

9. Observing that 13 goes into both: $13 \div 13 = 1$, $52 \div 13 = 4$. The ratio is $1:4$.

10: Using 8, their shared factor: $56 \div 8 = 7$, $72 \div 8 = 9$. It simplifies to $7:9$.

11. Recognizing 21 as the largest common divisor: $42 \div 21 = 2$, $63 \div 21 = 3$. The ratio is $2:3$.

12. 32 itself is the common factor here: $32 \div 32 = 1$, $96 \div 32 = 3$. The simplified ratio is $1:3$.

13. To find the ratio, you'll want to write the number of red cards to green cards. Bob has 16 red cards and 20 green cards. This can be represented as $16:20$.

To simplify this ratio, you need to find the common factor of these two numbers. The common factor of 16 and 20 is 4. Now, divide both numbers by 4: $16 \div 4 = 4$, and $20 \div 4 = 5$. Thus, the simplest form of the ratio is $4{:}5$.

14. The question tells us that for every 20 guests, 34 soft drinks are required. To determine the number of soft drinks for 260 guests, let's first find out the soft drinks required for 1 guest.

$Soft\ drinks\ per\ guest = Total\ soft\ drinks\ divided\ by\ 20\ guests\ = 34 \div 20 = 1.7$

So, 1 guest requires 1.7 soft drinks.

Now, for 260 guests, you multiply the number of guests by the soft drinks each guest requires.

$Total\ soft\ drinks = 260\ guests \times 1.7 \frac{soft\ drinks}{guest} = 442\ soft\ drinks$

Therefore, for 260 guests, 442 soft drinks are required.

15. To find the ratio of Sara's black pens to her blue pens, you can simply divide the number of black pens by the number of blue pens:

$Ratio\ of\ black\ pens\ to\ blue\ pens = \frac{Number\ of\ black\ pens}{Number\ of\ blue\ pens}$

$Ratio\ of\ black\ pens\ to\ blue\ pens = \frac{28\ black\ pens}{56\ blue\ pens}$

Now, simplify this ratio by dividing both the numerator and denominator by their greatest common divisor, which is 28:

$Ratio\ of\ black\ pens\ to\ blue\ pens = \frac{28\ black\ pens}{56\ blue\ pens} = \frac{1\ black\ pens}{2\ blue\ pens}$

So, the ratio of Sara's black pens to her blue pens is $1{:}2$.

16. To determine which class has a higher ratio of tall to short students, you need to calculate the ratios for both classes and then compare them.

In Jack's class: (Tall students: 48), and (Short students: 20)

$Ratio\ of\ tall\ to\ short\ students\ in\ Jack's\ class = \frac{Tall\ students}{Short\ students} \Rightarrow Ratio = \frac{48}{20}$

In Michael's class: (Tall students: 28), and (Short students: 12)

$Ratio\ of\ tall\ to\ short\ students\ in\ Michael's\ class = \frac{Tall\ students}{Short\ students} \Rightarrow Ratio = \frac{28}{12}$

Now, compare the two ratios. Jack's class ratio is $\frac{48}{20} = 2.4$. Michael's class ratio is $\frac{28}{12} = 2.333\cdots$.

So, Jack's class has a higher ratio of tall to short students.

17. To determine which place is the better buy for apples, you can calculate the price per apple at each store:

At Quick Market: Price for 6 apples = $1.52.

$Price\ per\ apple = \frac{\$1.52}{6\ apples} = \$0.25\overline{3}$ (rounded to two decimal places)

At Walmart: Price for 5 apples = $1.32.

$Price\ per\ apple = \frac{\$1.32}{5\ apples} = \$0.264$ (rounded to two decimal places)

So, the price per apple at Quick Market is approximately $0.25, while the price per apple at Walmart is approximately $0.26. Therefore, Quick Market is the better buy for apples.

18. To find out how many bagels the bakers can bake in 16 hours and the rate per hour, you can use the information that they can make 180 bagels in 6 hours.

First, let's find out how many bagels they can make in 1 hour:

$Bagels\ per\ hour = \frac{Total\ bagels}{Total\ hours} \Rightarrow Bagels\ per\ hour = \frac{180}{6} = 30$

Now, to find out how many bagels they can make in 16 hours, simply multiply the rate per hour by the number of hours:

$Bagels\ in\ N\ hours = Bagels\ per\ hour \times N\ hours$

$Bagels\ in\ 16\ hours = 30\ bagels\ per\ hour \times 16\ hours = 480\ bagels$

So, the bakers can bake 480 bagels in 16 hours, and their rate is 30 bagels per hour.

19. To find out how much it costs to buy 38 cans of green beans at the supermarket, you can first find the cost per can and then multiply it by the number of cans. Cost for 6 cans = $3.48.

$Cost\ per\ can = \frac{\$3.48}{6\ cans} = \$0.58\ per\ can$

Now, multiply the cost per can by the number of cans you want to buy (38 cans):

$Cost\ for\ 38\ cans = \$0.58\ per\ can \times 38\ cans = \$22.04$

So, it would cost $22.04 to buy 38 cans of green beans at the supermarket.

20. To solve for x, we can cross-multiply: $3x = 2 \times 9 \Rightarrow 3x = 18$. Now, divide both sides by 3 to isolate x: $x = \frac{18}{3} \Rightarrow x = 6$.

21. Cross-multiply to solve for x: $7 \times 4 = 2x \Rightarrow 28 = 2x$. Divide both sides by 2: $x = \frac{28}{2} \Rightarrow x = 14$.

22. Cross-multiply: $1 \times x = 3 \times 2 \Rightarrow x = 6$.

23. Cross-multiply: $1 \times x = 4 \times 5 \Rightarrow x = 20$.

24. Cross-multiply: $9 \times 2 = 6x \Rightarrow 18 = 6x$. Divide both sides by 6: $x = \frac{18}{6} \Rightarrow x = 3$.

25. Simplify $\frac{3}{6}$ to $\frac{1}{2}$: $\frac{1}{2} = \frac{5}{x}$. Cross-multiply: $1 \times x = 2 \times 5 \Rightarrow x = 10$.

26. Cross-multiply: $7 \times 6 = 2 \times x \Rightarrow 42 = 2x$. Divide both sides by 2: $x = \frac{42}{2} \Rightarrow x = 21$.

27. Cross-multiply: $2 \times 10 = 4 \times x \Rightarrow 20 = 4x$. Divide both sides by 4: $x = \frac{20}{4} \Rightarrow x = 5$.

28. Cross-multiply: $3 \times 8 = 2 \times x \Rightarrow 24 = 2x$. Divide both sides by 2: $x = \frac{24}{2} \Rightarrow x = 12$.

29. Cross-multiply: $x \times 3 = 6 \times 5 \Rightarrow 3x = 30$. Divide both sides by 3: $x = \frac{30}{3} \Rightarrow x = 10$.

30. Simplify $\frac{3}{9}$ to $\frac{1}{3}$: $\frac{1}{3} = \frac{5}{x}$. Cross-multiply: $1 \times x = 3 \times 5 \Rightarrow x = 15$.

31. Simplify $\frac{4}{18}$ to $\frac{2}{9}$: $\frac{2}{9} = \frac{2}{x}$. Cross-multiply: $2 \times x = 2 \times 9 \Rightarrow 2x = 18$. Divide both sides by 2: $x = \frac{18}{2} \Rightarrow x = 9$.

32. Simplify $\frac{6}{16}$ to $\frac{3}{8}$: $\frac{3}{8} = \frac{3}{x}$. Cross-multiply: $3 \times x = 3 \times 8 \Rightarrow 3x = 24$. Divide both sides by 3: $x = \frac{24}{3} \Rightarrow x = 8$.

33. Cross-multiply: $2 \times 20 = 5 \times x \Rightarrow 40 = 5x$. Divide both sides by 5: $x = \frac{40}{5} \Rightarrow x = 8$.

34. Cross-multiply: $28 \times 2 = 8 \times x \Rightarrow 56 = 8x$. Divide both sides by 8: $x = \frac{56}{8} \Rightarrow x = 7$.

35. Cross-multiply: $3 \times 15 = 5 \times x \Rightarrow 45 = 5x$. Divide both sides by 5: $x = \frac{45}{5} \Rightarrow x = 9$.

36. Cross-multiply: $2 \times 14 = 7 \times x \Rightarrow 28 = 7x$. Divide both sides by 7: $x = \frac{28}{7} \Rightarrow x = 4$.

37. Cross-multiply: $x \times 2 = 18 \times 3 \Rightarrow 2x = 54$. Divide both sides by 2: $x = \frac{54}{2} \Rightarrow x = 27$.

38. Simplify $\frac{2}{6}$ to $\frac{1}{3}$: $\frac{x}{24} = \frac{1}{3}$. Cross-multiply: $x \times 3 = 24 \times 1 \Rightarrow 3x = 24$. Divide both sides by 3: $x = \frac{24}{3} \Rightarrow x = 8$.

39. Simplify $\frac{4}{20}$ to $\frac{1}{5}$: $\frac{5}{x} = \frac{1}{5}$. Cross-multiply: $5 \times 5 = 1 \times x \Rightarrow x = 25$.

40. Simplify $\frac{20}{80}$ to $\frac{1}{4}$: $\frac{10}{x} = \frac{1}{4}$. Cross-multiply: $10 \times 4 = 1 \times x \Rightarrow x = 40$.

41. Simplify $\frac{90}{6}$ to 15: $15 = \frac{x}{2}$. Multiply both sides by 2 to isolate x: $x = 15 \times 2 \Rightarrow x = 30$.

42. If two rectangles are similar, the ratios of their corresponding sides are equal.

$$\frac{Width\,1}{Width\,2} = \frac{Length\,1}{Length\,2} \Rightarrow \frac{8}{12} = \frac{32}{L_2}$$

Cross-multiplying: $8 \times L_2 = 12 \times 32 \Rightarrow 8L_2 = 384 \Rightarrow L_2 = 48$. Length of the second rectangle is 48 feet.

43. Given the longer side of the second rectangle is 28 meters, and considering the longer side of the first rectangle is 7 meters, we can set up a proportion to find the shorter side (width) of the second rectangle. $\frac{Length\,1}{Length\,2} = \frac{Width\,1}{Width\,2} \Rightarrow \frac{7}{28} = \frac{4.6}{w_2}$.

Cross-multiplying:

$7 \times w_2 = 4.6 \times 28 \Rightarrow 7w_2 = 128.8 \Rightarrow w_2 = 18.4$

The other side (width) of the second rectangle is 18.4 meters.

CHAPTER

5 Percentage

Math topics in this chapter:

- ☑ Percent Problems
- ☑ Percent of Increase and Decrease
- ☑ Discount, Tax and Tip
- ☑ Simple Interest

Practices

Solve each problem.

1) What is 15% of 60? ____
2) What is 55% of 800? ____
3) What is 22% of 120? ____
4) What is 18% of 40? ____
5) 90 is what percent of 200? ____%
6) 30 is what percent of 150? ____%
7) 14 is what percent of 250? ____%
8) 60 is what percent of 300? ____%
9) 30 is 120 percent of what number? ____
10) 120 is 20 percent of what number? ____
11) 15 is 5 percent of what number? ____
12) 22 is 20% of what number? ____

Solve each problem.

13) Bob got a raise, and his hourly wage increased from $15 to $21. What is the percent increase? _____ %
14) The price of a pair of shoes increases from $32 to $36. What is the percent increase? ___ %
15) At a Coffee Shop, the price of a cup of coffee increased from $1.35 to $1.62. What is the percent increase in the cost of the coffee? _____ %
16) A $45 shirt now selling for $36 is discounted by what percent? _____ %
17) Joe scored 30 out of 35 marks in Algebra, 20 out of 30 marks in science and 58 out of 70 marks in mathematics. In which subject his percentage of marks is best? _____
18) Emma purchased a computer for $420. The computer is regularly priced at $480. What was the percent discount Emma received on the computer? _____
19) A chemical solution contains 15% alcohol. If there is 54 ml of alcohol, what is the volume of the solution? _____

Find the selling price of each item.

20) Original price of a computer: $600

    Tax: 8%, Selling price: $_______

21) Original price of a laptop: $450

    Tax: 10%, Selling price: $_______

22) Nicolas hired a moving company. The company charged $500 for its services, and Nicolas gives the movers a 14% tip. How much does Nicolas tip the movers? $_________

23) Mason has lunch at a restaurant and the cost of his meal is $40. Mason wants to leave a 20% tip. What is Mason's total bill, including tip? $_________

Determine the simple interest for the following loans.

24) $1,000 *at* 5% *for* 4 *years*. $__

25) $400 *at* 3% *for* 5 *years*. $__

26) $240 *at* 4% *for* 3 *years*. $__

27) $500 at 4.5% for 6 years. $__

Solve.

28) A new car, valued at $20,000, depreciates at 8% per year. What is the value of the car one year after purchase? $_____________

29) Sara puts $7,000 into an investment yielding 3% annual simple interest; she left the money in for five years. How much interest does Sara get at the end of those five years? $_____________

Answers

1) 9
2) 440
3) 26.4
4) 7.2
5) 45%
6) 20%
7) 5.6%
8) 20%
9) 25
10) 600
11) 300
12) 110
13) 40%
14) 12.5%
15) 20%
16) 20%
17) Algebra
18) 12.5%
19) 360 ml
20) $648.00
21) $495.00
22) $70.00
23) $48.00
24) $200.00
25) $60.00
26) $28.80
27) $135.00
28) $18.400
29) $1,050

Answers and Explanations

1. To find 15% of 60, multiply 60 by 0.15: $60 \times 0.15 = 9$.

2. Think of "percent" as "out of 100." Thus, 55% is equivalent to $\frac{55}{100}$. Multiply 800 by $\frac{55}{100}$: $800 \times \frac{55}{100} = 440$.

3. Another way to find a percentage is to divide the percentage by 100 and then multiply by the number: $\frac{22}{100} \times 120 = 26.4$.

4. Multiply 40 by 18 divided by 100: $40 \times \frac{18}{100} = 7.2$.

5. To find the percentage, divide 90 by 200 and then multiply by 100: $\frac{90}{200} \times 100 = 45\%$.

6. This means "30 is how much out of 150 in terms of 100": $\frac{30}{150} \times 100 = 20\%$.

7. Divide 14 by 250, and then turn it into a percentage: $\frac{14}{250} \times 100 = 5.6\%$.

8. Find the ratio of 60 to 300 and then express it as a percentage: $\frac{60}{300} \times 100 = 20\%$.

9. Set up the equation: $number \times 1.20 = 30$. Divide both sides by 1.20 to solve for the number: $number = \frac{30}{1.2} = 25$.

10. Using a proportion: $\frac{20}{100} = \frac{120}{number}$. Cross-multiplying gives: $20 \times number = 120 \times 100$. Thus, $number = \frac{120 \times 100}{20} = 600$.

11. Imagine the number you seek is a big container. 5% of this container is 15 units. The equation becomes: $number \times 0.05 = 15$. Solve for the number: $number = \frac{15}{0.05} = 300$.

12. Reverse the thought: If 22 is 20% (or 0.20), what's the full value (100%)? Using the equation:

$\frac{20}{100} = \frac{22}{number} \Rightarrow number = \frac{22}{0.2} = 110.$

13. Use this formula:

$$Percent\ of\ change = \frac{new\ number - original\ number}{original\ number} \times 100 = \frac{21-15}{15} \times 100 = 40\%$$

14. Use this formula:

$$Percent\ of\ change = \frac{new\ number - original\ number}{original\ number} \times 100 = \frac{36-32}{32} \times 100 = 12.5\%$$

15. Use this formula:

$$Percent\ of\ change = \frac{new\ number - original\ number}{original\ number} \times 100 = \frac{1.62-1.35}{1.35} \times 100 = 20\%$$

16. Using the formula:

$$Percent\ of\ decrease = \frac{original\ number - new\ number}{original\ number} \times 100 = \frac{45-36}{45} \times 100 = 20\%$$

17. $Algebra\ percentage = 30/35 \times 100 \approx 85.71\%$

$$Science\ percentage = \frac{20}{30} \times 100 \approx 66.67\%$$

$$Mathematics\ percentage = \frac{58}{70} \times 100 \approx 82.86\%$$

So, Joe's percentage of marks is best in Algebra.

18. Using the formula for percent discount:

$$Percent\ of\ decrease = \frac{original\ number - new\ number}{original\ number} \times 100 = \frac{480-420}{480} \times 100 = 12.5\%$$

19. Given that the solution contains 15% alcohol and there is $54\ ml$ of alcohol, we can set up the equation: $0.15 \times Volume\ of\ Solution = 54\ ml$. To find the volume of the solution, divide both sides by 0.15: $Volume\ of\ Solution = \frac{54\ ml}{0.15} = 360\ ml$.

20. Tax amount = Original price × Tax rate ⇒ $\$600 \times 0.08 = \$48$.

Selling price = Original price + Tax amount ⇒ $\$600 + \$48 = \$648$.

So, the selling price of the computer is $648.

21. Tax amount = Original price × Tax rate ⇒ $\$450 \times 0.1 = \$45$.

Selling price = Original price + Tax amount ⇒ $\$450 + \$45 = \$495$.

So, the selling price of the laptop is $\$495$.

22. To find out how much Nicolas tips the movers; you need to calculate 14% of the service cost.

Tip amount = Cost of service × Tip percentage ⇒ $\$500 \times 0.14 = \$70$.

So, Nicolas tips the movers $70.

23. First, we need to find out how much Mason will tip for his meal.

Tip amount = Cost of meal × Tip percentage ⇒ $\$40 \times 0.20 = \$8$. To find the total bill including tip, you simply add the cost of the meal to the tip amount.

Total bill = Cost of meal + Tip amount ⇒ $\$40 + \$8 = \$48$. Therefore, Mason's total bill, including tip, is $\$48$.

24. Simple interest is calculated as the principal (or initial amount) multiplied by the rate of interest, and then by the time period (usually in years).

Simple Interest = $\$1{,}000 \times 0.05 \times 4 = \$200$. The interest for this loan is $\$200$.

25. Using the formula: $I = prt \Rightarrow 400 \times 0.03 \times 5 = 60$. The interest for this loan is $60.

26. Using the formula: $I = prt \Rightarrow 240 \times 0.04 \times 3 = 28.8$. The interest for this loan is $\$28.80$.

27. Using the formula: $I = prt \Rightarrow 500 \times 0.045 \times 6 = 135$. The interest for this loan is $\$135$.

28. Depreciation means the loss in value. To find out how much the car depreciates in one year, you'll multiply its value by the depreciation rate.

Depreciation for one year $= \$20{,}000 \times 0.08 = \$1{,}600$.

Now, to find the car's value after one year, subtract this depreciation from the original value:

Value after one year $=$ Original value $-$ Depreciation $\Rightarrow \$20{,}000 - \$1{,}600 = \$18{,}400$.

So, one year after purchase, the car is worth $\$18{,}400$.

29. Using the formula: $I = prt \Rightarrow 7{,}000 \times 0.03 \times 5 = 1{,}050$. At the end of five years, Sara gets $\$1{,}050$ in interest.

CHAPTER

6 Exponents and Variables

Math topics in this chapter:

- ☑ Multiplication Property of Exponents
- ☑ Division Property of Exponents
- ☑ Powers of Products and Quotients
- ☑ Zero and Negative Exponents
- ☑ Negative Exponents and Negative Bases
- ☑ Scientific Notation
- ☑ Addition and Subtraction in Scientific Notation
- ☑ Multiplication and division in Scientific Notation

Practices

Find the products.

1) $x^2 \times 4xy^2 =$
2) $3x^2y \times 5x^3y^2 =$
3) $6x^4y^2 \times x^2y^3 =$
4) $7xy^3 \times 2x^2y =$
5) $-5x^5y^5 \times x^3y^2 =$
6) $-8x^3y^2 \times 3x^3y^2 =$
7) $-6x^2y^6 \times 5x^4y^2 =$
8) $-3x^3y^3 \times 2x^3y^2 =$
9) $-6x^5y^3 \times 4x^4y^3 =$
10) $-2x^4y^3 \times 5x^6y^2 =$
11) $-7y^6 \times 3x^6y^3 =$
12) $-9x^4 \times 2x^4y^2 =$

Simplify.

13) $\frac{5^3 \times 5^4}{5^9 \times 5} =$
14) $\frac{3^3 \times 3^2}{7^2 \times 7} =$
15) $\frac{15x^5}{5x^3} =$
16) $\frac{16x^3}{4x^5} =$
17) $\frac{72y^2}{8x^3y^6} =$
18) $\frac{10x^3y^4}{50x^2y^3} =$
19) $\frac{13y^2}{52x^4y^4} =$
20) $\frac{50xy^3}{200x^3y^4} =$
21) $\frac{48x^2}{56x^2y^2} =$
22) $\frac{81y^6x}{54x^4y^3} =$

Solve.

23) $(x^3y^3)^2 =$
24) $(3x^3y^4)^3 =$
25) $(4x \times 6xy^3)^2 =$
26) $(5x \times 2y^3)^3 =$
27) $\left(\frac{9x}{x^3}\right)^2 =$
28) $(\frac{3y}{18y^2})^2 =$
29) $\left(\frac{3x^2y^3}{24x^4y^2}\right)^3 =$
30) $\left(\frac{26x^5y^3}{52x^3y^5}\right)^2 =$
31) $\left(\frac{18x^7y^4}{72\quad {}^5y^2}\right)^2 =$
32) $\left(\frac{12x^6\quad {}^4}{48x^5y^3}\right)^2 =$

Evaluate each expression.

33) $(\frac{1}{4})^{-2} =$

34) $(\frac{1}{3})^{-2} =$

35) $(\frac{1}{7})^{-3} =$

36) $(\frac{2}{5})^{-3} =$

37) $(\frac{2}{3})^{-3} =$

38) $(\frac{3}{5})^{-4} =$

Write each expression with positive exponents.

39) $x^{-7} =$

40) $3y^{-5} =$

41) $15y^{-3} =$

42) $-20x^{-4} =$

43) $12a^{-3}b^{5} =$

44) $25a^{3}b^{-4}c^{-3} =$

45) $-4x^{5}y^{-3}z^{-6} =$

46) $\frac{18y}{x^{3}y^{-2}} =$

47) $\frac{20a^{-2}b}{-12c^{-4}} =$

Write each number in scientific notation.

48) $0.00412 =$

49) $0.000053 =$

50) $66{,}000 =$

51) $72{,}000{,}000 =$

Write the answer in scientific notation.

52) $6 \times 10^{4} + 10 \times 10^{4} =$ ________

53) $7.2 \times 10^{6} - 3.3 \times 10^{6} =$ __________

54) $2.23 \times 10^{7} + 5.2 \times 10^{7} =$ ______

55) $8.3 \times 10^{9} - 5.6 \times 10^{8} =$ _____

56) $1.4 \times 10^{2} + 7.4 \times 10^{5} =$ _____

57) $9.6 \times 10^{6} - 3 \times 10^{4} =$ ______

Simplify. Write the answer in scientific notation.

58) $(5.6 \times 10^{12})(3 \times 10^{-7}) =$ ____

59) $(3 \times 10^{-8})(7 \times 10^{10}) =$ _____

60) $(9 \times 10^{-3})(4.2 \times 10^{6}) =$ ____

61) $\frac{125\times10^{9}}{50\times10^{12}} =$ ____

62) $\frac{2.8\times10^{12}}{0.4\times10^{20}} =$ ____

63) $\frac{9\times10^{8}}{3\times10^{7}} =$ ____

Answers

1) $4x^3y^2$
2) $15x^5y^3$
3) $6x^6y^5$
4) $14x^3y^4$
5) $-5x^8y^7$
6) $-24x^6y^4$
7) $-30x^6y^8$
8) $-6x^6y^5$
9) $-24x^9y^6$
10) $-10x^{10}y^5$
11) $-21x^6y^9$
12) $-18x^8y^2$
13) $\frac{1}{125}$
14) $\frac{243}{343}$
15) $3x^2$
16) $\frac{4}{x^2}$
17) $\frac{9}{x^3y^4}$
18) $\frac{xy}{5}$
19) $\frac{1}{4x^4y^2}$
20) $\frac{1}{4x^2y}$
21) $\frac{6}{7y^2}$
22) $\frac{3y^3}{2x^3}$
23) x^6y^6
24) $27x^9y^{12}$
25) $576x^4y^6$
26) $1{,}000x^3y^9$
27) $\frac{81}{x^4}$
28) $\frac{1}{36y^2}$
29) $\frac{y^3}{512x^6}$
30) $\frac{x^4}{4y^4}$
31) $\frac{x^4y^4}{16}$
32) $\frac{x^2y^2}{16}$
33) 16
34) 9
35) 343
36) $\frac{125}{8}$
37) $\frac{27}{8}$
38) $\frac{625}{81}$
39) $\frac{1}{x^7}$
40) $\frac{3}{y^5}$
41) $\frac{15}{y^3}$
42) $-\frac{20}{x^4}$
43) $\frac{12b^5}{a^3}$
44) $\frac{25a^3}{b^4c^3}$
45) $-\frac{4x^5}{y^3z^6}$
46) $\frac{18y^3}{x^3}$
47) $-\frac{5bc^4}{3a^2}$
48) 4.12×10^{-3}
49) 5.3×10^{-5}
50) 6.6×10^4
51) 7.2×10^7
52) 1.6×10^5
53) 3.9×10^6
54) 7.43×10^7
55) 7.74×10^9

56) 7.4014×10^5

57) 9.57×10^6

58) 1.68×10^6

59) 2.1×10^3

60) 3.78×10^4

61) 2.5×10^{-3}

62) 7×10^{-8}

63) 3×10^1

Answers and Explanations

1. Multiply the coefficients and then apply the product of powers rule for each term. Coefficients: $1 \times 4 = 4$. For x: $x^2 \times x = x^{2+1} = x^3$. Result: $4x^3y^2$.

2. Coefficients: $3 \times 5 = 15$. For x: $x^2 \times x^3 = x^{2+3} = x^5$. For y: $y \times y^2 = y^{1+2} = y^3$. Answer: $15x^5y^3$.

3. Coefficients remain as 6 (since multiplying by 1). For x: $x^4 \times x^2 = x^{4+2} = x^6$. For y: $y^2 \times y^3 = y^{2+3} = y^5$. Result: $6x^6y^5$.

4. Coefficients: $7 \times 2 = 14$. For x: $x \times x^2 = x^{1+2} = x^3$. For y: $y^3 \times y = y^{3+1} = y^4$. Result: $14x^3y^4$.

5. Coefficients: $-5 \times 1 = -5$. For x: $x^5 \times x^3 = x^{5+3} = x^8$. For y: $y^5 \times y^2 = y^{5+2} = y^7$. Answer: $-5x^8y^7$.

6. Numerically, -8 times 3 is -24. x^3 terms combine to x^6. y^2 terms stay as y^4. The product is $-24x^6y^4$.

7. For the numbers, -6 times 5 is -30. Combining x terms, x^2 and x^4 yield x^6. For y, y^6 and y^2 produce y^8. It's $-30x^6y^8$.

8. -3 multiplied by 2 gives -6. The x^3 terms result in x^6. For y, y^3 and y^2 become y^5. The answer is $-6x^6y^5$.

9. -6 times 4 is -24. Combining x's, $x^5$and x^4 produce x^9. y^3's yield y^6. Hence, $-24x^9y^6$.

10. Numerically, -2 times 5 is -10. x^4 and x^6 combine to x^{10}. y^3 and y^2 become y^5. It's $-10x^{10}y^5$.

11. -7 times 3 results in -21. The x term is just x^6. Combining y's, y^6 and y^3 give y^9. The answer is $-21x^6y^9$.

12. -9 times 2 is -18. For x, x^4 times x^4 yields x^8. The y term remains y^2. The result is $-18x^8y^2$.

13. To simplify this expression, you can combine the exponents of like bases by adding them when multiplying and subtracting them when dividing. Here, $5^3 \times 5^4$ becomes $5^{3+4} = 5^7$ and $5^9 \times 5$ becomes $5^{9+1} = 5^{10}$. Then you divide 5^7 by 5^{10}, which is $5^{7-10} = 5^{-3}$. This is the same as $\frac{1}{5^3}$, which simplifies to $\frac{1}{125}$.

14. Combine the exponents for the number 3: $3^3 \times 3^2 = 3^{3+2} = 3^5$. For the number 7, it's $7^2 \times 7 = 7^{2+1} = 7^3$. Now, divide 3^5 by 7^3. Since these are different bases, you cannot simplify further and are left with $\frac{3^5}{7^3}$ or $\frac{243}{343}$.

15. Divide the numerical coefficients: $\frac{15}{5} = 3$, hen, for the variable $\frac{x^5}{x^3} = x^{5-3} = x^2$. Combining these gives you $3x^2$.

16. First, divide the numbers: $\frac{16}{4} = 4$. For $\frac{x^3}{x^5}$ you subtract the exponents (since you're dividing like bases), which gives you $x^{3-5} = x^{-2}$. This means $\frac{4}{x^2}$.

17. Divide the numerical coefficients: $\frac{72}{8} = 9$. For $\frac{y^2}{y^6}$, subtract the exponents, giving $y^{2-6} = y^4$, which is $\frac{1}{y^4}$. Since the x^3 in the denominator does not have a corresponding x in the numerator, the final answer is $\frac{9}{x^3y^4}$.

18. Divide the numbers: $\frac{10}{50} = \frac{1}{5}$. For, $\frac{x^3}{x^2}$, subtract the exponents: $x^{3-2} = x$. Do the same for $\frac{y^4}{y^3} = y^{4-3} = y$. The final answer is $\frac{xy}{5}$.

19. Here, divide the coefficients: $\frac{13}{52} = \frac{1}{4}$. For the variables, $\frac{y^2}{y^4} = y^{2-4} = y^{-2}$, which is $\frac{1}{y^2}$. There is no x in the numerator to cancel out the x^4 in the denominator, so the answer is $\frac{1}{4x^4y^2}$.

20. Divide $\frac{50}{200}$ to get $\frac{1}{4}$. Now, $\frac{xy^3}{x^3y^4}$ means you have to deal with the exponents separately for x and y. For x, there's no exponent in the numerator, so $\frac{x}{x^3} = x^{1-3} = x^{-2}$. For y, it's $\frac{y^3}{y^4} = y^{3-4} = y^{-1}$. Combined, you have $\frac{1}{4x^2y}$.

21. First, divide $\frac{48}{56}$ which simplifies to $\frac{6}{7}$ when reduced. For the x^2 terms, since the exponents are the same, $\frac{x^2}{x^2}$ cancels out to 1. The y^2 in the denominator remains, so the final expression is $\frac{6}{7y^2}$.

22. Divide the numerical coefficients $\frac{81}{54}$, which reduces to $\frac{3}{2}$. Then for the variables: $\frac{x}{x^4} = \frac{1}{x^3}$, and $\frac{y^6}{y^3} = y^{6-3} = y^3$. Putting it all together gives $\frac{3y^3}{2x^3}$.

23. When raising a power to a power, you multiply the exponents. For x^3 raised to the power of 2, you multiply the exponents: $3 \times 2 = 6$ resulting in x^6. Similarly, for y^3 raised to the power of 2, you get y^6. The result is x^6y^6.

24. Start by raising the coefficient 3 to the power of 3, which is 27. Then, raise x^3 the power of 3 to get x^9. Finally, y^4 to the power of 3 gives y^{12}. Combined, you get $27x^9y^{12}$.

25. First, multiply the coefficients: $4 \times 6 = 24$. Then multiply $x \times x$ to get x^2. Finally, there's a y^3 term. Combining these gives $24x^2y^3$. Now, square the entire expression to get $576x^4y^6$.

26. Multiply the numbers first: $5 \times 2 = 10$. Combine x and y^3 to get $10xy^3$. Now, raise the entire expression to the third power. This results in $1{,}000x^3y^9$.

27. Here, divide $9x$ by x^3. The x's will reduce to x^{-2}. When you square the result, you get $\frac{81}{x^4}$.

28. Divide 3 by 18 to get $\frac{1}{6}$. Then, simplify $\frac{y}{y^2}$ to get y^{-1}. Squaring the result, you have $\frac{1}{36y^2}$.

29. Begin by simplifying the terms inside the parentheses before taking the cube. For the constants, 3 divided by 24 is $\frac{1}{8}$. For the terms involving x, x^2 divided by x^4 is x^{-2}. For the terms involving y, y^3 divided by y^2 is y. So, before taking the cube, the expression is $\frac{y}{8x^2}$. When you cube this, you get $\frac{y^3}{512x^6}$.

30. First, divide the coefficients, 26 by 52, which is $\frac{1}{2}$. Then, for x: $\frac{x^5}{x^3}$ results in x^2. For y: $\frac{y^3}{y^5}$ results in y^2. Combining these gives $\frac{x^2}{2y^2}$. Squaring this entire expression provides $\frac{x^4}{4y^4}$.

31. Dividing the numbers, 18 by 72 becomes $\frac{1}{4}$. Simplifying x: $\frac{x^7}{x^5}$ becomes x^2. Simplifying y: $\frac{y^4}{y^2}$ becomes y^2. Combining these results, you have $\frac{x^2y^2}{4}$. When squared, this gives $\frac{x^4y^4}{16}$.

32. Start by dividing the coefficients, 12 by 48, which results in $\frac{1}{4}$. For the terms involving x, $\frac{x^6}{x^5}$ is x. For y, $\frac{y^4}{y^3}$ becomes y. Combining them, the expression is $\frac{xy}{4}$. Squaring this entire expression, you get $\frac{x^2y^2}{16}$.

33. A negative exponent means to take the reciprocal of the base and then raise it to the positive value of that exponent. For $\frac{1}{4}$ the reciprocal is $\frac{4}{1}$ or just 4. Raising 4 to the power of 2 (because of the -2 exponent) results in $4^2 = 16$.

34. Take the reciprocal of $\frac{1}{3}$, which is 3 (or $\frac{3}{1}$). Now, square 3 (due to the -2 exponent), which equals $3^2 = 9$.

35. First, find the reciprocal of $\frac{1}{7}$, which is 7. Now, raise 7 to the power of 3 (because of the -3 exponent) to get $7^3 = 343$.

36. For $\frac{2}{5}$, the reciprocal is $\frac{5}{2}$. Raise this fraction to the power of 3. That means you'll cube both the numerator and the denominator separately: $5^3 = 125$, and $2^3 = 8$. Thus, the result is $\frac{125}{8}$.

37. The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$. Cube both the numerator and the denominator separately: $3^3 = 27$, and $2^3 = 8$. So, the answer is $\frac{27}{8}$.

38. For $\frac{3}{5}$, the reciprocal is $\frac{5}{3}$. When raised to the power of 4, this means you'll raise both the numerator and denominator to the 4th power: $5^4 = 625$, and $3^4 = 81$. The final result is $\frac{625}{81}$.

39. A negative exponent means you'll take the reciprocal. In terms of x, this means you'll move it from the numerator to the denominator. So, x^{-7} becomes $\frac{1}{x^7}$.

40. The negative exponent on y means we'll place y in the denominator. The coefficient 3 remains in the numerator. Thus, it becomes $\frac{3}{y^5}$.

41. The y term, because of its negative exponent, will go to the denominator. This gives $\frac{15}{y^3}$.

42. The x term moves to the denominator due to its negative exponent. This results in $\frac{-20}{x^4}$.

43. The a term's negative exponent means it will be placed in the denominator, while the b^5 remains in the numerator. This results in $\frac{12b^5}{a^3}$.

44. Here, both b and c have negative exponents, so they'll move to the denominator. a^3 remains in the numerator. The expression becomes $\frac{25a^3}{b^4c^3}$.

45. Both y and z will move to the denominator because of their negative exponents, while x^5 remains in the numerator. This gives $\frac{-4\,x^5}{y^3z^6}$.

46. The negative exponent on y means it moves to the numerator, thus multiplying with the already existing y in the numerator. This results in $\frac{18y^3}{x^3}$.

47. The negative exponent on a sends it to the denominator, while b remains in the numerator. The c term's negative exponent brings it to the numerator. Combining these adjustments gives $\frac{-20bc^4}{12a^2}$. Simplifying further, this results in $-\frac{5bc^4}{3a^2}$.

48. Scientific notation involves representing a number as a product of two numbers: a coefficient between 1 and 10, and a power of 10. To express 0.00412 in scientific notation, we shift the decimal point two places to the right to make it 4.12. In doing so, we're multiplying by 10^3 or 1000. However, since the original number was smaller, we must divide by 10^3 to balance the operation, resulting in the exponent being -3. Hence, 0.00412 in scientific notation is 4.12×10^{-3}.

49. Shift the decimal point five places to the right to get 5.3. This is equivalent to multiplying by 10^5. But since the original number was smaller, our exponent will be -5. Thus, the scientific notation for 0.000053 is 5.3×10^{-5}.

50. Move the decimal point from the end of the number four places to the left to make it 6.6. In this process, you're effectively dividing the number by 10^4 or $10{,}000$. To balance the operation, we multiply by 10^4. Therefore, $66{,}000$ in scientific notation is 6.6×10^4.

51. Shift the decimal seven places to the left to get 7.2. We've divided the number by 10^7 or $10{,}000{,}000$. To counteract that division, we multiply by 10^7. Hence, $72{,}000{,}000$ in scientific notation is 7.2×10^7.

52. When adding numbers in scientific notation with the same exponent, simply add their coefficients. So, $6 + 10 = 16$. Your answer is 1.6×10^5 .

53. For numbers with the same exponent, subtract their coefficients. $7.2 - 3.3 = 3.9$. The result is 3.9×10^6.

54. Combine the coefficients by adding them: $2.23 + 5.2 = 7.43$. So, your answer is 7.43×10^7.

55. To subtract these, you need to express them with the same exponent. Rewrite 5.6×10^8 as 0.56×10^9. Then subtract: $8.3 - 0.56 = 7.74$. The answer is 7.74×10^9.

56. The exponents are different, so focus on the one with the larger exponent. You can rewrite 1.4×10^2 as 0.0014×10^5. Now, add the coefficients: $0.0014 + 7.4 = 7.4014$. The result is 7.4014×10^5.

57. Rewrite 3×10^4 as 0.03×10^6. Subtracting the coefficients gives $9.6 - 0.03 = 9.57$. The result is 9.57×10^6.

58. When you multiply numbers in scientific notation, you multiply the coefficients (the numbers in front) together, and then you add the exponents. Coefficient multiplication: $5.6 \times 3 = 16.8$. Exponent addition: $12 + (-7) = 5$. Answer: 1.68×10^6.

59. Multiply the coefficients and add the exponents. Coefficient multiplication: $3 \times 7 = 21$.

Exponent addition: $(-8) + 10 = 2$. Answer: 2.1×10^3.

60. Coefficient multiplication: $9 \times 4.2 = 37.8$. Exponent addition: $(-3) + 6 = 3$. Answer: 3.78×10^4.

61. For division, you divide the coefficients and subtract the second exponent from the first. Coefficient division: $125 \div 50 = 2.5$. Exponent subtraction: $9 - 12 = -3$. Answer: 2.5×10^{-3}.

62. Divide the coefficients and subtract the exponents. Coefficient division: $2.8 \div 0.4 = 7$. Exponent subtraction: $12 - 20 = -8$.Answer: 7×10^{-8}.

63. Coefficient division: $9 \div 3 = 3$. Exponent subtraction: $8 - 7 = 1$. Answer: 3×10^{1}.

CHAPTER

7 Expressions and Equations

Math topics in this chapter:

Practices

Simplify each expression.

1) $(3 + 4x - 1) =$

2) $(-5 - 2x + 7) =$

3) $(12x - 5x - 4) =$

4) $(-16x + 24x - 9) =$

5) $(6x + 5 - 15x) =$

6) $2 + 5x - 8x - 6 =$

7) $5x + 10 - 3x - 22 =$

8) $-5 - 3x^2 - 6 + 4x =$

9) $-6 + 9x^2 - 3 + x =$

10) $5x^2 + 3x - 10x - 3 =$

11) $4x^2 - 2x - 6x + 5 - 8 =$

12) $3x^2 - 5 - 7x + 2 - 4 =$

13) $9x^2 - x - 5x + 3 - 9 =$

14) $2x^2 - 7x - 3x^2 + 4x + 6 =$

Evaluate each expression using the value given.

15) $x = 4 \rightarrow 10 - x =$ ____

16) $x = 6 \rightarrow x + 8 =$ ____

17) $x = 3 \rightarrow 2x - 6 =$ ____

18) $x = 2 \rightarrow 10 - 4x =$ ____

19) $x = 7 \rightarrow 8x - 3 =$ ____

20) $x = 9 \rightarrow 20 - 2x =$ ____

21) $x = 5 \rightarrow 10x - 30 =$ ___

22) $x = -6 \rightarrow 5 - x =$ ____

23) $x = -3 \rightarrow 22 - 3x =$ ____

24) $x = -7 \rightarrow 10 - 9x =$ ____

25) $x = -10 \rightarrow 40 - 3x =$ ____

26) $x = -2 \rightarrow 20x - 5 =$ ____

27) $x = -5 \rightarrow -10x - 8 =$ ___

28) $x = -4 \rightarrow -1 - 4x =$ ___

Evaluate each expression using the values given.

29) $x = 2, y = 1 \rightarrow 2x + 7y =$ _____

30) $a = 3, b = 5 \rightarrow 3a - 5b =$ _____

31) $x = 6, y = 2 \rightarrow 3x - 2y + 8 =$ _____

32) $a = -2, = 3 \rightarrow -5a + 2b + 6 =$ _____

33) $x = -4, y = -3 \rightarrow -4x + 10 - 8y =$ _____

Solve each equation.

34) $x + 6 = 3 \rightarrow x =$ ____

35) $5 = 11 - x \rightarrow x =$ ____

36) $-3 = 8 + x \rightarrow x =$ ____

37) $x - 2 = -7 \rightarrow x =$ ____

38) $-15 = x + 6 \rightarrow x =$ ____

39) $10 - x = -2 \rightarrow x =$ ____

40) $22 - x = -9 \rightarrow x =$ ____

41) $-4 + x = 28 \rightarrow x =$ ____

42) $11 -= -7 \rightarrow x =$ ____

43) $35 - x = -7 \rightarrow x =$ ____

Solve each equation.

44) $4(x + 2) = 12 \rightarrow x =$ ____

45) $-6(6 - x) = 12 \rightarrow x =$ ____

46) $5 = -5(x + 2) \rightarrow x =$ ____

47) $-10 = 2(4 + x) \rightarrow x =$ ____

48) $4(x + 2) = -12 \rightarrow x =$ ____

49) $-6(3 + 2x) = 30 \rightarrow x =$ ____

50) $-3(4 - x) = 12 \rightarrow x =$ ____

51) $-4(6 - x) = 16 \rightarrow x =$ ____

Solve.

52) $q = 2l + 2w$ for w.

53) $x = 2yw$ for w.

54) $pv = nRT$ for T.

55) $a = b + c + d$ for d.

Find the midpoint of the line segment with the given endpoints.

56) $(5,0), (1,4)$

57) $(2,3), (4,7)$

58) $(8,1), (2,5)$

59) $(5,10), (3,6)$

60) $(4, -1), (-2,7)$

61) $(2, -5), (4,1)$

62) $(7, 6), (-5, 2)$

63) $(-2, 8), (4, -6)$

Find the distance between each pair of points.

64) $(-2,8), (-6,8)$

65) $(4,-4), (14,20)$

66) $(-1,9), (-5,6)$

67) $(0,3), (4,3)$

68) $(0,-2), (5,10)$

69) $(4,3), (7,-1)$

70) $(2,6), (10,-9)$

71) $(3,3), (6,-1)$

72) $(-2,-12), (14,18)$

73) $(2,-2), (12,22)$

Answers

1) $4x + 2$
2) $-2x + 2$
3) $7x - 4$
4) $8x - 9$
5) $-9x + 5$
6) $-3x - 4$
7) $2x - 12$
8) $-3x^2 + 4x - 11$
9) $9x^2 + x - 9$
10) $5x^2 - 7x - 3$
11) $4x^2 - 8x - 3$
12) $3x^2 - 12x - 2$
13) $9x^2 - 6x - 6$
14) $-x^2 - 3x + 6$
15) 6
16) 14
17) 0
18) 2
19) 53
20) 2
21) 20
22) 11
23) 31
24) 73
25) 70
26) -45
27) 42
28) 15
29) 11
30) -16
31) 22
32) 22
33) 50
34) -3
35) 6
36) -11
37) -5
38) -21
39) 12
40) 31
41) 32
42) 18
43) 42
44) 1
45) 8
46) -3
47) -9

48) -5

49) -4

50) 8

51) 10

52) $\frac{1}{2}q - l = w$

53) $w = \frac{x}{2y}$

54) $T = \frac{PV}{nR}$

55) $d = a - b -$

56) $(3, 2)$

57) $(3, 5)$

58) $(5,3)$

59) $(4,8)$

60) $(1, 3)$

61) $(3, -2)$

62) $(1, 4)$

63) $(1, 1)$

64) 4

65) 26

66) 5

67) 4

68) 13

69) 5

70) 17

71) 5

72) 34

73) 26

Answers and Explanations

1. Combine like terms. There are no like terms to the variable x and the constants can be combined. Answer: $4x + 2$.

2. Combining the constants, $-5 + 7$ gives 2. The term $-2x$ remains as it is. Result: $-2x + 2$.

3. Combine the like terms (terms that have x). $12x$ minus $5x$ equals $7x$. The constant -4 remains unchanged. Simplified: $7x - 4$.

4. When we combine the x terms, $-16x + 24x$ gives $8x$. The constant -9 is unaltered. Answer: $8x - 9$.

5. Combine the x coefficients. $6x$ minus $15x$ is $-9x$. The constant 5 remains the same. Simplified: $-9x + 5$.

6. Combine the x coefficients: $5x$ minus $8x$ results in $-3x$. Next, combine the constants: 2 minus 6 is -4. Answer: $-3x - 4$.

7. $5x$ minus $3x$ gives $2x$. And, 10 minus 22 equals -12. Answer: $2x - 12$.

8. The term $-3x^2$ remains as it is. The x term, $4x$, remains unchanged. Combining the constants, $-5 -\ 6$ gives -11. Simplified: $-3x^2 + 4x - 11$.

9. The term $9x^2$ stands alone. Combining the x terms, x remains unchanged. -6 minus 3 equals -9. Simplified: $9x^2 + x - 9$.

10. The term $5x^2$ is unaltered. $3x$ minus $10x$ is $-7x$. The constant -3 remains the same. Answer: $5x^2 - 7x - 3$.

11. The term $4x^2$ stands alone. $-2x$ minus $6x$ results in $-8x$. 5 minus 8 gives -3. Result: $4x^2 - 8x - 3$.

12. The term $3x^2$ is unaltered. $-5x$ minus $7x$ equals $-12x$. 2 minus 4 results in -2. Result: $3x^2 - 12x - 2$.

13. The term $9x^2$ is unchanged. $-x$ minus $5x$ gives $-6x$. 3 minus 9 is -6. Answer: $9x^2 - 6x - 6$.

14. Combine the x^2 terms: $2x^2$ minus $3x^2$ equals $-x^2$. $-7x$ plus $4x$ is $-3x$. The constant 6 remains unchanged. Result: $-x^2 - 3x + 6$.

15. Subtract the value of x (which is 4) from 10 to get the result, 6.

16. Plugging in 6 for x: $6 + 8 = 14$. Hence, with x as 6, $x + 8$ is 14.

17. Multiply 2 by 3 and then subtract 6: $(2 \times 3) - 6 = 6 - 6 = 0$. Thus, if x is 3, $2x - 6$ is 0.

18. Multiply 4 by 2 and subtract from 10: $10 - (4 \times 2) = 10 - 8 = 2$. Here, with x being 2, $10 - 4x$ equals 2.

19. Multiply 8 by 7 and then subtract 3: $(8 \times 7) - 3 = 56 - 3 = 5$. So, if x is 7, $8x - 3$ becomes 53.

20. Multiplying 2 by 9 and subtracting from 20: $20 - (2 \times 9) = 20 - 18 = 2$. This means, for x as 9, $20 - 2x$ is 2.

21. By multiplying 10 by 5 and then subtracting 30: $(10 \times 5) - 30 = 50 - 30 = 20$. Thus, when x is 5, $10x - 30$ gives 20.

22. Subtracting -6 from 5 gives: $5 - (-6) = 5 + 6 = 11$. So, with x as -6, $5 - x$ results in 11.

23. Multiplying 3 by -3 and adding to 22: $22 - (3 \times (-3)) = 22 + 9 = 31$. Hence, for x equal to -3, $22 - 3x$ equals 31.

24. Multiply 9 by -7 and add to 10: $10 - (9 \times (-7)) = 10 + 63 = 73$. Thus, when x is -7, $10 - 9x$ is 73.

25. Multiply 3 by -10 and add to 40: $40 - (3 \times (-10)) = 40 + 30 = 70$. With x as -10, $40 - 3x$ results in 70.

26. Multiply 20 by -2 and subtract 5: $(20 \times (-2)) - 5 = -40 - 5 = -45$. Hence, for x being -2, $20x - 5$ is -45.

27. Multiply -10 by -5 and subtract 8: $(-10 \times (-5)) - 8 = 50 - 8 = 42$. So, if x equals -5, $-10x - 8$ gives 42.

28. Multiplying 4 by -4 and subtracting from -1: $-1 - (4 \times (-4)) = -1 + 16 = 15$. With x as -4, $-1 - 4x$ results in 15.

29. For this expression, replace x with 2 and y with 1: $2(2) + 7(1) = 4 + 7 = 11$. With the values provided, the expression evaluates to 11.

30. Insert the values for a and b into the equation: $3(3) - 5(5) = 9 - 25 = -16$. Using the assigned values, the expression calculates to -16.

31. Place the given values of x and y into the equation: $3(6) - 2(2) + 8 = 18 - 4 + 8 = 22$. By substituting the values for x and y, we get the result as 22.

32. Add the given values of a and b to the formula: $-5(-2) + 2(3) + 6 = 10 + 6 + 6 = 22$. By inputting -2 for a and 3 for b, the sum becomes 22.

33. Plug the provided x and y into the equation: $-4(-4) + 10 - 8(-3) = 16 + 10 + 24 = 50$. Employing the given values for x and y results in a sum of 50.

34. To solve for x, you need to isolate x on one side of the equation. First, subtract 6 from both sides of the equation to get: $x + 6 - 6 = 3 - 6$, $x = -3$.

35. To solve for x in this equation, you want to isolate x on one side. Begin by subtracting 11 from both sides: $5 - 11 = 11 - x - 11 \rightarrow -6 = -x$. Now, to find x, multiply both sides by -1 to get: $-1 \times (-6) = -1 \times (-x) \rightarrow 6 = x$.

36. Start by subtracting 8 from both sides of the equation: $-3 - 8 = 8 + x - 8 \rightarrow -11 = x$.

37. To solve for x, add 2 to both sides of the equation: $x-2+2=-7+2 \rightarrow x=-5$.

38. To isolate x, subtract 6 from both sides of the equation: $-15-6=x+6-6 \rightarrow -21=x$.

39. Start by adding x to both sides of the equation: $10-x+x=-2+x \rightarrow 10=x-2$. Now, add 2 to both sides: $10+2=x-2+2 \rightarrow 12=x$.

40. Add x to both sides of the equation: $22-x+x=-9+x \rightarrow 22=-9+x$. Now, subtract -9 from both sides: $22+9=-9+x+9 \rightarrow 31=x$.

41. To solve for x, add 4 to both sides of the equation: $-4+x+4=28+4 \rightarrow x=32$.

42. Add x to both sides of the equation: $11-x+x=-7+x \rightarrow 11=-7+x$. Now, add 7 to both sides: $11+7=-7+x+7 \rightarrow 18=x$.

43. Add x to both sides of the equation: $35-x+x=-7+x \rightarrow 35=-7+x$. Now, add 7 to both sides: $35+7=-7+x+7 \rightarrow 42=x$.

44. Multiply out the brackets: $4x+8=12$. Subtract 8 from both sides:$4x=4$. Divide by 4:$x=1$.

45. Distribute the -6: $-36+6x=12$. Add 36 to both sides: $6x=48$. Divide by 6: $x=8$.

46. Multiply out the brackets: $5=-5x-10$. Add $5x$ to both sides: $5x+5=-10$. Subtract 5 from both sides: $5x=-15$. Divide by 5: $x=-3$.

47. Distribute the 2: $-10=8+2x$. Subtract 8 from both sides: $-18=2x$. Divide by 2: $x=-9$.

48. Multiply out the brackets: $4x+8=-12$. Subtract 8 from both sides: $4x=-20$. Divide by 4: $x=-5$.

49. Distribute the -6: $-18-12x=30$. Add 18 to both sides: $-12x=48$. Divide by -12: $x=-4$.

50. Distribute the -3: $-12+3x=12$. Add 12 to both sides: $3x=24$. Divide by 3: $x=8$.

51. Distribute the -4: $-24+4x=16$. Add 24 to both sides: $4x=40$. Divide by 4: $x=10$.

52. First, isolate the terms with w by subtracting $2l$ from both sides: $q-2l=2w$. Now, divide both sides by 2 to solve for w. $w=\frac{q-2l}{2}$. When simplified further: $w=\frac{1}{2}q-l$.

53. Initially, to free w from being multiplied, divide both sides by $2y$: $w=\frac{x}{2y}$.

54. Begin by isolating T. To do this, divide both sides by nR: $T=\frac{pv}{nR}$.

55. To get d on its own, subtract both b and c from each side: $d=a-b-c$.

56. The x-coordinate of the midpoint is the average of 5 and 1, which is $\frac{5+1}{2}=3$. Similarly, the y-coordinate of the midpoint is the average of 0 and 4, which is $\frac{0+4}{2}=2$. Midpoint: (3,2).

57. For the x-values: $\frac{2+4}{2}=3$. For the y-values: $\frac{3+7}{2}=5$. Midpoint: (3,5).

58. The central x-coordinate is $\frac{8+2}{2} = 5$ and the y-coordinate is $\frac{1+5}{2} = 3$. Midpoint: $(5,3)$.

59. For x, you have $\frac{5+3}{2} = 4$. For y, $\frac{10+6}{2} = 8$. Midpoint: $(4,8)$.

60. Taking the middle for x: $\frac{4+(-2)}{2} = 1$ and for y: $\frac{-1+7}{2} = 3$. Midpoint: $(1,3)$.

61. Average the x-values: $\frac{2+4}{2} = 3$. For y: $\frac{-5+1}{2} = -2$. Midpoint: $(3,-2)$.

62. The x's midpoint is $\frac{7-5}{2} = 1$. For y's, it's $\frac{6+2}{2} = 4$. Midpoint: $(1,4)$.

63. For the x-axis: $\frac{-2+4}{2} = 1$. For the y-axis: $\frac{8-6}{2} = 1$. Midpoint: $(1,1)$.

64. Use the distance formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-6-(-2)^2 - (8-8)^2} = 4$

65. Calculate the differences in x and y, square them, and add. Then take the square root: $d = \sqrt{(14-4)^2 - (20-(-4)^2} = \sqrt{100+576} = \sqrt{675}$. $d = 26$.

66. Find the differences in x and y, square, add, and root: $d = \sqrt{(-5-(-1))^2 + (6-9)^2} = \sqrt{16+9} = \sqrt{25} = 5$.

67. Use the distance formula: $d = \sqrt{(4-0)^2 + (3-3)^2} = 4$

68. Calculate the x and y differences, square, add, and root: $d = \sqrt{(5-0)^2 + (10-(2))^2} = \sqrt{25+144} = \sqrt{169} = 13$.

69. Find the x and y differences, square, add, and root: $d = \sqrt{(7-4)^2 + (-1-3)^2} = \sqrt{9+16} = \sqrt{25} = 5$.

70. To calculate the distance, we use the distance formula: $d = \sqrt{(10-2)^2 + (-9-6)^2} = \sqrt{64+225} = \sqrt{289} = 17$.

71. Calculate x and y differences, square, add, and root: $d = \sqrt{(6-3)^2 + (-1-3)^2} = \sqrt{9+16} = \sqrt{25} = 5$.

72. Find the x and y differences, square, add, and root: $d = \sqrt{(14-(-2))^2 + (18-(-12))^2} = \sqrt{256+900} = \sqrt{1156} = 34$.

73. Find x and y differences, square, add, and root: $d = \sqrt{(12-2)^2 + (22-(-2)^2} = \sqrt{100+576} = \sqrt{676} = 26$.

CHAPTER

8 Linear Functions

Math topics in this chapter:

- ☑ Finding Slope
- ☑ Writing Linear Equations
- ☑ Graphing Linear Inequalities
- ☑ Write an Equation from a Graph
- ☑ Slope-intercept Form and Point-slope Form
- ☑ Write a Point-slope Form Equation from a Graph
- ☑ Find $x-$ and $y-$intercepts in the Standard Form of Equation
- ☑ Graph an Equation in the Standard Form
- ☑ Equations of Horizontal and Vertical Lines
- ☑ Graph a Horizontal or Vertical line
- ☑ Graph an Equation in Point-Slope Form
- ☑ Equation of Parallel or Perpendicular Lines
- ☑ Compare Linear Function's Graph and Linear Equations
- ☑ Graphing Absolute Value Equations
- ☑ Two-variable Linear Equation Word Problems

Practices

Find the slope of each line.

1) $y = x - 5$
2) $y = 2x + 6$
3) $y = -5x - 8$
4) Line through (2,6) and (5,0)
5) Line through (8,0) and (−4,3)
6) Line through (−2, − 4) and (−4,8)

Solve.

7) What is the equation of a line with slope 4 and intercept 16? ______________

8) What is the equation of a line with slope 3 and passes through point (1,5)? ______________

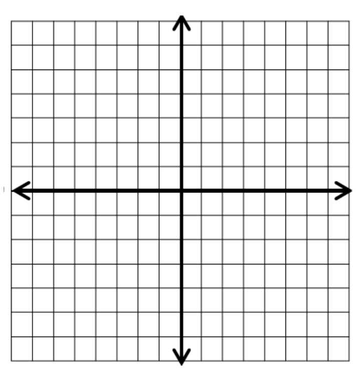
9) What is the equation of a line with slope −5 and passes through point (−2,7)? ______________

10) The slope of a line is −4 and it passes through point (−6,2). What is the equation of the line? ______________

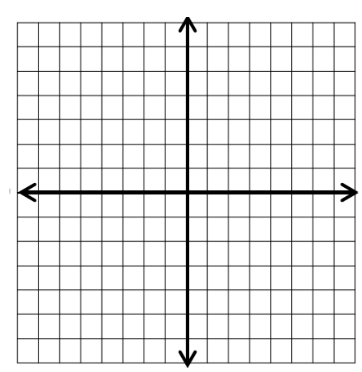
11) The slope of a line is −3 and it passes through point (−3, −6). What is the equation of the line? ______________

Sketch the graph of each linear inequality.

12) $y > 4x + 2$

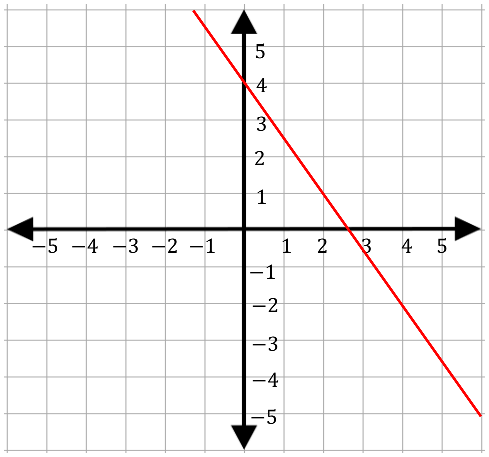


13) $y < -2x + 5$

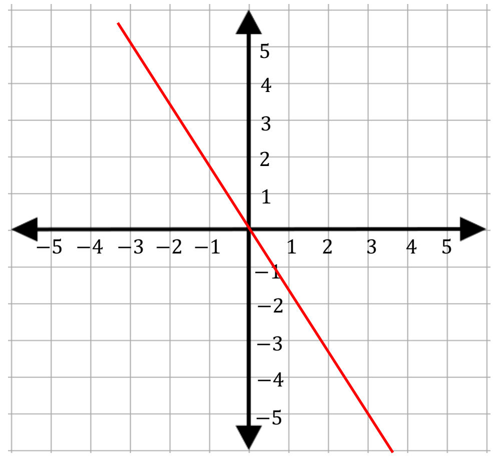


Write an equation of each of the following line in slope-intercept from.

14) ___________



15) _________



Find the equation of each line.

16) Through: $(6, -6)$, slope $= -2$

Point-slope form: ________

Slope-intercept form: ________

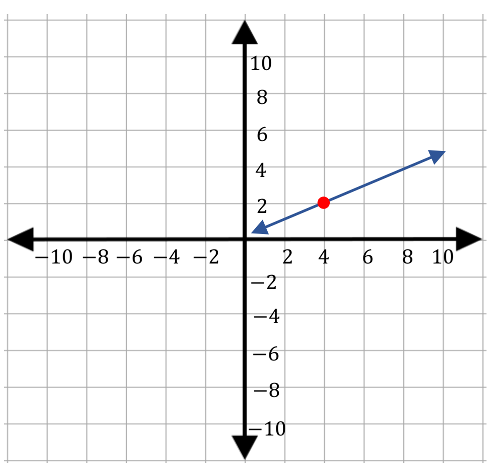
17) Through: $(-7,7)$, slope $= 4$

Point-slope form: _________

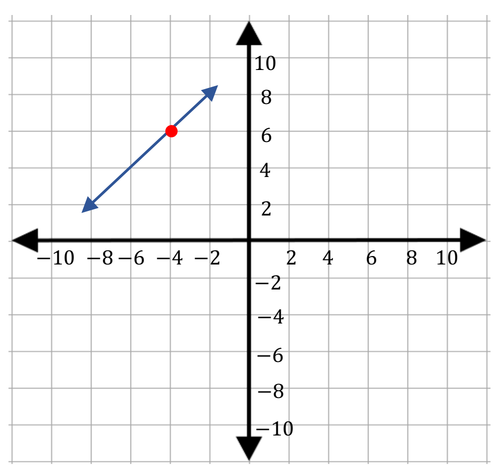
Slope-intercept form: ________

Write equation of the line in point-slope form.

18)



19)



Find the $x -$ intercept of each line.

20) $21x - 3y = -18$

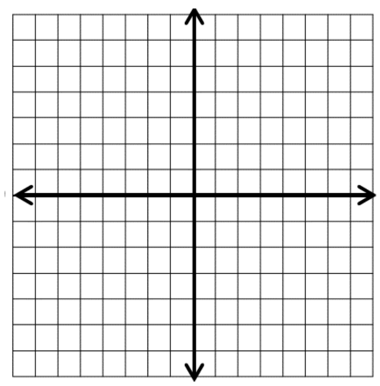
21) $20x + 20y = -10$

22) $8x + 6y = 16$

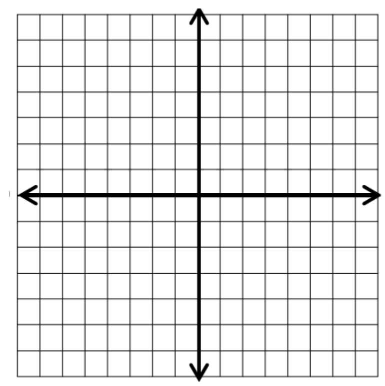
23) $2x - 4y = -12$

Graph each equation.

24) $4x - 5y = 40$



25) $9x - 8y = -72$



Find the equation of the following lines.

26) Write an equation for the horizontal line that passes through $(3, -5)$.

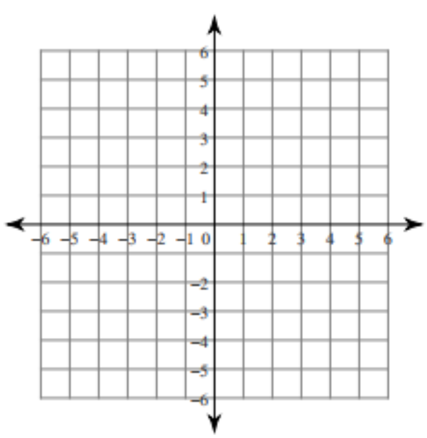
27) Write an equation for the horizontal line that passes through $(-4, 7)$.

28) Write an equation for the vertical line that passes through $(4, 0)$.

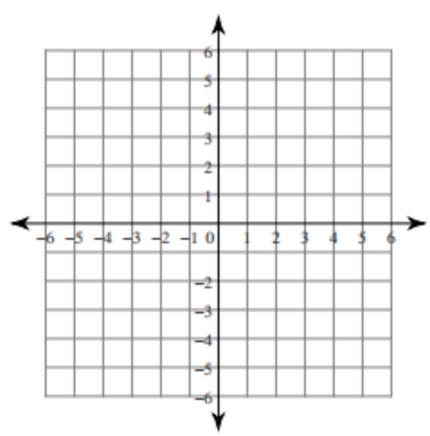
29) Write an equation for the vertical line that passes through $(0, -7)$.

Sketch the graph of each line.

30) Vertical line that passes through $(2, 6)$.

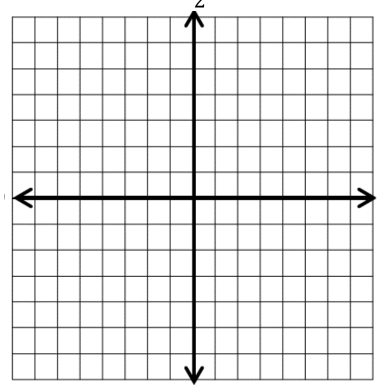


31) Horizontal line that passes through $(5, 3)$.

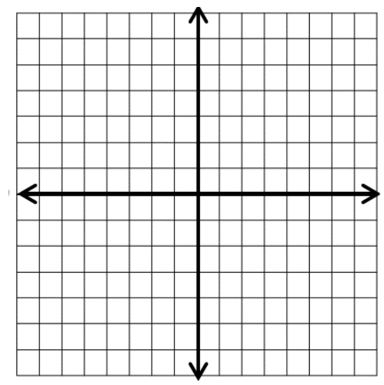


Graph each equation.

32) $y + 3 = -\frac{1}{2}(x - 8)$



33) $y - 8 = -2(x - 1)$



Find the equation of each line with the given information.

34) Through: $(4, 4)$

Parallel to $y = -6x + 5$

Equation: _________

35) Through: $(7,1)$

Perp. to $y = -\frac{1}{2}x - 4$

Equation: ________

36) Through: $(2,0)$

Parallel to $y = x$

Equation: _________

37) Through: $(0, -4)$

Perp. to $y = 2x + 3$

Equation: _________

38) Through: $(-1,1)$

Parallel to $y = 2$

Equation: _________

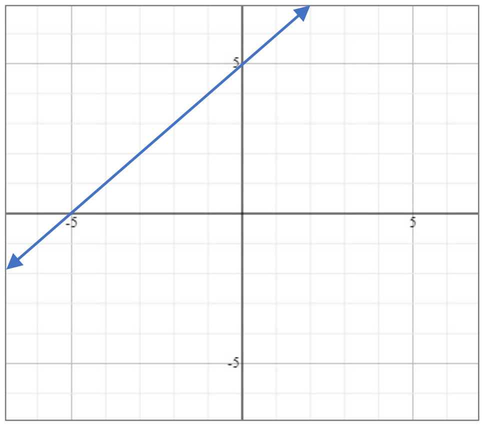
39) Through: $(3,4)$

Perp. to $y = -x$

Equation: _________

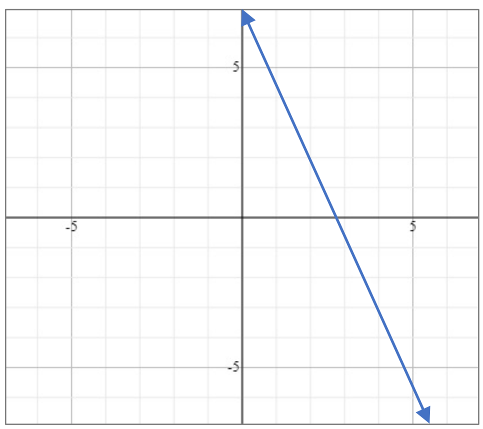
Compare the slope of the function A and function B.

40) Function A:



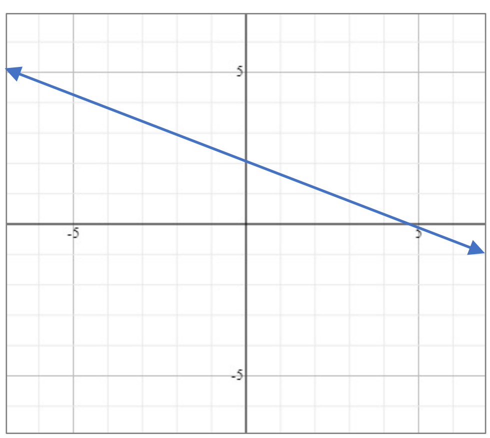
Function B: $y = 6x - 3$

41) Function A:



Function B: $y = -2.5 - 1$

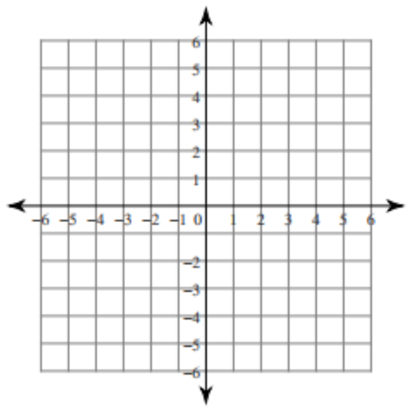
42) Function A:



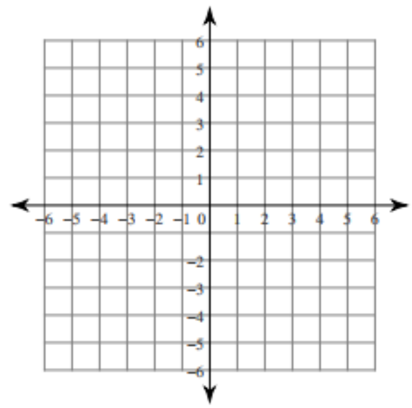
Function B: $y = 2x - 1$

Graph each equation.

43) $y = -|x| - 1$



44) $y = -|x - 3|$



Solve.

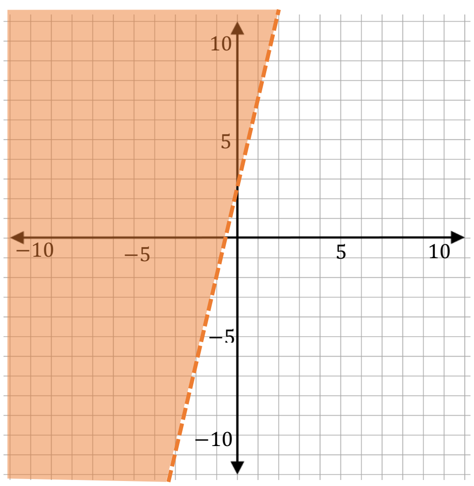
45) John has an automated hummingbird feeder. He fills it to capacity, 8 fluid ounces. It releases 1 fluid ounce of nectar every day. Write an equation that shows how the number of fluid ounces of nectar left, y, depends on the number of days John has filled it, x.

46) The entrance fee to Park City is $9. Additionally, skate rentals cost $4 per hour. Write an equation that shows how the total cost, , depends on the length of the rental in hours, x.

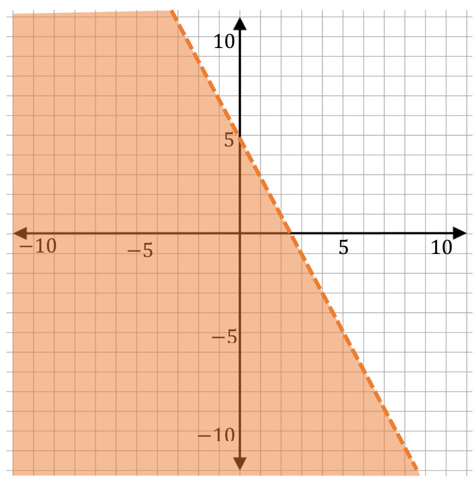
Answers

1) 1
2) 2
3) -5
4) -2
5) $-\frac{1}{4}$
6) -6
7) $y = 4x + 16$
8) $y = 3x + 2$
9) $y = -5x - 3$
10) $y = -4x - 22$
11) $y = -3 - 15$

12) $y > 4x + 2$



13) $y < -2x + 5$



14) $y = -\frac{3}{2}x + 4$

15) $y = -\frac{5}{3}x$

16) Point-slope form: $y + 6 = -2(x - 6)$

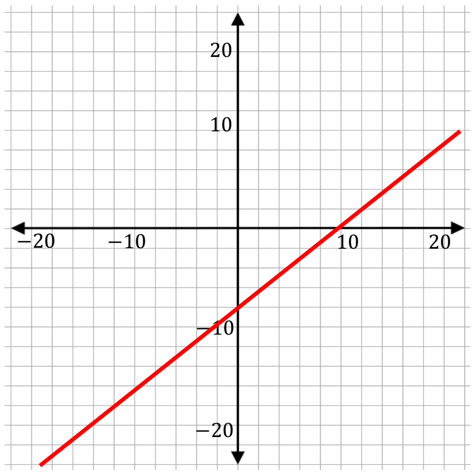
Slope-intercept form: $y = -2x + 6$

17) Point-slope form: $y - 7 = 4(x + 7)$

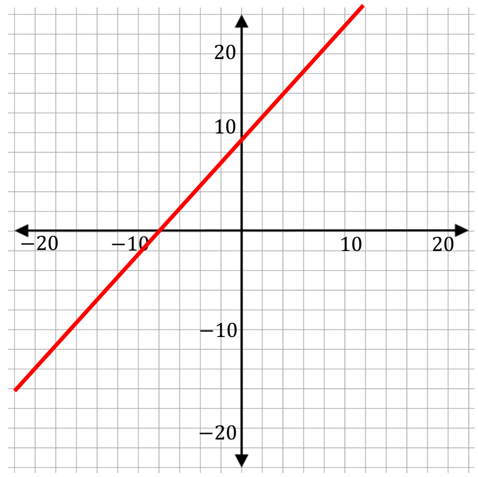
Slope-intercept form: $y = 4 + 35$

18) $(y - 2) = \frac{1}{2}(x - 4)$
19) $(y - 6) = (x + 4)$
20) $-\frac{6}{7}$
21) $-\frac{1}{2}$
22) 2
23) -6

24)



25)



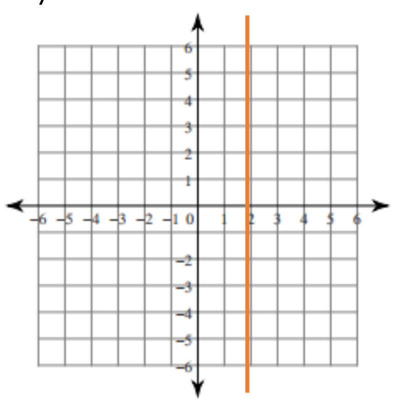
26) $y = -5$

27) $y = 7$

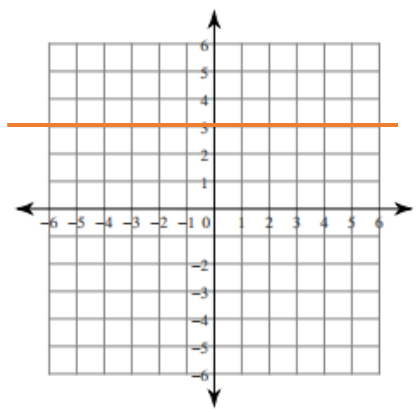
28) $= 4$

29) $x = 0$

30)



31)



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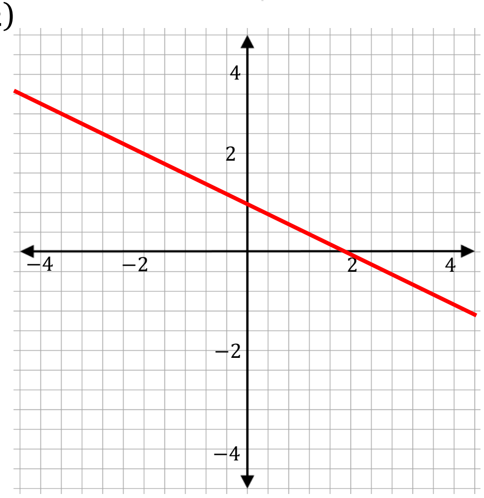

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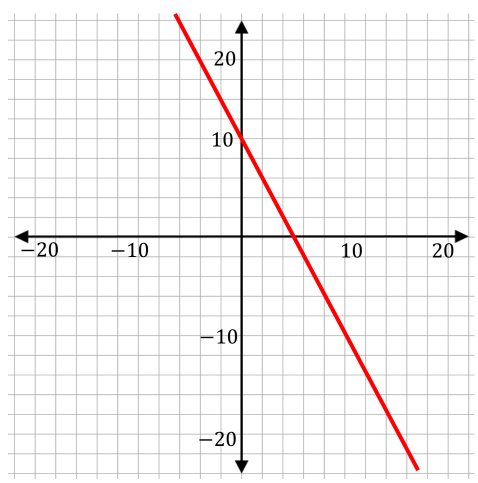


34) $y = -6x + 28$

35) $y = 2x - 13$

36) $y = x - 2$

37) $y = -\frac{1}{2}x - 4$

38) $y = 1$

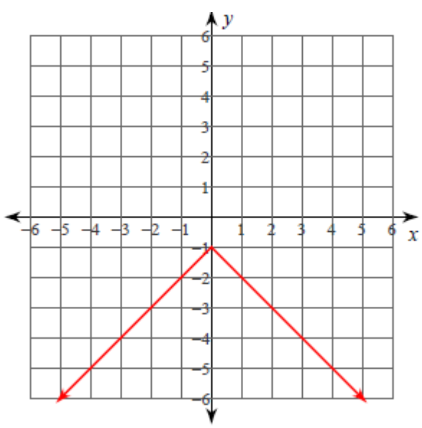
39) $y = x + 1$

40) The slope of function A is 1 and is lower that than the slope of function B (6).

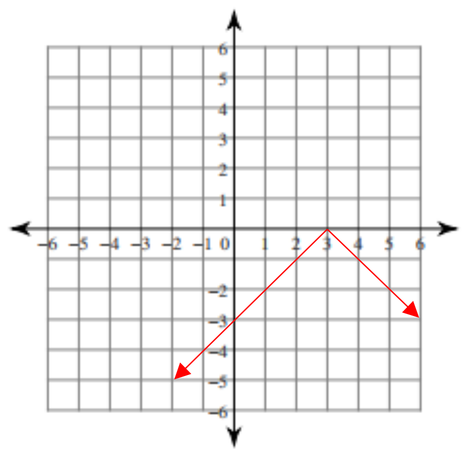
41) Two functions are parallel.

42) Two functions are intersecting.

43) $y = -|x| - 1$



44) $y = -|x - 3|$



45) $y = -x + 8$

46) $y = 4x + 9$

Answers and Explanations

1. This equation is already in the slope-intercept form, which is $y = mx + b$, where m is the slope. The slope here is the coefficient of x, so the slope is 1.

2. Similarly, this is in slope-intercept form. The number in front of x tells you how steep the line is and in which direction it goes. Here, the slope is 2, meaning for every step right on the x-axis, we go 2 steps up on the y-axis.

3. In the slope-intercept form, the slope is the number before x. This time it's -5, indicating a steep decline; for each step right, it goes 5 steps down.

4. Here we have two points, so we'll use the slope formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$. That gives us $m = \frac{0-6}{5-2} = -\frac{6}{3} = -2$.

5. Here, we find the slope with the formula: $= \frac{3-0}{-4-8} = \frac{3}{-12} = -\frac{1}{4}$.

6. Using the slope formula again, we calculate: $m = \frac{8-(-4)}{-4-(-2)} = \frac{12}{-2} = -6$.

7. Plugging these values into the slope-intercept form $y = mx + b$, where m is the slope and b is the y-intercept, we get $y = 4x + 16$.

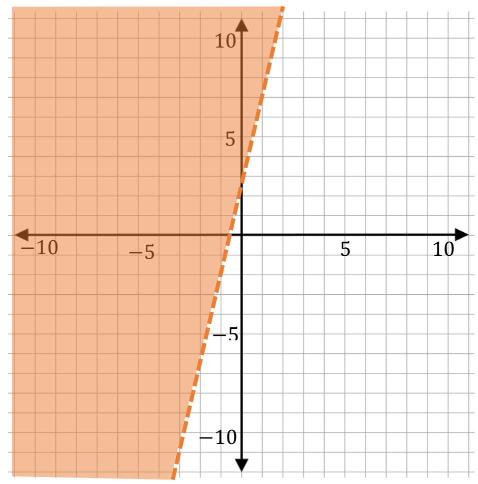
8. First, use the slope and point to find the y-intercept (b) with the formula $y = mx + b$. Plug in the values: $5 = 3(1) + b$, so $b = 2$. The equation is $y = 3x + 2$.

9. We apply the point to find b: $7 = -5(-2) + b$, simplifying to $7 = 10 + b$, which means $b = -3$. The line's equation is $y = -5x - 3$.

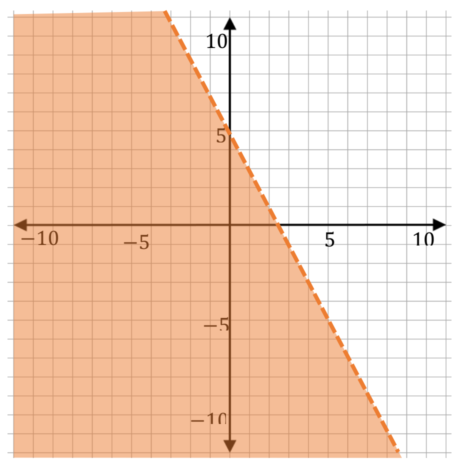
10. Insert the point into the formula: $2 = -4(-6) + b$ gives us $2 = 24 + b$, leading to $b = -22$. The line's equation is $y = -4x - 22$.

11. Use the point for b: $-6 = -3(-3) + b$, which is $-6 = 9 + b$, so $b = -15$. The equation of this line is $y = -3x - 15$.

12. Start with the related equation $y = 4x + 2$ to find the intercepts. The y-intercept is where $x = 0$, which is $(0, 2)$. The x-intercept is where $y = 0$; set $y = 0$ and solve for x to get $x = -0.5$ (or $-\frac{1}{2}$). Now, plot these two points on your graph. Draw a dashed line through them to represent that points on the line aren't included (since it's a 'greater than' inequality). Then, shade above the dashed line because y is greater than $4x + 2$ on that side.



13. Begin with $y = -2x + 5$. The y-intercept is $(0, 5)$, and for the x-intercept, set y to 0 and solve for x, giving $x = 2.5$ (or $\frac{5}{2}$). Plot these points and draw a dashed line through them because it's a 'less than' inequality, and we don't include the line itself. Then shade below the dashed line. This shading indicates where y is less than $-2x + 5$.



14. To determine the slope, pick two points on the line. Let's choose the points $(0, 4)$ and $(\frac{8}{3}, 0)$. Use the formula: $m = \frac{0-4}{\frac{8}{3}-0} = -4 \times \frac{3}{8} = -\frac{3}{2}$. The y-intercept is the point where the line crosses the y-axis. For the given line, the y-intercept is the y-coordinate of the point $(0, 4)$. So, $b = 4$. Using the slope and y-intercept, the equation of line is: $y = -\frac{3}{2}x + 4$.

15. To write the equation of a line in slope-intercept form, which is $y = mx + b$, we need to identify the slope m and the y-intercept b from the graph.

Looking at the line, we can determine the slope by finding two points on the line and calculating the rise over run. Similarly, the y −intercept is the point where the line crosses the y −axis.

Let's choose the points $(0, 0)$ and $(3, -5)$. Calculate the slope m using the formula:

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5-0}{3-0} = -\frac{5}{3}$.

Look at where the line crosses the y —axis to determine the y —intercept m. For the given line, the y-intercept is the y-coordinate of the point $(0, 0)$. So, $b = 0$.

Once you have m and b, you can write the equation of the line: $y = -\frac{5}{3}x$

16. Using the formula $y - y_1 = m(x - x_1)$, where m is the slope and (x_1, y_1) is the given point. $y + 6 = -2(x - 6)$. To convert to slope-intercept form (which is of the form $y = mx + b$), we can simplify the point-slope form equation: $y = -2x + 6$.

17. Applying the same formula and using the point $(-7,7)$ and slope 4: $y - 7 = 4(x + 7)$. To rewrite in slope-intercept form, simplify the point-slope form equation: $y = 4x + 35$.

18. To determine the slope (m) between these two points, use the slope formula: $m = \frac{y_2-y_1}{x_2-x_1}$. Plugging points $(4,2)$ and $(2,1)$ in the formula: $m = \frac{2-1}{4-2} = \frac{1}{2}$. Using the point-slope formula $y - y_1 = m(x - x_1)$ and the given point $(4, 2)$: $y - 2 = \frac{1}{2}(x - 4)$.

19. To find the slope (m) between these two points, use the slope formula: $m = \frac{y_2-y_1}{x_2-x_1}$. Plugging points $(-4,6)$ and $(-6,4)$ in the formula: $m = \frac{4-6}{-6-(-4)} = \frac{-2}{-2} = 1$. Using the point-slope formula $y - y_1 = m(x - x_1)$ and the given point $(-4, 6)$: $y - 6 = (x + 4)$.

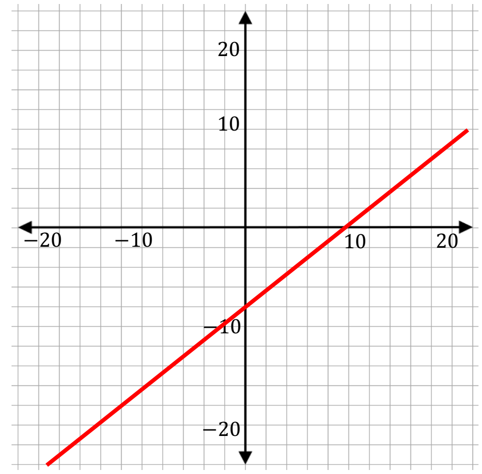
20. To find the x-intercept, set $y = 0$ and solve for x. $21x = -18 \rightarrow x = \frac{-18}{21} \rightarrow x = -\frac{6}{7}$.

21. For the x-intercept, make $y = 0$ and rearrange to get x on its own. $20x = -10$. Divide both sides by 20: $x = -\frac{10}{20}$. $x = -0.5$.

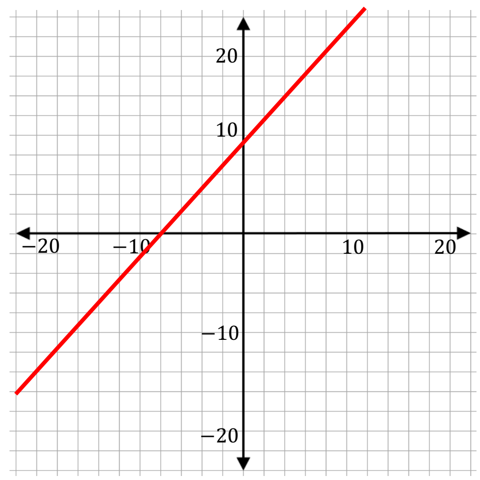
22. Finding the x-intercept requires letting y be zero. $8x = 16$. Divide each side by 8: $x = 2$.

23. For the x-intercept, substitute y with zero. $2x = -12$. Now, divide through by 2: $x = -6$.

24. To graph the equation $4x - 5y = 40$, let's find two points by choosing values for x and solving for y, and then plot these points on a graph. First point: Let $x = 0$, then the equation becomes $-5y = 40$. Solving for y gives $y = -8$. So, our first point is $(0, -8)$. Second point: Let $x = 10$, then the equation becomes $40 - 5y = 40$. This simplifies to $-5y = 0$, hence $y = 0$. Our second point is $(10, 0)$. Plot these points $(0, -8)$ and $(10, 0)$ on a graph, then draw a straight line through them. This line represents all solutions to the equation $4x - 5y = 40$.



25. For the equation $9x - 8y = -72$, we'll also find two points: First point: Let $x = 0$, then the equation becomes $-8y = -72$. Solving for y gives $y = 9$. So, our first point is $(0, 9)$. Second point: Let $y = 0$, then the equation becomes $9x - 0 = -72$. This simplifies to $9x = -72$, hence $y = -8$. Our second point is $(-8, 0)$. Plot these points $(0, 9)$ and $(-8, 0)$ on a graph, and draw a line through them. This line represents all solutions to the equation $9x - 8y = -72$.



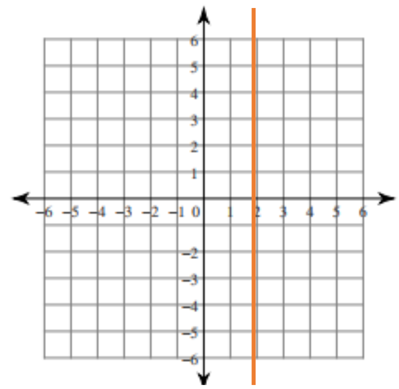
26. For a horizontal line passing through $(3, -5)$, remember that in a horizontal line, the y-coordinate remains constant while the x-coordinate can change. So, every point on this line has a y-coordinate of -5, no matter what the x-coordinate is. The equation that represents this line is simply $y = -5$. This equation tells us that no matter what value x takes, y will always be -5.

27. Similarly, a horizontal line passing through $(-4, 7)$ maintains a constant y-coordinate across all its points, which in this case is 7. This is because a horizontal line doesn't climb or fall as it moves to the right or left. Thus, the equation of this line is $y = 7$, indicating that the y-value remains at 7 for any value of x.

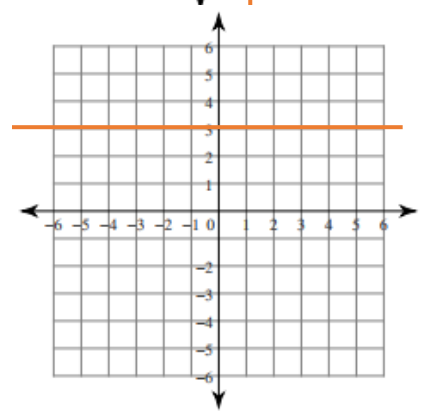
28. For the vertical line passing through $(4, 0)$, focus on how in a vertical line, it's the x-coordinate that stays constant. Here, every point on the line will have an x-coordinate of 4. Since vertical lines run up and down without moving left or right, the equation for this line is $x = 4$. This equation means that x is always 4, regardless of the y-value.

29. A vertical line passing through $(0,-7)$ keeps its x-coordinate constant. In this case, every point on the line has an x-coordinate of 0. Vertical lines are like walls or edges that don't move forward or backward but extend infinitely up and down. Hence, the equation representing this line is $x=0$, which is actually the y-axis on a graph. It signifies that the line lies exactly at $x=0$, for any value of y.

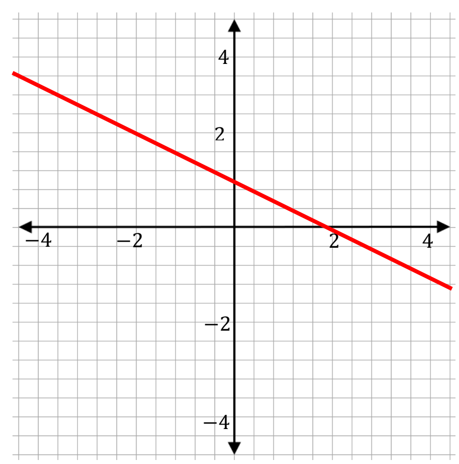
30. To sketch the vertical line that passes through $(2,6)$, first understand that a vertical line means the x-coordinate is constant. For this line, x is always 2, no matter what y is. This is represented by the equation $x=2$. To draw this, find the point on the x-axis where x is 2. From this point, draw a straight line straight up and down (perpendicular to the x-axis). This line doesn't depend on the y-coordinate given in the point $(2,6)$; it will be the same vertical line whether it passes through $(2,6)$ or $(2,-3)$ or any point where x is 2.



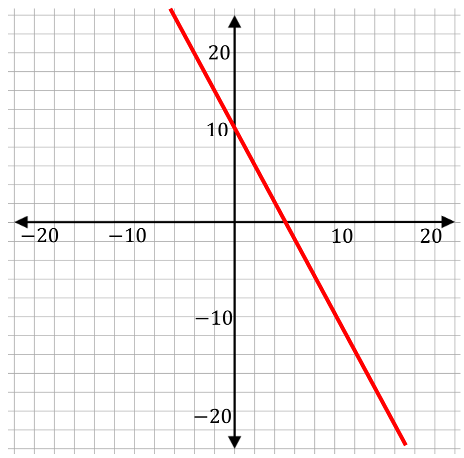
31. For the horizontal line passing through $(5,3)$, you need to know that in a horizontal line, the y-coordinate is the same everywhere. Here, y is always 3, which is shown in the equation $y=3$. Start by locating the point on the y-axis where y is 3. From there, draw a straight line that goes left and right (parallel to the x-axis). This line remains at height 3 above the x-axis all along, showing that for any x-coordinate, whether it's 5 or -2 or any other number, y is always 3. The fact that the given point is $(5,3)$ simply confirms that this point lies on the line; the line itself extends infinitely in both directions.



32. To graph the equations, we need to rewrite them in a more familiar format and then plot points or use key features like slope and y-intercept.For $y+3=-\frac{1}{2}(x-8)$. First, simplify the equation. Expand the right-hand side: $-\frac{1}{2}(x-8)=-\frac{1}{2}x+4$ (since $-\frac{1}{2}$ times -8 is 4). Next, bring the equation to the form $y=mx+b$ by isolating y: $y=-\frac{1}{2}x+4-3$ (subtract 3 from both sides). So, $y=-\frac{1}{2}x+1$. Plot the y-intercept $(0,1)$ on the graph. The -intercept is $(2,0)$. Draw the line through these points.



33. Begin by expanding the equation: $-2(x-1)=-2x+2$. Rearrange it to get y on one side: $y=-2x+2+8$. Simplify it to $y=-2x+10$. This is again in $y=mx+b$ form, with a slope (m) of -2 and a y-intercept (b) of 10. Plot the y-intercept $(0,10)$. Then, plot the x-intercept $(5,0)$. Draw the line through these points.



34. Parallel lines have the same slope. The slope of $y=-6x+5$ is -6. Using the point $(4,4)$ and the slope -6 in the point-slope form: $y-4=-6(x-4)$. Simplify to get the equation: $y=-6x+28$.

35. Perpendicular lines have slopes that are negative reciprocals. The negative reciprocal of $-\frac{1}{2}$ is 2. Use the point $(7,1)$ and the slope 2: $y-1=2(x-7)$. Simplify to get the equation: $y=2x-13$.

36. The slope of $y=x$ is 1. Parallel lines have the same slope. Using the point $(2,0)$ and the slope 1: $y-0=1(x-2)$. Simplify this equation: $y=x-2$.

37. The negative reciprocal of 2 (the slope of the given line) is $-\frac{1}{2}$. Use the point $(0,-4)$ and the slope $-\frac{1}{2}$: $y+4=-\frac{1}{2}\ (x-0)$. Simplify to get the equation: $y=-\frac{1}{2}x-4$.

38. The slope of $y=2$ (a horizontal line) is 0. Parallel lines have the same slope. Since the slope is 0, the line is horizontal, and the equation is simply $y=1$.

39. The negative reciprocal of -1 (the slope of the given line) is 1. Use the point $(3,4)$ and the slope 1: $y-4=1(x-3)$. Simplify this equation. $y=x+1$.

40. For function A, we can determine the slope by finding two points on the line where it crosses the grid lines exactly and then calculate the rise over run. Two points are $(-5,0)$ and $(0,5)$. So, slope is: $m=\frac{5-0}{0-(-5)}=1$. For function B, the equation is given as $y=6x-3$. In this linear equation format, $y=mx+b$, the coefficient of x is the slope. Therefore, the slope of function B is 6. Comparing the slopes of function A and function B shows that function B has a steeper slope than function A. The slope of function A is 1, while the slope of function B is 6

41. Let's calculate the slope of function A by choosing two points where the line crosses the grid exactly, then use the rise over run formula to find the slope. Two points are $(2.4,0)$ and $(0,6)$. So, slope is: $m=\frac{6-0}{0-2.4}=-2.5$. From the equation of function B, $y=-2.5x-1$, we know that the slope is -2.5. The slopes are the same, so the two functions are parallel.

42. For Function A, the slope can be calculated by finding two points on the line and using the formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$

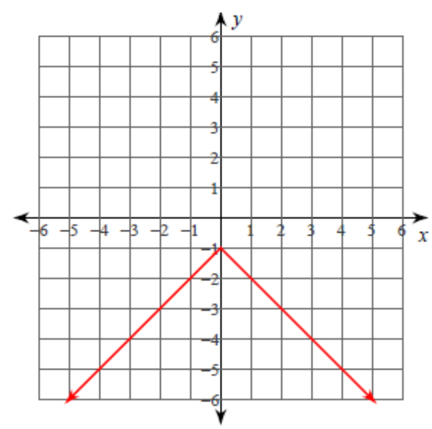
Let's choose the points $(-2, 3)$ and $(0, 2)$. Calculate the slope m using the formula:

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-3}{0-(-2)} = -\frac{1}{2}$.

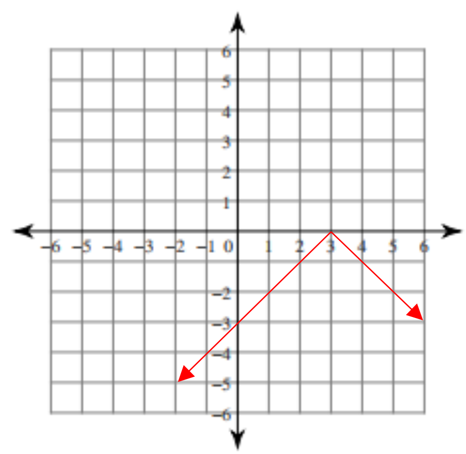
The slope (m) of a linear function in the form $y = mx + b$ is the coefficient of x, which represents the rate of change of y with respect to x. For Function B, the slope is 2, since that's the coefficient of x in the equation.

Considering that the product of the slope of function A and the slope of function B is equal to 1, then the two functions are perpendicular to each other.

43. Start by understanding the absolute value function $|x|$. This function takes the input x and gives its positive magnitude. Now, let's consider the negative sign in front of the absolute value: $-|x|$. This means that whatever positive value the absolute value function returns, we make it negative. So, for $x > 0$, $-|x|$ will be negative, and for $x < 0, -|x|$ will be positive. The -1 at the end of the equation means that we shift the entire graph downward by 1 unit. To graph this equation, start by drawing the graph of $y = |x|$, which is a V-shaped graph centered at the origin $(0,0)$. Then, for all points above the x-axis, make the y-values negative (reflect them below the x-axis) and shift the entire graph down by 1 unit. Your final graph should resemble an upside-down V shape with its vertex at $(0, -1)$.



44. Start by understanding the absolute value function $|x - 3|$. This function takes the input x and subtracts 3 before taking its positive magnitude. So, if $x = 5$, then $|x - 3| = |5 - 3| = |2| = 2$. Now, let's consider the negative sign in front of the absolute value: $-|x - 3|$. Just like in the previous equation, this means that whatever positive value the absolute value function returns, we make it negative. To graph this equation, start with the graph of $y = |x - 3|$. This is also a V-shaped graph, but it will be centered at $x = 3$ because of the "$x - 3$" inside the absolute value. Now, for all points to the right of $x = 3$, make the y-values negative (reflect them below the x-axis). Your final graph should look like an upside-down V shape, centered at $x = 3$, with its vertex at $(3, 0)$, and extending to the left side of the graph.



45. We know that the feeder starts with 8 fluid ounces of nectar, and each day, 1 fluid ounce is released. So, as the number of days, x, increases, the amount of nectar left, y, decreases. We can represent this relationship with the equation: $y = -x + 8$.

46. The total cost, y, depends on two factors: the entrance fee, which is a fixed cost of $\$9$, and the skate rental cost, which is $\$4$ per hour. As the length of the rental in hours, x, increases, the rental cost increases proportionally. We can represent this relationship with the equation: $y = 4x + 9$.

CHAPTER

9 Inequalities and System of Equations

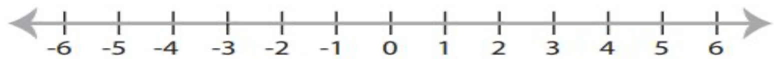
Math topics in this chapter:

- ☑ One–Step Inequalities
- ☑ Multi–Step Inequalities
- ☑ Compound Inequalities
- ☑ Write a Linear Inequality from a Graph
- ☑ Graph Solutions to Linear Inequalities
- ☑ Solve Advanced Linear Inequalities in Two-Variables
- ☑ Graph Solutions to Advanced Linear Inequalities
- ☑ Absolute Value Inequalities
- ☑ Systems of Equations
- ☑ Find the Number of Solutions to a Linear Equation
- ☑ Write a System of Equations Given a Graph
- ☑ Systems of Equations Word Problems
- ☑ Solve Linear Equations Word Problems
- ☑ Systems of Linear Inequalities
- ☑ Write Two-variable Inequalities Word Problems

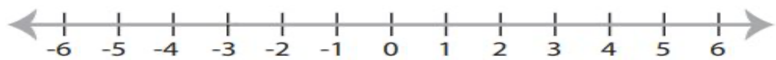
Practices

Solve each inequality and graph it.

1) $x - 2 \geq -2$



2) $2x - 3 < 9$



Solve each inequality.

3) $x + 13 > 4$

4) $x + 6 > 5$

5) $-12 + 2x \leq 26$

6) $-2 + 8x \leq 14$

7) $6 + 4x \leq 18$

8) $4(x + 3) \geq -12$

9) $2(6 + x) \geq -12$

10) $3(x - 5) < -6$

11) $10 + 5x < -15$

12) $6(6 + x) \geq -18$

13) $2(x - 5) \geq -14$

14) $6(x + 4) < -12$

15) $3(x - 8) \geq -48$

16) $-(6 - 4x) > -30$

17) $2(2 + 2x) > -60$

18) $-3(4 + 2x) > -24$

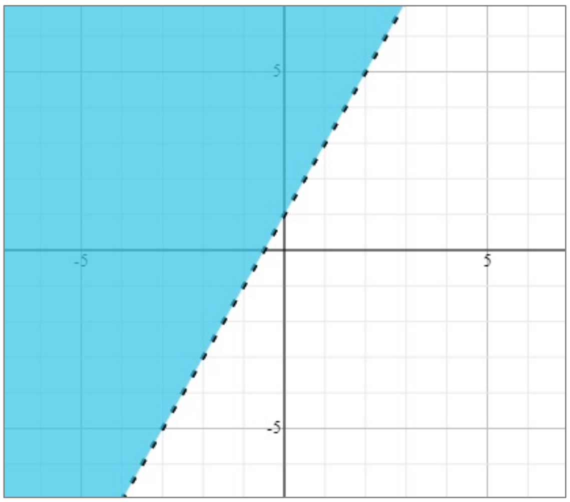
Solve each inequality.

19) $5x \leq 45$ and $x - 11 > -21$

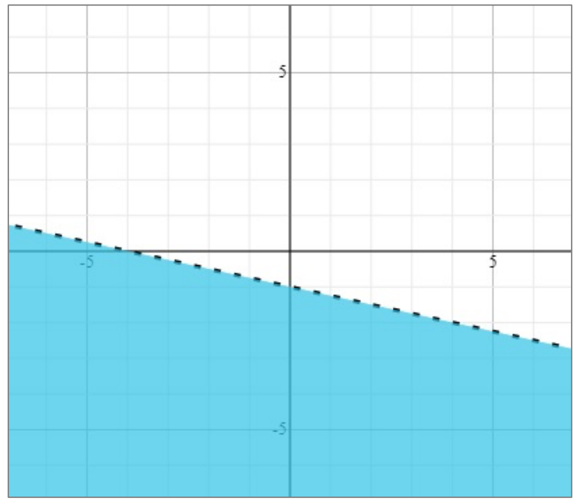
20) $-7 < x - 9 < 8$

Write the slope-intercept from equation of the following graph.

21)



23)



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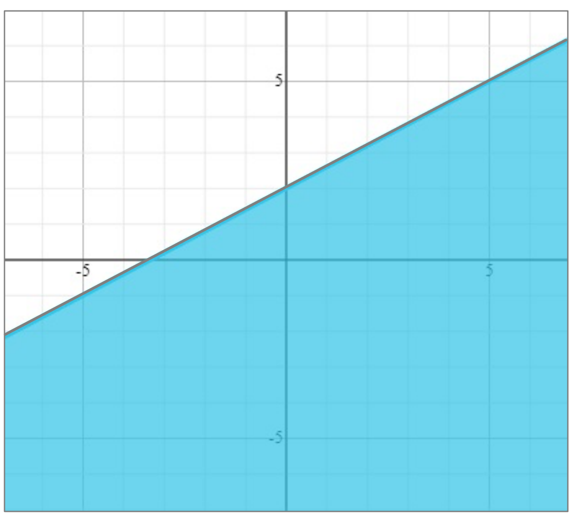

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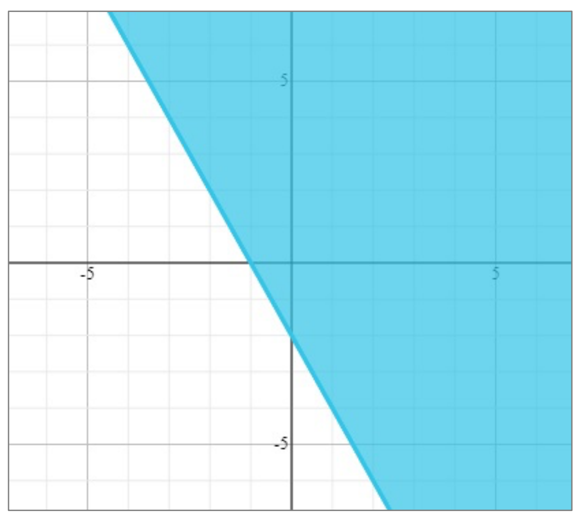


Solve the following inequality and graph the solution.

25) $10 + 6 \leq -2$

27) $-2f + 10 \geq 6$

26) $-r + 8 \leq 4$

28) $1 + 3p > 7$

Solve each inequality.

29) $8x - 3 \geq 4y + 2$

30) $4x - 3 \geq 5y + x$

31) $y \leq \frac{3}{2} + 4$

32) $5x - 2y \leq 10$

Graph the solution of each inequality.

33) $7x + 3 \geq 1 - 2x$

34) $\frac{x+4}{-4} > 8x + 2$

Solve each inequality.

35) $|x| - 4 < 17$

36) $6 + |x - 8| > 15$

37) $\left|\frac{x}{2} + 3\right| > 6$

38) $\left|\frac{x+5}{4}\right| < 7$

Solve each system of equations.

39) $\begin{cases} -2x + 2y = -4 \\ 4x - 9y = 28 \end{cases}$ $x =$ $y =$

40) $\begin{cases} x + 8y = -5 \\ 2x + 6y = 0 \end{cases}$ $x =$ $y =$

41) $\begin{cases} 4x - 3y = -2 \\ x - y = 3 \end{cases}$ $x =$ $y =$

42) $\begin{cases} 2x + 9y = 17 \\ -3x + 8y = 39 \end{cases}$ $x =$ $y =$

How many solutions does the following equation have?

43) $4n = 8 + 5n$

44) $5 - 9f = -9f$

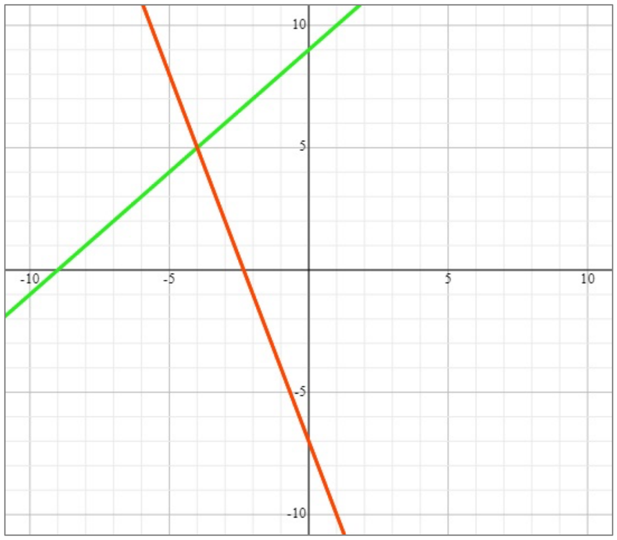
45) $0 = 3z - 3z$

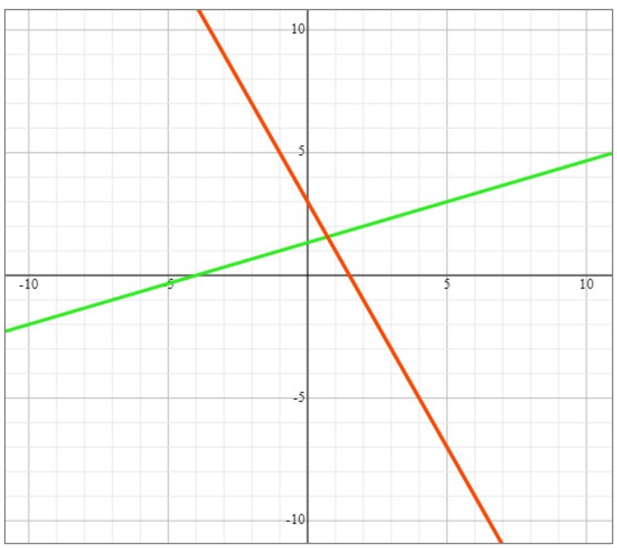
46) $-9x + 2 = -9x$

47) $20 + 12y = 11y$

48) $10h - 2 = -4h$

Write a system of equations for the following graph.

49) 

50) 

Solve each word problem.

51) The equations of two lines are $3x - y = 7$ and $2x + 3y = 1$. What is the value of x in the solution for this system of equations?

52) The perimeter of a rectangle is 100 feet. The rectangle's length is 10 feet less than 5 times its width. What are the length and width of the rectangle?

Solve each word problem.

53) A golf club charges $150 to join the club and $15 for every hour using the driving range. Write an equation to express the cost C in terms of h hours playing tennis.

54) Susan is twice as old as Jane. In 4 years, Susan will be 24 years old. How old is Jane now?

55) A movie ticket costs $7. Popcorn costs $3 more than the ticket. If Alex bought 1 movie ticket and 1 popcorn, how much did he spend in total?

Solve each system of inequalities and graph them.

56) $\begin{cases} x + 2y \leq 3 \\ y - x \geq 0 \\ y \geq -2 \end{cases}$

57) $\begin{cases} x < 3 \\ x + y > -2 \\ y - 1 \leq x \end{cases}$

Solve each word problem.

58) James used his first 2 tokens in Glimmer Arcade to play a Roll-and-Score game. Then he played his favorite game, Balloon Bouncer, over and over until he ran out of tickets. Balloon Bouncer costs 4 tokens per game and James started the game with a bucket of 38 tokens. Write an equation James can use to find how many games of Balloon Bouncer, g, he played.

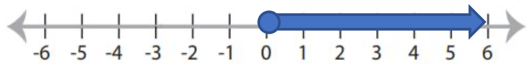
59) Sara buys juice and soda for the party and wants to spend no more than $46. The price of each bottle of soda is 3 dollars and each bottle of fruit juice is 1 dollar. Write the inequality in a standard form that describes this situation. Use the given numbers and variables below.

x = the number of bottles of soda

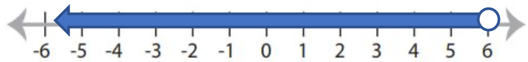
y = the number of bottles of juice

Answers

1) $x \geq 0$



2) $x < 6$



3) $x > -9$

4) $x > -1$

5) $x \leq 19$

6) $x \leq 2$

7) $x \leq 3$

8) $x \geq -6$

9) $x \geq -12$

10) $x < 3$

11) $x < -5$

12) ≥ -9

13) $x \geq -2$

14) $x < -6$

15) $x \geq -8$

16) $x > -6$

17) $x > -16$

18) $x < 2$

19) $-10 < x \leq 9$

20) $2 < x < 17$

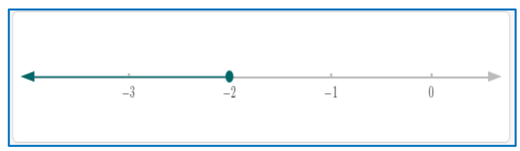
21) $y > 2x + 1$

22) $y \leq \frac{3}{5}x + 2$

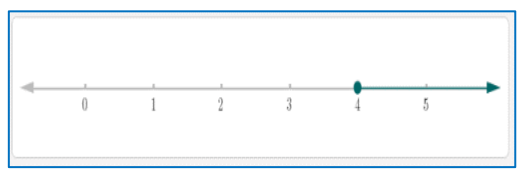
23) $y < -\frac{1}{4}x - 1$

24) $y \geq -2x - 2$

25) $p \leq -2$

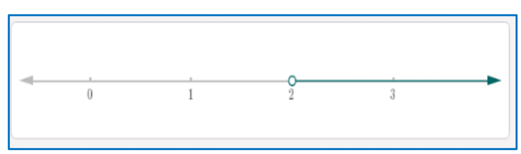


26) $r \geq 4$



27) $f \leq 2$

28) $p > 2$



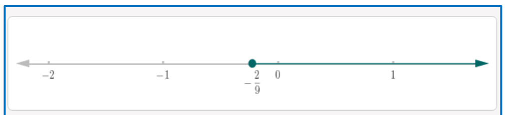
29) $\{(x, y) | y \in R, x \geq \frac{4y+5}{8}\}$

30) $\{(x, y) | y \in R, x \geq \frac{5y+3}{3}\}$

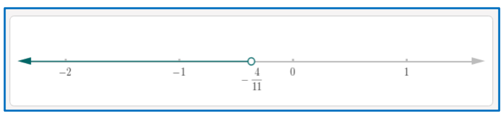
31) $\{(x, y) | y \in R, x \geq \frac{2y-8}{3}\}$

32) $\{(x, y) | y \in R, x \leq \frac{2y+10}{5}\}$

33) $x \geq -\frac{2}{9}$



34) $x < -\frac{4}{11}$



35) $-21 < x < 21$

36) $x > 17$ or $x < -1$

37) $x > 6$ or $x < -18$

38) $-33 < x < 23$

39) $x = -2, y = -4$

40) $x = 3, y = -1$

41) $x = -11, y = -14$

42) $x = -5, y = 3$

43) One solution

44) No solution

45) Infinitely solutions

46) No solution

47) One solution

48) One solution

49) $\begin{cases} y = -3x - 7 \\ y = x + 9 \end{cases}$

50) $\begin{cases} 2x + y = 3 \\ -x + 3y = 4 \end{cases}$

51) $x = 2$

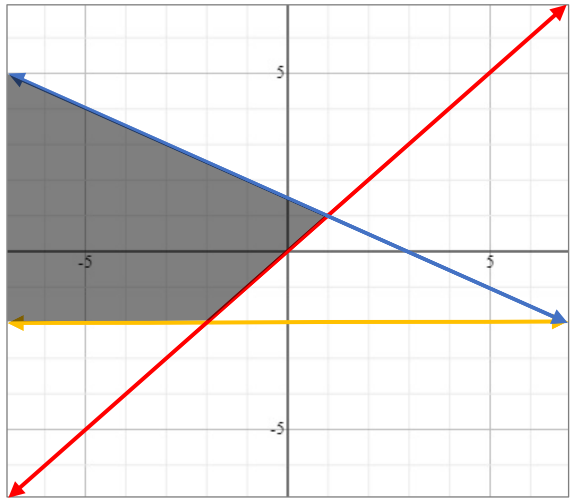
52) 10, 40

53) $C = 15h + 150$

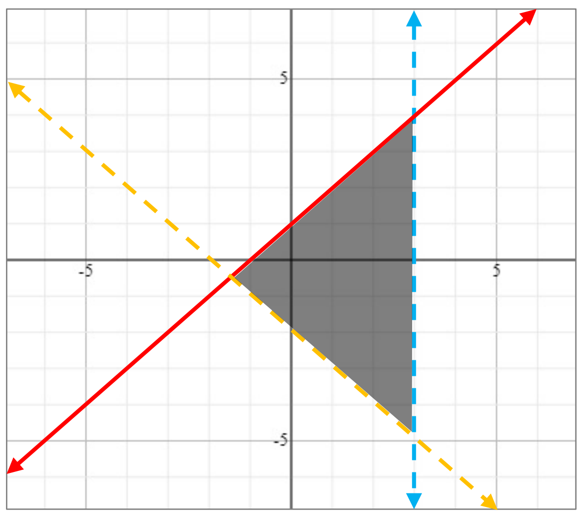
54) 10

55) $17

56) $\begin{cases} y \leq -\frac{1}{2}x + \frac{3}{2} \\ y \geq x \\ y \geq -2 \end{cases}$



57) $\begin{cases} x < 3 \\ y > -x - 2 \\ y \leq x + 1 \end{cases}$

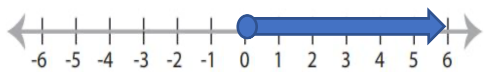


58) $4g + 2 = 38$

59) $3x + y \leq 46$

Answers and Explanations

1. To solve this inequality, we want to isolate x on one side. Here are the steps: Add 2 to both sides of the inequality: $x - 2 + 2 \geq -2 + 2$. This simplifies to $x \geq 0$, because $-2 + 2 = 0$. To graph this inequality, you draw a number line, make a solid circle on 0 to show that 0 is included in the solution (because it's "greater than or equal to"), and shade all the numbers to the right of 0 to indicate that all those numbers are part of the solution.



2. Solving this inequality is about finding the range of x values that make the inequality true: first, add 3 to both sides to cancel out the -3: $2x - 3 + 3 < 9 + 3$. This gives us $2x < 12$. Now, divide both sides by 2 to find the value of x:$\frac{2x}{2} < \frac{12}{2}$. We end up with $x < 6$. For graphing this inequality, you'd draw a number line and place an open circle on 6 (since 6 is not included in the solution), and shade all the numbers to the left of 6, indicating that the solution includes all numbers less than 6.

3. To find the value of x, we reduce the inequality. Subtract 13 from both sides: $x > 4 - 13$, resulting in $x > -9$.

4. Here, we again simplify by moving 6 to the other side. So, $x > 5 - 6$, giving $x > -1$.

5. Bring -12 to the other side, then divide the result by 2: $2x \leq 38$, so $x \leq 19$.

6. Shift -2 to the right side of the inequality and then divide by 8: $x \leq \frac{16}{8}$, simplifying to $x \leq 2$.

7. Move 6 across and divide by 4 to isolate $x \leq \frac{12}{4}$, which simplifies to $x \leq 3$.

8. Start by expanding the left side: $4x + 12 \geq -12$. Next, subtract 12 from both sides to isolate the term with $4x \geq -24$. Finally, divide everything by 4 to solve for $x \geq -6$.

9. First, expand the left side: $12 + 2x \geq -12$. Then, move 12 to the right side by subtracting it: $2x \geq -24$. Divide by 2 to find $x \geq -12$.

10. Expand the multiplication: $3x - 15 < -6$. Add 15 to both sides to bring x terms on one side: $3x < 9$. Divide by 3 to isolate $x < 3$.

11. First, move 10 to the other side by subtracting it: $5x < -25$. Then divide everything by 5 to get $x < -5$.

12. Expand the left side: $36 + 6x \geq -18$. Subtract 36 from both sides: $6x \geq -54$. Divide by 6 to solve for $x \geq -9$.

13. Expand the multiplication: $2x - 10 \geq -14$. Add 10 to both sides: $2x \geq -4$. Divide by 2 to find x: $x \geq -2$.

14. Expand the left side: $6x + 24 < -12$. Subtract 24 from both sides: $6x < -36$. Divide by 6 to isolate $x < -6$.

15. First, expand the left side: $3x - 24 \geq -48$. Add 24 to both sides: $3x \geq -24$. Finally, divide by 3: $x \geq -8$.

16. Simplify the left side: $-6 + 4x > -30$. Add 6 to both sides: $4x > -24$. Divide by 4 to solve for x: $x > -6$.

17. First, expand the multiplication: $4 + 4x > -60$. Subtract 4 from both sides: $4x > -64$. Divide everything by 4 to find $x > -16$.

18. Expand the multiplication: $-12 - 6x > -24$. Add 12 to both sides: $-6x > -12$. Divide by -6 and reverse the inequality sign: $x < 2$.

19. For the first part, $5x \leq 45$, divide both sides by 5 to isolate x: $x \leq 9$. For the second part, $x - 11 > -21$, add 11 to both sides to solve for x: $x > -10$. Combining these two, the solution is x values that satisfy both conditions, which is $-10 < x \leq 9$.

20. This inequality can be split into two parts: $x - 9 > -7$ and $x - 9 < 8$. For the first part, $x - 9 > -7$, add 9 to both sides: $x > 2$. For the second part, $x - 9 < 8$, again add 9 to both sides: $x < 17$. The solution is the range of x that satisfies both parts: $2 < x < 17$.

21. The inequality is likely in the form of $y > mx + b$ or $y \geq mx + b$, where m is the slope of the line and b is the y-intercept. Given two points from the graph, $(0, 1)$ and $(-0.5, 0)$, we can first determine the slope of the line (m) using the slope formula: $m = \frac{0-1}{-0.5-0} = 2$. The slope of the line (m) is 2. Therefore, the equation of the line using the slope-intercept form $y = mx + b$ with the given y-intercept at point $(0, 1)$ is: $y = 2x + 1$. The inequality will be $y > 2x + 1$.

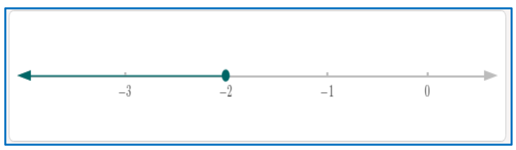
22. To create the inequality from the graph, we first need to determine the slope of the line, which tells us how steep the line is. Given two points from the graph, $(0,2)$ and $(-3.3,0)$, we can first determine the slope of the line (m) using the slope formula: $m = \frac{0-2}{-3.3-0} = 0.6$. Once we have the slope, we use it along with the y-intercept to write the line's equation. The y-intercept is where the line crosses the y-axis, which in this case is at $y = 2$ (the point $(0, 2)$). Putting the slope and y-intercept together, we get the equation of the line: $y = 0.60x + 2$. But for the inequality representing all the points in the shaded area below the line, it will be: $y \leq 0.60x + 2$.

23. To find the inequality, we first need to determine the slope of the line. Given the points $(0, -1)$ and $(-4, 0)$, the slope is: $m = \frac{0-(-1)}{-4-0} = -\frac{1}{4}$. The y-intercept is where the line crosses the y-axis. For the point $(0, -1)$, the x-value is 0, so this point is the y-intercept.

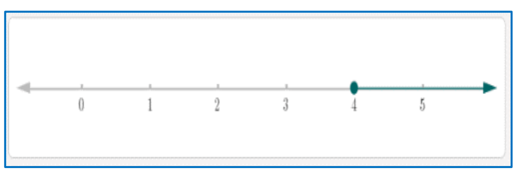
Therefore, the equation of the line is $y = -\frac{1}{4}x - 1$. The shading below the dashed line tells us that we are looking for values of y that are less than the values on the line, but not equal to them, because the line is not included in the solution set. Thus, the inequality representing the shaded region below the line is: $y < -\frac{1}{4}x - 1$.

24. First, we find the slope of the line. We have two points: $(0, -2)$ and $(-1, 0)$. The slope is $m = \frac{0-(-2)}{-1-0} = 2$. Next, we use the y-intercept to write the line's equation. The y-intercept is where the line hits the y-axis. Our line crosses at $(0, -2)$, so the y-intercept is -2. In this case, the equation of the line is: $y = -2x - 2$. The shaded area is above this line, the inequality is: $y \geq -2x - 2$.

25. To find the value of p, we need to isolate it on one side of the inequality. We start by subtracting 10 from both sides to get $6p \leq -12$. Then, we divide both sides by 6 to find p. This will give us $p \leq -2$. The graph of this solution would show a number line with everything to the left of -2 shaded and a closed circle at -2 because p can also be equal to -2.

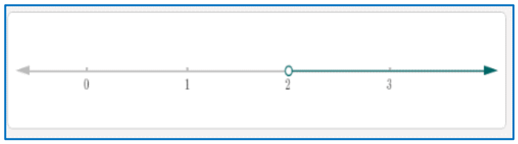


26. Here, we want to find out what r can be. We'll subtract 8 from both sides to get $-r \leq -4$. Multiplying or dividing by a negative number flips the inequality sign, so when we multiply both sides by -1 to get r, the inequality sign flips, giving us $r \geq 4$. On the number line, we'd shade everything to the right of 4 and put a closed circle on 4 since r can be 4 as well.



27. To solve for f, we'll subtract 10 from both sides first, getting $-2f \geq -4$. Next, we divide by -2, which reverses the inequality direction, so we end up with $f \leq 2$. The graph for this would be similar to the first one, with a closed circle at 2 and shading to the left.

28. For this inequality, subtract 1 from both sides to isolate the term with p, giving us $3p > 6$. Dividing by 3 then gives us $p > 2$. Since this is a strict inequality (not including the number 2), we'd draw an open circle at 2 on the number line and shade everything to the right of 2.



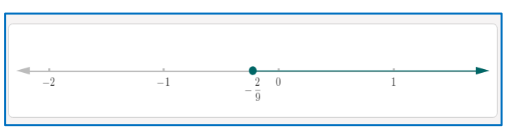
29. To solve for x, we need to get x by itself on one side. First, add 3 to both sides to eliminate the -3: $8x \geq 4y + 5$. Next, divide everything by 8 to find x: $x \geq \frac{4y+5}{8}$. The final answer is: $\left\{(x, y)\middle| y \in R, x \geq \frac{4y+5}{8}\right\}$.

30. Here, we first want to get all the x-terms on one side, so subtract x from both sides: $3x - 3 \geq 5y$. Now, add 3 to both sides: $3x \geq 5y + 3$. Finally, divide by 3 to solve for x: $x \geq \frac{5y+3}{3}$. The final answer is: $\left\{(x, y)\middle| y \in R, x \geq \frac{5y+3}{3}\right\}$.

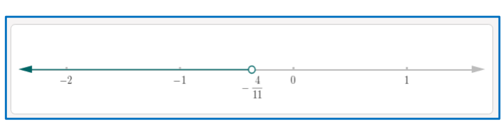
31. To solve for x, we need to reverse the operations affecting x. First, subtract 4 from both sides to get rid of the 4 next to x: $y - 4 \leq \frac{3}{2}x$. Now, multiply everything by $\frac{2}{3}$ to isolate x: $x \geq \frac{2y-8}{3}$. The final answer is: $\left\{(x, y)\middle| y \in R, x \geq \frac{2y-8}{3}\right\}$.

32. Begin by adding $2y$ to both sides to move the y-term to the other side: $5x \leq 10 + 2y$. Then, divide by 5 to solve for x: $x \leq \frac{2y+10}{5}$. The final answer is: $\left\{(x, y)\middle| y \in R, x \leq \frac{2y+10}{5}\right\}$.

33. First, we'll collect all the x-terms on one side by adding $2x$ to both sides: $9x + 3 \geq 1$. Next, we'll move the constant term (3) to the other side by subtracting 3 from both sides: $9x \geq -2$. Then, divide by 9 to isolate x: $x \geq -\frac{2}{9}$. To graph this, we'll draw a number line, place a closed dot on $-\frac{2}{9}$, and shade to the right, indicating that x can be any number greater than or equal to $-\frac{2}{9}$.



34. We want to isolate x, so multiply both sides by -4, which will reverse the inequality because we're multiplying by a negative: $x + 4 < -32x - 8$. Now, get all x-terms to one side by adding $32x$ to both sides: $33x + 4 < -8$. Subtract 4 from both sides to have only x-terms on one side: $33x < -12$. Finally, divide by 33 to solve for x: $x < -\frac{12}{33}$, which simplifies to $x < -\frac{4}{11}$. For the graph, draw a number line, place an open dot on $-\frac{4}{11}$, and shade to the left, showing that x can be any number less than $-\frac{4}{11}$, but not equal to it.



35. First, add 4 to both sides to isolate the absolute value: $|x| < 21$. The inequality now says the distance of x from 0 is less than 21. This means x can be less than 21 and greater than -21. So, we have two inequalities: $x < 21$ and $x > -21$, which we can write together as $-21 < x < 21$.

36. Subtract 6 from both sides first: $|x - 8| > 9$. This means the distance from x to 8 is more than 9. This leads to two scenarios: $x - 8 > 9$, which simplifies to $x > 17$, or $x - 8 < -9$, which simplifies to $x < -1$.

37. The absolute value being greater than 6 means the expression inside is either more than 6 units away from zero on the positive side or less than -6 on the negative side. Splitting into two cases, we get $\frac{x}{2} + 3 > 6$, which simplifies to $x > 6$, or $\frac{x}{2} + 3 < -6$, which simplifies to $x < -18$.

38. This tells us that the quantity $\frac{(x+5)}{4}$ is less than 7 units away from zero in both the positive and negative directions. We consider two inequalities: $\frac{(x+5)}{4} < 7$, which simplifies to $x < 23$, and $\frac{(x+5)}{4} > -7$, which simplifies to $x > -33$. So, the solution to this inequality is $-33 < x < 23$.

39. We can multiply the first equation by 2 to align the coefficients of x for elimination. This gives us $-4x + 4y = -8$ and $4x - 9y = 28$. Adding these equations eliminates x, resulting in $-5y = 20$, and solving for y gives $y = -4$. Substituting $y = -4$ into one of the original equations and solving for x gives $x = -2$. So, the solution is $x = -2, y = -4$.

40. We first multiply the first equation by -2, which gives $-2x - 16y = 10$, and then add it to the second equation $2x + 6y = 0$. This eliminates x, resulting in $-10y = 10$, and solving for y gives $y = -1$. Substituting $y = -1$ back into one of the original equations, we find $x = 3$. Therefore, the solution is $x = 3, y = -1$.

41. We first multiply the second equation by 3, getting $3x - 3y = 9$. Next, we subtract the second equation from the first one: $(4x - 3y) - (3x - 3y) = -2 - 9$. This simplifies to $x = -11$. Finally, we substitute $x = -11$ into one of the original equations, like $x - y = 3$. Solving for y gives us $y = -14$.

42. We multiply the first equation by 3 and the second by 2 to align the coefficients of x for elimination. This results in $6x + 27y = 51$ and $-6x + 16y = 78$. Adding these equations eliminates x, leading to $43y = 129$, and solving for y gives $y = 3$. Substituting $y = 3$ into one of the original equations, we find $x = -5$. Thus, the solution is $x = -5, y = 3$.

43. Subtract $4n$ from both sides to get $0 = 8 + n$. This simplifies to $-8 = n$, indicating a unique solution, $n = -8$.

44. Add $9f$ to both sides, resulting in $5 = 0$. This is a contradiction, as there's no value of f that can satisfy this equation. Hence, there are no solutions.

45. Simplifying the right side, $3z - 3z$ becomes 0, leading to $0 = 0$. This is a true statement for all values of z, meaning there are infinitely many solutions.

46. Subtract $-9x$ from both sides, yielding $2 = 0$, which is a contradiction. Thus, this equation has no solutions.

47. Subtract $12y$ from both sides, resulting in $20 = -y$. This simplifies to $y = -20$, indicating a unique solution, $y = -20$.

48. Add 2 to both sides, getting $10h = -2$. Dividing both sides by 10 gives $h = -0.2$, a unique solution, $h = -0.2$.

49. We need to find two points for each line to determine its slope and equation. For the green line: Point 1: $(-9,0)$ and point 2: $(0,9)$. For the red line: Point 1: $(-4,5)$ and point 2: $(0,-7)$. The slope of a line is calculated with the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$. For the green line: $m = \frac{9-0}{0-(-9)} = 1$. For the red line: $m = \frac{-7-5}{0-(-4)} = -3$. The equation of a line can be found using the point-slope formula $y - y_1 = m(x - x_1)$, where x_1 and y_1 is a point on the line. For the green line using point $(0,9)$: $y - 9 = 1(x - 0) \rightarrow y = x + 9$. For the red line using point $(0,-7)$: $y + 7 = -3(x - 0) \rightarrow y = 3x - 7$. Thus, the system of equations representing the lines in the graph is: $\begin{cases} y = -3x - 7 \\ y = x + 9 \end{cases}$.

50. To write a system of equations for the lines shown in the graph, we need to determine the slope-intercept form of each line, which is $y = mx + b$, where m is the slope and b is the y-intercept of the line. For the red line, we use the two points given: $(0,3)$ and $(1.5,0)$. The slope is: $m = \frac{0-3}{1.5-0} = -2$. The y-intercept (b) is the y-value where the line crosses the y-axis, which is 3 for the red line. So, the equation for the red line is $y = -2x + 3$. For the green line, the points are $(-4,0)$ and $(0,\frac{4}{3})$. Following the same method, the slope m is the change in y divided by the change in x. Here, the change in y is $\frac{4}{3} - 0 = \frac{4}{3}$, and the change in x is $0 - (-4) = 4$. So, the slope $m = \frac{\frac{4}{3}}{4} = \frac{1}{3}$. Since one of the points is on the y-axis $(0,\frac{4}{3})$, this is also the y-intercept (b) for the green line. Thus, the equation for the green line is $y = \frac{1}{3}x + \frac{4}{3}$. Putting it all together, the system of equations for the two lines is: $\begin{cases} 2x + y = 3 \\ -x + 3y = 4 \end{cases}$.

51. We manipulate the equations to align either the x or y coefficients. In this case, we can multiply the first equation by 3 and the second equation by 1, which aligns the y coefficients. This manipulation gives us $9x - 3y = 21$ and $2x + 3y = 1$. Adding these equations together, $3y$ is eliminated, leaving us with $11x = 22$. Solving for x by dividing both sides of the equation by 11, we find that $x = 2$.

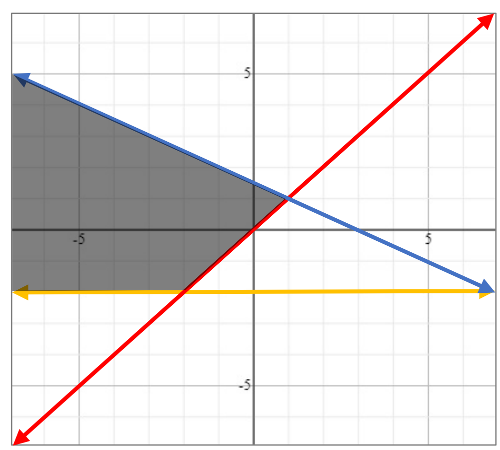
52. The perimeter of a rectangle is the total distance around it, which is twice its length plus twice its width $(2L + 2W)$. We're also told that the length (L) is 10 feet less than 5 times the width (W). So, $L = 5W - 10$. Now, we use the perimeter formula: $2L + 2W = 100$. Replace L with $5W - 10$ in this formula, getting $2(5W - 10) + 2W = 100$. Simplifying this equation, we get $10W - 20 + 2W = 100$. Combine like terms to get $12W - 20 = 100$. Add 20 to both sides to isolate the variable term, resulting in $12W = 120$. Finally, divide both sides by 12 to find W, which is 10 feet. Knowing W, we can find L using $L = 5W - 10$, which gives $L = 40$ feet. So, the width is 10 feet, and the length is 40 feet.

53. The total cost C consists of a fixed part (the membership fee) and a variable part (the hourly fee multiplied by the number of hours). The equation is $C = 150 + 15h$. Here, 150 represents the fixed membership fee, and $15h$ represents the total cost of playing tennis for h hours at \$15 per hour.

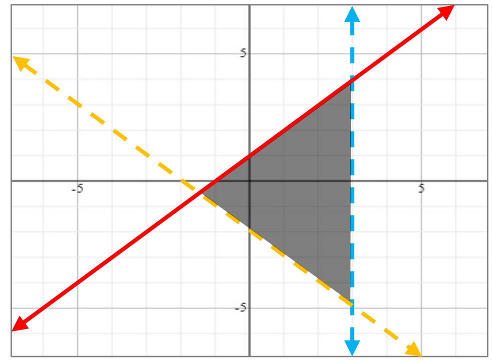
54. To find Jane's current age, we first determine Susan's current age. If Susan is 24 years old in 4 years, she must be $24 - 4 = 20$ years old now. Since Susan is currently twice as old as Jane, we divide Susan's age by 2 to find Jane's age: $\frac{20}{2} = 10$. Therefore, Jane is currently 10 years old.

55. To find the total amount Alex spent on a movie ticket and popcorn, we add the costs of each item. The movie ticket costs \$7. The popcorn costs \$3 more than the ticket, so its price is $7 + 3 = \$10$. Adding these together, $7 + 10 = \$17$, we find that Alex spent a total of \$17.

56. We rearrange each inequality to solve for y. For $x + 2y \leq 3$, the rearranged form is $y \leq -\frac{1}{2}x + \frac{3}{2}$. For $y - x \geq 0$, it becomes $y \geq x$. The third inequality, $y \geq -2$, remains the same. We then plot these inequalities on a graph. The shaded region where all these conditions overlap is the solution.



57. Again, we rearrange each inequality for y. For $x + y > -2$, it becomes $y > -x - 2$. For $y - 1 \leq x$, the rearranged form is $y \leq x + 1$. We plot these on the graph along with $x < 3$. The solution is where the shaded regions of these inequalities intersect.



58. To write an equation for how many games of Balloon Bouncer James played at Glimmer Arcade, we need to account for all his tokens. James started with 38 tokens. He used 2 tokens for the Roll-and-Score game and the rest on Balloon Bouncer, which costs 4 tokens per game. If g represents the number of Balloon Bouncer games he played, the total tokens used for these games would be $4g$ tokens. The total number of tokens used for both games is $4g$ for Balloon Bouncer plus 2 for Roll-and-Score. Therefore, the equation that represents this situation is $4g + 2 = 38$.

59. Sara's budget for buying soda and juice is $\$46$. Each bottle of soda costs $\$3$, and each bottle of juice costs $\$1$. If x represents the number of soda bottles and y represents the number of juice bottles, the total cost can be represented by $3x + y$. Since she wants to spend no more than $\$46$, the inequality to describe this situation is $3x + y \leq 46$. Here, $3x$ accounts for the total cost of the soda, and y for the juice, and the inequality ensures that the combined cost doesn't exceed her budget.

CHAPTER

10 Quadratic

Math topics in this chapter:

- ☑ Solving a Quadratic Equations
- ☑ Graphing Quadratic Functions
- ☑ Solve a Quadratic Equation by Factoring
- ☑ Transformations of Quadratic Functions
- ☑ Quadratic Formula and the Discriminant
- ☑ Characteristics of Quadratic Functions: Equations
- ☑ Characteristics of Quadratic Functions: Graphs
- ☑ Complete a Function Table: Quadratic Functions
- ☑ Domain and Range of Quadratic Functions: Equations
- ☑ Factor Quadratics: Special Cases
- ☑ Factor Quadratics Using Algebra Tiles
- ☑ Write a Quadratic Function from Its Vertex and Another Point

Practices

Solve each equation by factoring or using the quadratic formula.

1) $x^2 - x - 2 = 0$

2) $x^2 - 6x + 8 = 0$

3) $x^2 - 4x + 3 = 0$

4) $x^2 + x - 12 = 0$

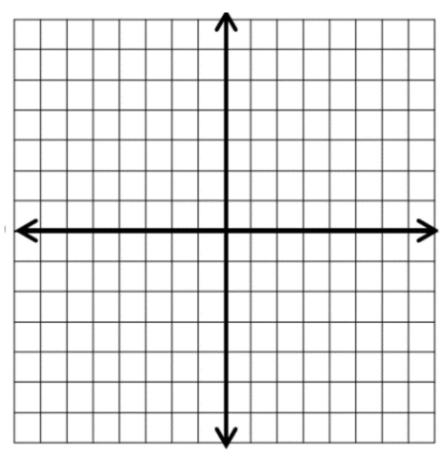
5) $x^2 + 7x - 18 = 0$

6) $x^2 - 2x - 15 = 0$

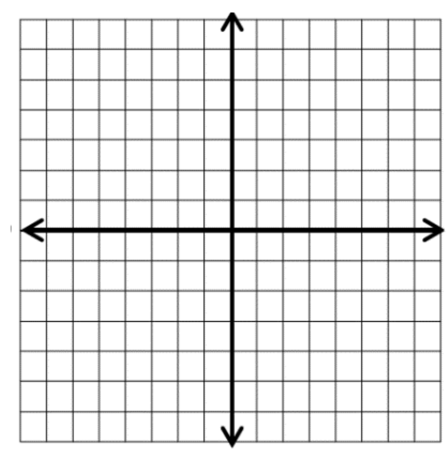
7) $x^2 + 6x - 40 = 0$

8) $x^2 - 9x - 36 = 0$

Sketch the graph of each function.



9) $y = (x - 4)^2 - 2$



10) $y = 2(x + 2)^2 - 3$

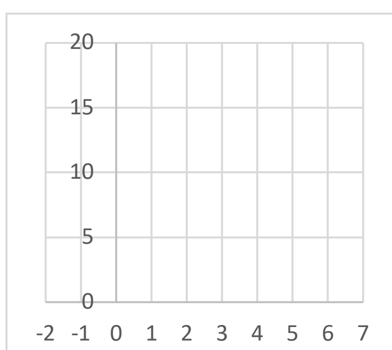
Solve each equation by factoring or using the quadratic formula.

11) $x^2 - 2x - 3 = 0$

12) $x^2 + 9x + 20 = 0$

State the transformations and sketch the graph of the following function.

13) $y = 2(x - 3)^2 + 1$



Find the answer to the equation.

14) $2x^2 - 7x + 3 = 0$

15) $x^2 + 8x - 9 = 0$

16) $2x^2 + 5x - 3 = 0$

17) $x^2 + 6x + 9 = 0$

Solve.

18) Find the equation of the axis of symmetry for the parabola $y = x^2 + 7x + 3$.

19) Find the y-intercept of the parabola $x^2 + 25x + 7$.

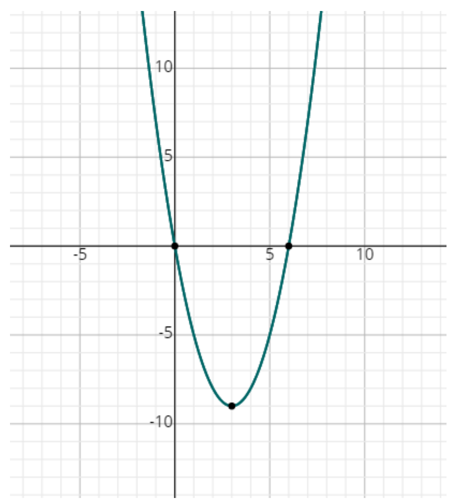
20) Find the vertex of the parabola $y = x^2 - 4x + 3$.

Considering the following graph, determine the following:

21) vertex

22) axis of symmetry

23) y –intercepts



Complete the table.

24)

$g(t) = t^2 + 7$	
t	$g(t)$
-1	
0	
1	

25)

$f(p) = 4p^2$	
p	$f(p)$
-2	
0	
2	

Determine the domain and range of each function.

26) $y = x^2 + 5x + 6$

27) $y = x^2 + 3$

28) $y = -x^2 + 4$

Factor.

29) $25x^2 + 20x + 4$

30) $9x^2 - 1$

31) $3 + 6x + 3x^2$

32) $b^4 - 36$

Use algebra tiles to factor.

33) $x^2 - 3x + 2$

34) $x^2 + 5x + 6$

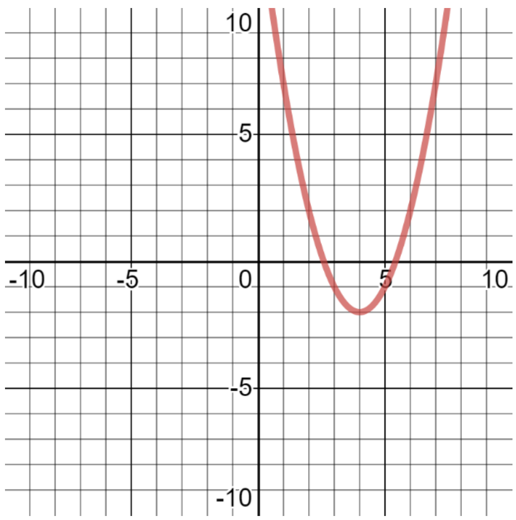
Write each quadratic function as a vertex form.

35) A parabola opening or down has vertex (0,0) and passes through $(8, -16)$.

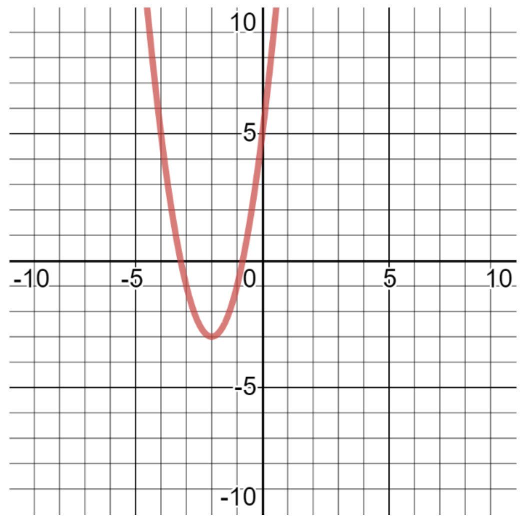
36) A parabola opening up or down has vertex $(0, 2)$ and passes through $(-2,5)$.

Answers

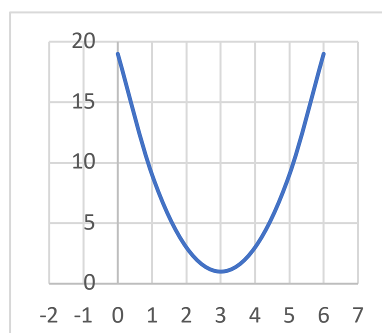
1) $x = 2, x = -1$
2) $x = 2, x = 4$
3) $x = 3, x = 1$
4) $x = 3, x = -4$
5) $x = 2, x = -9$
6) $x = 5, x = -3$
7) $x = 4, x = -10$
8) $x = 12, x = -3$
9) $y = (x - 4)^2 - 2$



10) $y = 2(x + 2)^2 - 3$



11) $\{3, -1\}$
12) $\{-4, -5\}$
13) The graph stretches vertically by a factor of 2.
Move 3 units to the right and 1 unit up.



14) $x_1 = 3, x_2 = \frac{1}{2}$
15) $x_1 = -9, x_2 = 1$
16) $_1 = -3, x_2 = \frac{1}{2}$
17) $x_1 = x_2 = -3$

18) $x = -\frac{7}{2}$

19) 7

20) $(2, -1)$

21) $(3, -9)$

22) 3

23) 0

24)

$g(t) = t^2 + 7$	
t	$g(t)$
-1	8
0	7
1	8

25)

$f(p) = 4p^2$	
p	$f(p)$
-2	16
0	0
2	16

26) $D = \{x|x \in R\}, R = \{y \in R|y \geq -0.25\}$

27) $= \{x|x \in R\}, R = \{R|y \geq 3\}$

28) $D = \{x|x \in R\}, R = \{y \in R|y \leq 4\}$

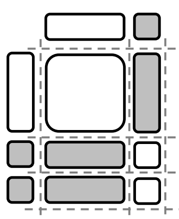
29) $(5x + 2)^2$

30) $(3x - 1)(3 + 1)$

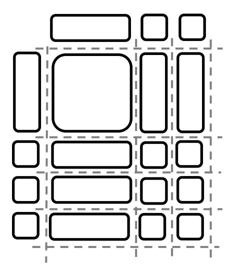
31) $3(1 + x)^2$

32) $(b^2 + 6)(b^2 - 6)$

33) $(x - 1)(x - 2)$



34) $(x + 2)(x + 3)$



35) $y = -\frac{1}{4}x^2$

36) $y = \frac{3}{4}x^2 + 2$

Answers and Explanations

1. The factors of -2 that add up to -1 (the coefficient of x) are -2 and 1. So, we rewrite the equation as $x^2 - 2x + x - 2 = 0$. Factoring by grouping, we get $x(x-2) + 1(x-2) = 0$, which simplifies to $(x-2)(x+1) = 0$. Setting each factor to zero gives the solutions: $x = 2$ and $x = -1$.

2. We look for factors of 8 that add up to -6. These are -2 and -4. Rewriting the equation as $x^2 - 4x - 2x + 8 = 0$, and factoring by grouping, we get $x(x-4) - 2(x-4) = 0$. This simplifies to $(x-4)(x-2) = 0$, giving the solutions: $x = 4$ and $x = 2$.

3. Here, we need factors of 3 that sum up to -4. These factors are -1 and -3. Rewriting the equation as $x^2 - 3x - x + 3 = 0$ and factoring by grouping, we get $x(x-3) - 1(x-3) = 0$. This factors into $(x-3)(x-1) = 0$, yielding solutions: $x = 3$ and $x = 1$.

4. We need factors of -12 that add to 1. These are 4 and -3. Rewrite the equation as $x^2 + 4x - 3x - 12 = 0$. Factoring by grouping, we get $x(x+4) - 3(x+4) = 0$. This factors into $(x+4)(x-3) = 0$, resulting in solutions: $x = -4$ and $x = 3$.

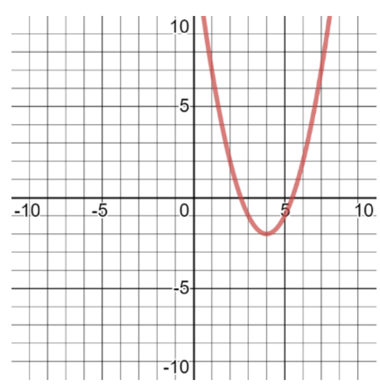
5. Looking for factors of -18 that sum to 7, we find 9 and -2. Rewrite as $x^2 + 9x - 2x - 18 = 0$. Factoring by grouping, we have $x(x+9) - 2(x+9) = 0$. This factors into $(x+9)(x-2) = 0$, giving solutions: $x = -9$ and $x = 2$.

6. Factors of -15 that add up to -2 are -5 and 3. Rewrite as $x^2 - 5x + 3x - 15 = 0$. Factoring by grouping, we get $x(x-5) + 3(x-5) = 0$. This factors into $(x-5)(x+3) = 0$, resulting in solutions: $x = 5$ and $x = -3$.

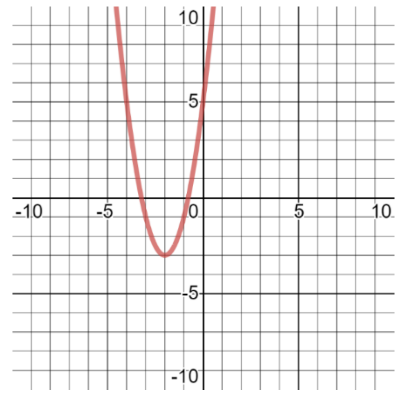
7. We need factors of -40 that sum to 6. These are 10 and -4. Rewrite as $x^2 + 10x - 4x - 40 = 0$. Factoring by grouping, we have $x(x+10) - 4(x+10) = 0$. This simplifies to $(x+10)(x-4) = 0$, giving solutions: $x = -10$ and $x = 4$.

8. Seek numbers that multiply to -36 and add to -9. These are -12 and 3. Factor as $(x-12)(x+3) = 0$. Solving $x - 12 = 0$ and $x + 3 = 0$ gives $x = 12$ and $x = -3$.

9. This graph represents a parabola. The basic shape of the parabola is $y = x^2$, which opens upwards. The term $(x-4)$ shifts this parabola 4 units to the right, as it changes the x-coordinate of the vertex. The "-2" at the end lowers the parabola by 2 units, moving the vertex down. The vertex of this parabola is at $(4, -2)$, which is the lowest point since the parabola opens upwards.



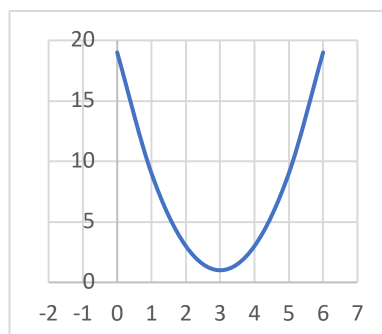
10. This graph also represents a parabola. The "2" multiplying the square term causes the parabola to be narrower than the standard $y = x^2$. The term $(x + 2)$ shifts the parabola 2 units to the left (opposite direction of the sign). The "-3" moves the parabola down by 3 units, altering the y-coordinate of the vertex. The vertex of this parabola is at $(-2, -3)$, and it opens upwards, making the vertex the lowest point of the parabola.



11. For factoring, we need to find two numbers that multiply to -3 (the constant term) and add to -2 (the coefficient of x). These numbers are -3 and 1. So, we can rewrite the equation as $(x - 3)(x + 1) = 0$. Now, apply the zero-product property, which states that if a product equals zero, then at least one of the factors must be zero. Setting each factor equal to zero gives us $x - 3 = 0$ and $x + 1 = 0$, leading to solutions $x = 3$ and $x = -1$.

12. To factor this, look for two numbers that multiply to 20 (the constant term) and add up to 9 (the coefficient of x). These numbers are 4 and 5. So, we can rewrite the equation as $(x + 4)(x + 5) = 0$. Applying the zero-product property, we set each factor to zero: $x + 4 = 0$ and $x + 5 = 0$. This gives us the solutions $x = -4$ and $x = -5$.

13. The term $(x - 3)$ indicates a horizontal shift of the basic parabola. It moves the parabola 3 units to the right. The coefficient 2 in front of the square term indicates a vertical stretch. This makes the parabola narrower than the standard parabola $y = x^2$. There's an upward shift of 1 unit, as indicated by the "$+1$" in the function. This moves the entire graph up.



14. Here, $a = 2$, $b = -7$, and $c = 3$. Substituting these into the formula gives $x_{1,2} = \frac{-(-7)\pm\sqrt{(-7)^2-4\times 2\times 3}}{2\times 2}$. Solving this, we get two solutions: $x_1 = \frac{1}{2}$ and $x_2 = 3$.

15. Coefficients are $a = 1$, $b = 8$, and $c = -9$. Plugging these into the quadratic formula: $x_{1,2} = \frac{-8\pm\sqrt{8^2-4\times1\times(-9)}}{2\times1}$. This results in the roots $x_1 = -9$ and $x_2 = 1$.

16. Coefficients: $a = 2$, $b = 5$, $c = -3$. Apply the formula: $x_{1,2} = \frac{-5\pm\sqrt{5^2-4\times2\times(-3)}}{2\times2}$. The solutions are $x_1 = -3$ and $x_2 = \frac{1}{2}$.

17. Coefficients: $a = 1, b = 6, c = 9$. Using the formula $x_{1,2} = \frac{-6 \pm \sqrt{6^2 - 4 \times 1 \times 9}}{2 \times 1}$. Here, the discriminant (part under the square root) is zero, indicating one real root, $x_{1,2} = -3$.

18. The axis of symmetry of a parabola in the form $ax^2 + bx + c$ is given by $x = -\frac{b}{2a}$. Here, $a = 1$ and $b = 7$, so the axis of symmetry is $x = -\frac{7}{2 \times 1} = -\frac{7}{2}$. This means the line $x = -\frac{7}{2}$ is the axis of symmetry for the parabola.

19. The y-intercept of a parabola is found when $x = 0$. Substituting $x = 0$ in the equation gives the y-intercept. For $y = x^2 + 25x + 7$ substituting $x = 0$ yields $y = 0^2 + (25 \times 0) + 7 = 7$. Therefore, the y-intercept of this parabola is $y = 7$.

20. The vertex of a parabola $y = ax^2 + bx + c$ is at $(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right))$, where $f(x)$ is the parabola's equation. Here, $a = 1$ and $b = -4$, so the x −coordinate of the vertex is $x = -\frac{-4}{2 \times 1} = 2$. Substituting $x = 2$ into the parabola's equation gives the y-coordinate: $y = 2^2 - 4 \times 2 + 3 = -1$. Therefore, the vertex of this parabola is at the point $(2, -1)$.

21. The vertex is the point at the bottom of the parabola since it opens upwards. It's at the "tip" or the lowest point on the graph. You can locate the vertex by finding the point where the parabola turns around, which, on this graph, appears to be at the $(3, -9)$.

22. The axis of symmetry is the vertical line that goes through the vertex and splits the parabola into two symmetrical halves. In this graph, the axis of symmetry would be $x = 3$.

23. The y-intercept is where the parabola crosses the y-axis. From the graph, it's clear that this parabola crosses the y-axis at the point (0,0). There are no other points where the parabola crosses the y-axis.

24. To complete the table for each function, you substitute the given values of t into the functions and calculate the corresponding output. When $t = -1, g(t) = (-1)^2 + 7 = 1 + 7 = 8$. When $t = 0, g(t) = (0)^2 + 7 = 0 + 7 = 7$. When $t = 1, g(t) = (1)^2 + 7 = 1 + 7 = 8$.

$g(t) = t^2 + 7$	
t	$g(t)$
-1	8
0	7
1	8

25. When $p = -2, f(p) = 4(-2)^2 = 4 \times 4 = 16$. When $p = 0, f(\ \ ) = 4(0)^2 = 4 \times 0 = 0$. When $p = 2$, $f(p) = 4(2)^2 = 4 \times 4 = 16$.

$f(p) = 4p^2$	
p	$f(p)$
-2	16
0	0
2	16

26. The domain of any quadratic function, like this one, is all real numbers because there's no restriction on the values that x can take. Let's calculate the vertex to determine the minimum value of y, which will help us in determining the range. The vertex of function is at $(-2.5, -0.25)$. This means the lowest point on the graph of the function is at $y = -0.25$. Therefore, the range is $y \geq -0.25$.

27. Similar to the first function, this is also a quadratic function, where x can be any real number. There's no number that doesn't work in the equation. So, the domain is all real numbers. This equation also makes a U-shaped curve. The lowest point is the vertex. Here, the vertex is simpler because there's no x term to shift it left or right. So, the lowest point is just when $x = 0$, giving $y = 3$. Therefore, the smallest value y can be is 3, and it increases from there. The range is $y \geq 3$.

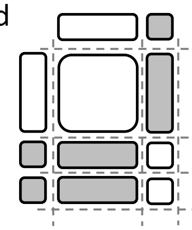
28. As with the other quadratic functions, there are no limitations on x here. You can choose any number for x. Hence, the domain is all real numbers. This function is also a parabola, but it opens downwards (because of the negative sign before x^2). The highest point on the curve is the vertex. Since there's no x term, the vertex is at $x = 0$, giving $y = 4$. That's the highest point, and the curve goes down from there. So, the range is all values of y that are less than or equal to 4, written as $y \leq 4$.

29. This equation can be factored by looking for a perfect square trinomial. A perfect square trinomial is formed when a binomial is squared. We notice that $(5x)^2 = 25x^2$, $2 \times 5x \times 2 = 20x$, and $2^2 = 4$. Therefore, $25x^2 + 20x + 4$ can be factored as $(5x + 2)^2$.

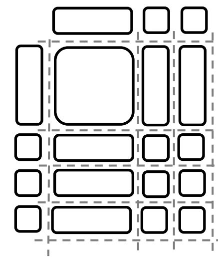
30. This equation is a difference of squares. Here,$9x^2$ is $(3x)^2$ and 1 is 1^2. So, $9x^2 - 1$ factors to $(3x + 1)(3x - 1)$.

31. It's often easier to factor when the terms are in descending order of their exponents. Rearranging the equation gives us $3x^2 + 6x + 3$. Next, we look for common factors. In this case, each term is divisible by 3. Factoring out 3, we get: $3(x^2 + 2x + 1)$. Now, we need to factor the quadratic expression inside the parentheses. The expression $x^2 + 2x + 1$ is a perfect square trinomial, which factors into the square of a binomial. The pattern for a perfect square trinomial is $a^2 + 2ab + b^2 = (a + b)^2$. Here, $x^2 = (x)^2$, $2 \times x \times 1 = 2x$, and $1^2 = 1$. Putting it all together, the factored form of $3 + 6x + 3x^2$ is: $3(x + 1)^2$.

32. In $b^4 - 36, b^4$ is a perfect square because it can be written as $(b^2)^2$and 36 is also a perfect square because it can be written as 6^2. Applying the difference of squares formula, we rewrite $b^4 - 36$ as $(b^2 + 6)(b^2 - 6)$.



33. For $x^2 - 3x + 2$, imagine a set of tiles representing x^2, $-x$ and $+1$. You need to arrange these tiles into a rectangle. The tiles for x^2 form the large square, the $-x$ tiles are rectangles, and $+1$ tiles are small squares. The goal is to create a rectangle where one side is made up of x tiles and the other side with constant number tiles. For this expression, you'll use one x^2 tile, three $-x$ tiles, and two $+1$ tiles. The arrangement that forms a rectangle is one that makes two groups: $(x-1)$ and $(x-2)$. So, $x^2 - 3x + 2$ factors into $(x-1)(x-2)$.



34. For $x^2 + 5x + 6$ use the same method. This time, you have one x^2 tile, five $+x$ tiles, and six $+1$ tiles. Arrange these to form a rectangle. The setup that works is creating two groups: $(x+2)$ and $(x+3)$. So, $x^2 + 5x + 6$ factors into $(x+2)(x+3)$.

35. The vertex form of a quadratic function is $y = a(x-h)^2 + k$, where (h,k) is the vertex. For a parabola with vertex $(0,0)$, the equation simplifies to $y = ax^2$. We need to find the value of a. We know the parabola passes through $(8,-16)$, so we substitute these values into the equation to find a. Substituting $x = 8$ and $y = -16$ gives $-16 = 64a$. Solving for a gives $a = -\frac{16}{64} = -\frac{1}{4}$. So, the equation in vertex form is $y = -\frac{1}{4}x^2$.

36. Starting with the vertex form $y = a(x-h)^2 + k$, we substitute the vertex $(0,2)$, yielding $y = a(x-0)^2 + 2$ or $y = ax^2 + 2$. To find a, we use the point $(-2,5)$. Substituting $x = -2$ and $y = 5$ gives $5 = a(-2)^2 + 2$, which simplifies to $5 = 4a + 2$. Solving for a results in $a = \frac{3}{4}$. Therefore, the quadratic function in vertex form is $y = \frac{3}{4}x^2 + 2$.

CHAPTER

11 Polynomials

Math topics in this chapter:

- ☑ Simplifying Polynomials
- ☑ Adding and Subtracting Polynomials
- ☑ Add and Subtract Polynomials Using Algebra Tiles
- ☑ Multiplying Monomials
- ☑ Multiplying and Dividing Monomials
- ☑ Multiplying a Polynomial and a Monomial
- ☑ Multiply Polynomials Using Area Models
- ☑ Multiplying Binomials
- ☑ Multiply two Binomials Using Algebra Tiles
- ☑ Factoring Trinomials
- ☑ Factoring Polynomials
- ☑ Use a Graph to Factor Polynomials
- ☑ Factoring Special Case Polynomials
- ☑ Add Polynomials to Find Perimeter

Practices

Simplify each polynomial.

1) $3(6x+4)=$

2) $5(3x-8)=$

3) $x(7x+2)+9x=$

4) $6x(x+3)+5x=$

5) $6x(3x+1)-5x=$

6) $x(3x-4)+3x^2-6=$

7) $x^2-5-3x(x+8)=$

8) $2x^2+7-6x(2x+5)=$

Add or subtract polynomials.

9) $(x^2+3)+(2x^2-4)=$

10) $(3x^2-6x)-(x^2+8x)=$

11) $(4x^3-3x^2)+(2x^3-5x^2)=$

12) $(6x^3-7x)-(5x^3-3x)=$

13) $(10x^3+4x^2)+(14x^2-8)=$

14) $(4x^3-9)-(3x^3-7x^2)=$

15) $(9x^3+3x)-(6x^3-4x)=$

16) $(7x^3-5x)-(3x^3+5x)=$

Use algebra tiles to simplify polynomials.

17) $(2x^2-3x+3)-(x^2-x-1)$

18) $(2x^2+2x+5)+(x^2+2x+1)$

Find the products.

19) $3x^2\times 8x^3=$

20) $2x^4\times 9x^3=$

21) $-4a^4b\times 2ab^3=$

22) $(-7x^3yz)\times(3xy^2z^4)=$

23) $-2a^5bc\times 6a^2b^4=$

24) $9u^3t^2\times(-2ut)=$

25) $12x^2z\times 3xy^3=$

26) $11x^3z\times 5xy^5=$

27) $-6a^3bc\times 5a^4b^3=$

28) $-4x^6y^2\times(-12xy)=$

Simplify each expression.

29) $(7x^2y^3)(3x^4y^2) =$

30) $(6x^3y^2)(4x^4y^3) =$

31) $(10x^8y^5)(3x^5y^7) =$

32) $(15a^3b^2)(2a^3b^8) =$

33) $\frac{42x^4y^2}{6x^3y} =$

34) $\frac{49x^5y^6}{7x^2y} =$

35) $\frac{63x^{15}y^{10}}{9x^8y^6} =$

36) $\frac{35x^8y^{12}}{5x^4y^8} =$

Find each product.

37) $3x(5x - y) =$

38) $2x(4x + y) =$

39) $7x(x - 3y) =$

40) $x(2x^2 + 2x - 4) =$

41) $5x(3x^2 + 8x + 2) =$

42) $7x(2x^2 - 9x - 5) =$

Use the area model to find each product.

43) $3x(x + 2)$

44) $(a - 3)(2a + 2)$

Find each product.

45) $(x - 3)(x + 3) =$

46) $(x - 6)(x + 6) =$

47) $(x + 10)(x + 4) =$

48) $(x - 6)(x + 7) =$

49) $(x + 2)(-5) =$

50) $(x - 10)(x + 3) =$

Use algebra tiles to simplify.

51) $(x+1)(x+6)$

52) $(2x+1)(x-4)$

Factor each trinomial.

53) $x^2+6x+8=$

54) $x^2+3x-10=$

55) $x^2+2x-48=$

56) $x^2-10x+16=$

57) $2x^2-10x+12=$

58) $3x^2-10x+3=$

Factor each expression.

59) $4x^2-4x-8$

60) $6x^2+37x+6$

61) $16x^2+60x-100$

62) $4x^2-17x+4$

Use a graph to factor the following polynomial.

63) x^2-4

64) $-(x+2)^2$

Factor each completely.

65) $36x^2-121$

66) $-36x^4+4x^2$

67) $-36\quad^2+400=$

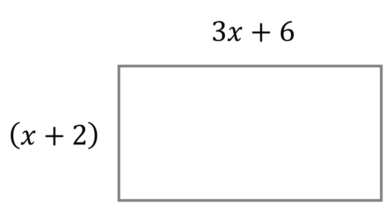
68) $49x^2-56x+16=$

69) $1-\quad^2=$

70) $81x^4-900x^2=$

Find the perimeter.

71)



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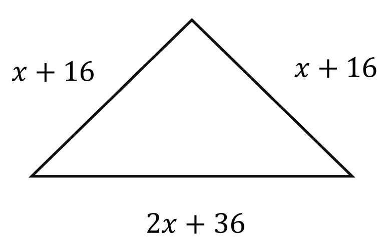
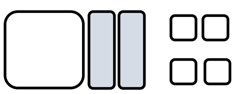


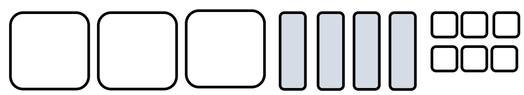
Answers

1) $18x + 12$
2) $15x - 40$
3) $7x^2 + 11x$
4) $6x^2 + 23x$
5) $18x^2 + x$
6) $6x^2 - 4x - 6$
7) $-2x^2 - 24x - 5$
8) $-10x^2 - 30x + 7$
9) $3x^2 - 1$
10) $2x^2 - 14x$
11) $6x^3 - 8x^2$
12) $x^3 - 4x$
13) $10x^3 + 18x^2 - 8$
14) $x^3 + 7x^2 - 9$
15) $3x^3 + 7x$
16) $4x^3 - 10x$

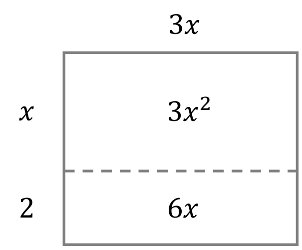
17) $x^2 - 2x + 4$



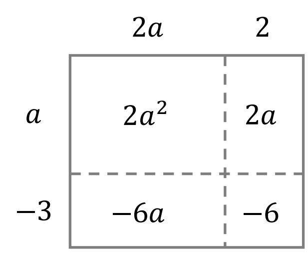
18) $3x^2 + 4x + 6$



19) $24x^5$
20) $18x^7$
21) $-8a^5b^4$
22) $-21x^4y^3z^5$
23) $-12a^7b^5c$
24) $-18u^4t^3$
25) $36x^3y^3z$
26) $55x^4y^5z$
27) $-30\ \ {}^7b^4c$
28) $48x^7y^3$
29) $21x^6y^5$
30) $24x^7y^5$
31) $30x^{13}y^{12}$
32) $30a^6b^{10}$
33) $7xy$
34) $7x^3y^5$
35) $7x^7y^4$
36) $7x^4y^4$
37) $15x^2 - 3xy$
38) $8x^2 + 2xy$
39) $7x^2 - 21xy$
40) $2x^3 + 2x^2 - 4x$
41) $15x^3 + 40x^2 + 10x$
42) $14x^3 - 63x^2 - 35x$
43) $3x^2 + 6x$



44) $2a^2 - 4a - 6$



45) $x^2 - 9$
46) $x^2 - 36$

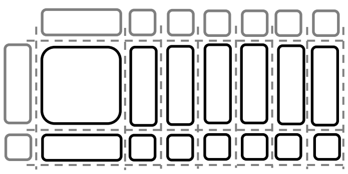
47) $x^2 + 14x + 40$

48) $x^2 + x - 42$

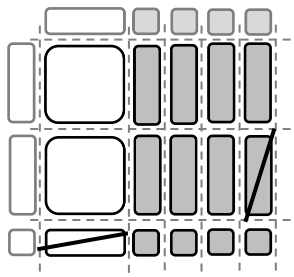
49) $x^2 - 3x - 10$

50) $x^2 - 7x - 30$

51) $x^2 + 7x + 6$



52) $2x^2 - 7x - 4$



53) $(x + 4)(x + 2)$

54) $(x + 5)(x - 2)$

55) $(x - 6)(x + 8)$

56) $(x - 8)(x - 2)$

57) $(2x - 4)(x - 3)$

58) $(3x - 1)(x - 3)$

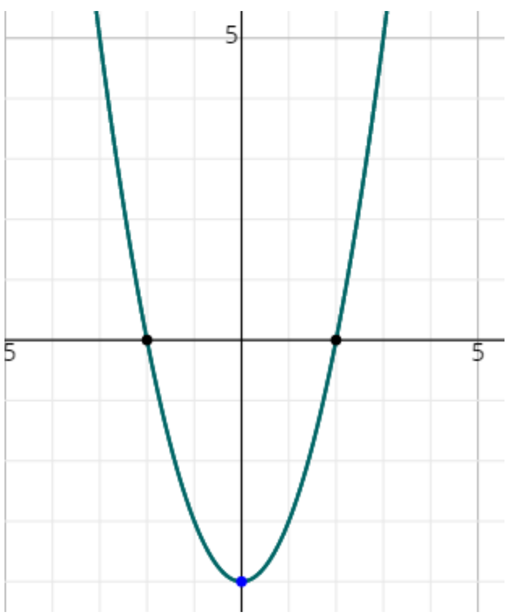
59) $4(x + 1)(x - 2)$

60) $(x + 6)(6x + 1)$

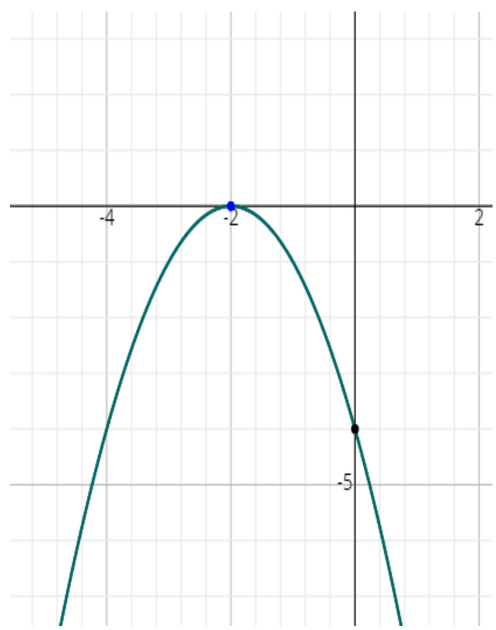
61) $4(x + 5)(4x - 5)$

62) $(x - 4)(4x - 1)$

63) $x = \pm 2$



64) $x = -2$



65) $(6x - 11)(6 + 11)$

66) $4x^2(1 - 3x)(1 + 3x)$

67) $4(10 + 3x)(10 - 3x)$

68) $(7x - 4)^2$

69) $(1 + x)(1 - x)$

70) $9x^2(3x + 10)(3x - 10)$

71) $8x + 16$

72) $4x + 68$

Answers and Explanations

1. First, distribute the 3 to both $6x$ and 4. This means multiplying 3 by each term inside the parentheses. $3 \times 6x$ gives $18x$, and 3×4 gives 12. So, the simplified form is $18x + 12$.

2. Here, you'll distribute 5 to $3x$ and -8. Multiplying 5 by $3x$ gives $15x$, and 5 times -8 gives -40. Thus, the polynomial simplifies to $15x - 40$.

3. Start by expanding $x(7x + 2)$. Multiply x with $7x$ to get $7x^2$, and x with 2 to get $2x$. Now, add $9x$ to the expanded form. So, it becomes $7x^2 + 11x$.

4. Multiply $6x$ by each term inside the parentheses (x and 3). This gives $6x^2$ and $18x$. Then, add $5x$ to these. The result is $6x^2 + 23x$.

5. Distribute $6x$ to $3x$ and 1. You get $18x^2$ and $6x$. Subtracting $5x$ from $6x$ results in $18x^2 + x$.

6. Multiply x by $3x$ and -4, getting $3x^2$ and $-4x$. Add these to $3x^2 - 6$. The simplified form is $6x^2 - 4x - 6$.

7. Start by expanding $3x(x + 8)$, which gives $3x^2$ and $24x$. Now, subtract these from $x^2 - 5$. The result is $-2x^2 - 24x - 5$.

8. First, expand $6x(2x + 5)$ to get $12x^2$ and $30x$. Then, subtract these from $2x^2 + 7$. You end up with $-10x^2 - 30x + 7$.

9. Add the like terms. Here, x^2 and $2x^2$ are like terms, so they combine to $3x^2$. The constants 3 and -4 also combine to give -1. Thus, the simplified expression is $3x^2 - 1$.

10. Subtract each term in the second polynomial from the corresponding term in the first. $3x^2 - x^2$ equals $2x^2$, and $-6x - 8x$ equals $-14x$. So, the answer is $2x^2 - 14x$.

11. Combine like terms. $4x^3$ and $2x^3$ add up to $6x^3$, and $-3x^2$ and $-5x^2$ add up to $-8x^2$. The result is $6x^3 - 8x^2$.

12. Subtract the terms in the second polynomial from the first. $6x^3 - 5x^3$ is x^3, and $-7x - (-3x)$ simplifies to $-4x$. The final expression is $x^3 - 4x$.

13. Add like terms. $10x^3$ has no like term, so it remains as is. $4x^2$ and $14x^2$ make $18x^2$. The constant −8 stands alone. So, the simplified form is $10x^3 + 18x^2 - 8$.

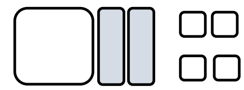
14. Here, $4x^3 - 3x^3$ equals x^3. There's no like term for $-7x^2$, so it's just subtracted, becoming $+7x^2$. Finally, -9 has no like term, so it remains. The answer is $x^3 + 7x^2 - 9$.

15. Subtract the second polynomial from the first. $9x^3 - 6x^3$ is $3x^3$, and $3x - (-4x)$ simplifies to $7x$. Thus, the result is $3x^3 + 7x$.

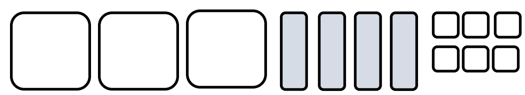
16. Subtract the terms in the second polynomial from the first. $7x^3 - 3x^3$ results in $4x^3$, and

$-5x - 5x$ equals $-10x$. The final expression is $4x^3 - 10x$.

17. First, represent $2x^2$ with two large squares, $-3x$ with three negative long rectangles, and $+3$ with three small squares. Next, for $-(x^2 - x - 1)$, use one negative large square for $-x^2$, one positive long rectangle for $+x$, and one positive small square for $+1$. Now, combine these tiles. The two large squares and one negative large square leave one large square (x^2). The three negative rectangles and one positive rectangle leave two negative rectangles $(-2x)$. The three small squares and one positive small square leave four small squares $(+4)$. So, the simplified form is $x^2 - 2x + 4$.



18. Represent $2x^2$ with two large squares, $2x$ with two long rectangles, and $+5$ with five small squares. For $x^2 + 2x + 1$, use one large square, two long rectangles, and one small square. Combine these tiles. The two large squares and one large square make three large squares $(3x^2)$. The four long rectangles represent $4x$. The five small squares and one small square make six small squares $(+6)$. The simplified expression is $3x^2 + 4x + 6$.



19. Multiply the coefficients 3 and 8 to get 24. For the variables, add the exponents of x (2 and 3), resulting in x^{2+3} . So, the product is $24x^5$.

20. First, multiply the coefficients 2 and 9, which equals 18. Then, combine x^4 and x^3 by adding their exponents $(4 + 3)$, leading to x^7. Thus, the product is $18x^7$.

21. Multiply the coefficients -4 and 2 to get -8. Combine a^4 and a (which is a^1) by adding exponents $(4 + 1 = 5)$ to get a^5. Similarly, combine b and b^3 to get b^4. So, the result is $-8a^5b^4$.

22. Here, multiply -7 and 3 to get -21. For x^3 and x, add their exponents $(3 + 1 = 4)$ to get x^4. For y, combine y and y^2 to y^3. Lastly, add the exponents of z and z^4 $(1 + 4 = 5)$ to get z^5. The final product is $-21x^4\, y^3z^5$.

23. Multiply -2 and 6 to get -12. Combine a^5 and a^2 to get a^7, and b and b^4 to get b^5. The c term remains as is. The result is $-12a^7b^5c$.

24. Multiply 9 and -2 to get -18. Add the exponents of u^3 and u to get u^4, and the exponents of t^2 and t to get t^3. So, the product is $-18\, u^4t^3$.

25. Multiply the coefficients 12 and 3 to get 36. Combine x^2 and x to x^3. The y^3 and z terms don't have like terms, so they are just appended. The result is $36x^3\, y^3z$.

26. Here, multiply 11 and 5 to get 55. Add the exponents of x^3 and x to get x^4. Since z and y^5 don't have like terms, they remain as is. The final product is $55x^4\,y^5z$.

27. First, multiply the coefficients (numerical values) together: $-6 \times 5 = -30$. Next, multiply the 'a' terms: $a^3 \times a^4$. When multiplying with the same base, add the exponents: $3 + 4 = 7$. So, this becomes a^7. Then, multiply the 'b' terms: $b \times b^3$. Similarly, add the exponents for 'b': $1 + 3 = 4$ (Note: 'b' is b^1). This results in b^4. Lastly, the 'c' term remains as is since there's no other 'c' term to multiply with. Combine all these: $-30a^7b^4c$.

28. Multiply the coefficients: $-4 \times (-12) = 48$. Multiply the 'x' terms: $x^6 \times x$. Add their exponents: $6 + 1 = 7$. This gives x^7. Multiply the 'y' terms: $y^2 \times y$. The exponents add up to $2 + 1 = 3$, resulting in y^3. Combine these results: $48x^7y^3$.

29. We multiply the coefficients (numbers) and add the exponents of like bases (x and y). So, $7 \times 3 = 21$ and $x^{2+4} = x^6$, $y^{3+2} = y^5$. The simplified expression is $21x^6y^5$.

30. Multiply the coefficients (6 and 4) and add the exponents of x and y separately. $6 \times 4 = 24$, $x^{3+4} = x^7$, and $y^{2+3} = y^5$. This gives $24x^7y^5$.

31. Here, multiply 10 and 3, and add the exponents of x and y. $10 \times 3 = 30$, $x^{8+5} = x^{13}$, $y^{5+7} = y^{12}$. The result is $30x^{13}y^{12}$.

32. Multiply 15 and 2, and add the exponents of a and b. $15 \times 2 = 30$, $a^{3+3} = a^6$, $b^{2+8} = b^{10}$. So, we get $30a^6b^{10}$.

33. Divide the coefficients and subtract the exponents of x and y in the denominator from those in the numerator. $42 \div 6 = 7$, $x^{4-3} = x$, and $y^{2-1} = y$. The simplified form is $7xy$.

34. Again, divide the coefficients and subtract the exponents. $49 \div 7 = 7$, $x^{5-2} = x^3$, and $y^{6-1} = y^5$. This simplifies to $7x^3y^5$.

35. Divide 63 by 9 and subtract the exponents of x and y in the denominator from the numerator. $63 \div 9 = 7$, $x^{15-8} = x^7$, $y^{10-6} = y^4$. Thus, the simplified expression is $7x^7y^4$.

36. Finally, divide 35 by 5 and subtract the exponents. $35 \div 5 = 7$, $x^{8-4} = x^4$, and $y^{12-8} = y^4$. The result is $7x^4y^4$.

37. Multiply $3x$ by each term inside the parentheses. First, $3x \times 5x = 15x^2$. (Multiplying the coefficients and adding the exponents of x). Next, $3x \times (-y) = -3xy$ (multiplying the coefficient of x by y and keeping the sign). The product is $15x^2 - 3xy$.

38. Here, multiply $2x$ by $4x$ to get $8x^2$ and $2x$ by y to get $2xy$. The result is $8x^2 + 2xy$.

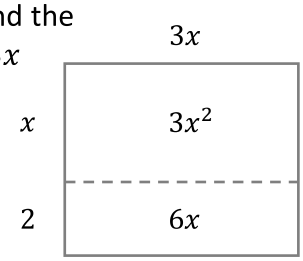
39. Distribute $7x$ across x and $-3y$. $7x \times x = 7x^2$and $7x \times (-3y) = -21xy$. This results in $7x^2 - 21xy$.

40. Multiply x with each term inside the parentheses. $x \times 2x^2 = 2x^3, x \times 2x = 2x^2$, and $x \times (-4) = -4x$. Combine these for $2x^3 + 2x^2 - 4x$.

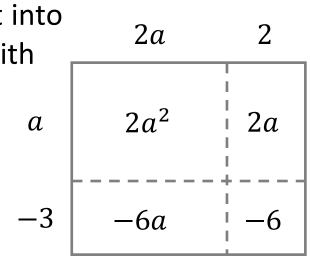
41. Apply distribution: $5x \times 3x^2 = 15x^3$, $5x \times 8x = 40x^2$, and $5x \times 2 = 10x$. The product is $15x^3 + 40x^2 + 10x$.

42. Here, multiply 7 by $2x$ to get $14x^2$, 7 by $-9x$ to get $-63x$ and 7 by -5 to get -35. This results in $14x^2 - 63x - 35$.

43. Picture a rectangle with one side as $3x$ and another as $x + 2$. To find the area, divide the rectangle into two smaller rectangles, one with side $3x$ and the other side x, and the second with side $3x$ and the other side 2. The area of the first smaller rectangle is $3x \times x = 3x^2$. (multiplying the lengths of its sides). The area of the second smaller rectangle is $3x \times 2 = 6x$. The total area, or the product, is the sum of these areas: $3x^2 + 6x$.



44. Visualize this as a rectangle with sides $a - 3$ and $2a + 2$. Split it into two smaller rectangles: one with sides $a - 3$ and $2a$, and the other with sides $a - 3$ and 2. The area of the first is $2a \times (a - 3) = 2a^2 - 6a$ (multiplying $2a$ with each term in $a - 3$). The area of the second is $2 \times (a - 3) = 2a - 6$ (multiplying 2 with each term in $a - 3$). The total area, or product, is $2a^2 - 6a + 2a - 6$, , which simplifies to $2a^2 - 4a - 6$.



45. Multiply each term in the first binomial by each term in the second. This gives $x \times x = x^2$, $x \times 3 = 3x$, $-3 \times x = -3x$, and $-3 \times 3 = -9$. Combining these, we get $x^2 + 3x - 3x - 9$. Notice that $3x$ and $-3x$ cancel out, so the final answer is $x^2 - 9$.

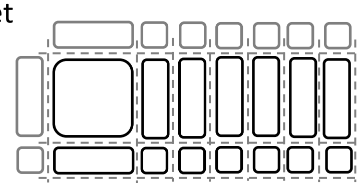
46. Following the same method, we multiply each term: $x \times x = x^2, x \times 6 = 6x, -6 \times x = -6x$, and $-6 \times 6 = -36$. Combining gives $x^2 + 6x - 6x - 36$. Again, $6x$ and $-6x$ cancel out, resulting in $x^2 - 36$.

47. The product is found by multiplying: $x \times x = x^2$, $x \times 4 = 4x$, $10 \times x = 10x$, and $10 \times 4 = 40$. Adding these up gives $x^2 + 4x + 10x + 40$. Combining like terms ($4x$ and $10x$), we get $x^2 + 14x + 40$.

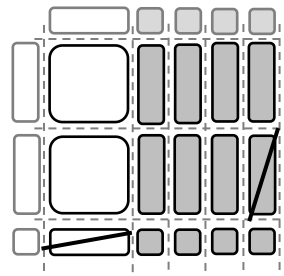
48. Multiply each term to get $x \times x = x^2, x \times 7 = 7x, -6 \times x = -6x$, and $-6 \times 7 = -42$. Summing these gives $x^2 + 7x - 6x - 42$, which simplifies to $x^2 + x - 42$.

49. Multiply each term: $x \times x = x^2, x \times (-5) = -5x, 2 \times x = 2x$, and $2 \times (-5) = -10$. Adding up these products gives $x^2 - 5x + 2x - 10$, , which simplifies to $x^2 - 3x - 10$.

50. The multiplication of each term yields $x \times x = x^2$, $x \times 3 = 3x$, $-10 \times x = -10x$, and $-10 \times 3 = -30$. Combining these results in $x^2 + 3x - 10x - 30$, which simplifies to $x^2 - 7x - 30$.



51. Imagine a set of tiles representing $x+1$ and another set representing $x+6$. To find the product, we arrange these tiles to form a rectangle. The tiles for $x+1$ form one side of the rectangle, and the tiles for $x+6$ form the other. The area of the rectangle is the product. In this case, the area consists of $x \times x$ (which is x^2) $x \times 6$ (which is $6x$), $1 \times x$ (which is x), and 1×6 (which is 6). Adding these areas together, we get $x^2+6x+x+6$, which simplifies to x^2+7x+6.



52. Here, we use algebra tiles for $2x+1$ and $x-4$ to form another rectangle. The area of this rectangle is found by multiplying the lengths of its sides. We have $2x \times x$ (which gives $2x^2$) $2x \times -4$ (which is $-8x$), $1 \times x$ (which is x), and $1 \times (-4)$ (which is -4). When these areas are combined, we get $2x^2-8x+x-4$, which simplifies to $2x^2-7x-4$.

53. We need two numbers that multiply to 8 and add up to 6. The numbers 2 and 4 fit this description, as $2 \times 4 = 8$ and $2+4=6$. So, the factorization is $(x+2)(x+4)$.

54. We look for numbers that multiply to -10 and add to 3. The numbers 5 and -2 work here, since $5 \times (-2) = -10$ and $5+(-2)=3$. Thus, the factorization is $(x+5)(x-2)$.

55. We need numbers that multiply to -48 and add to 2. The numbers 8 and -6 meet these criteria ($8 \times (-6) = -48$ and $8+(-6)=2$). Therefore, it factors to $(x+8)(x-6)$.

56. Here, we seek numbers that multiply to 16 and add to -10. The numbers -2 and -8 do the trick ($-2 \times (-8) = 16$ and $-2+(-8)=-10$). So, the factorization is $(x-2)(x-8)$.

57. First, notice all coefficients are even, so we can factor out a 2: $2(x^2-5x+6)$. Now, factor x^2-5x+6. We need numbers that add to -5 and multiply to 6. These are -3 and -2. The factored form is $2(x-3)(x-2)$.

58. Again, the coefficient of x^2 is not 1, so we look for numbers that multiply to $3 \times 3 = 9$ and add to -10. The numbers -9 and -1 fit ($-9 \times (-1) = 9$ and $-9+(-1)=-10$). Therefore, it factors to $(3x-1)(x-3)$.

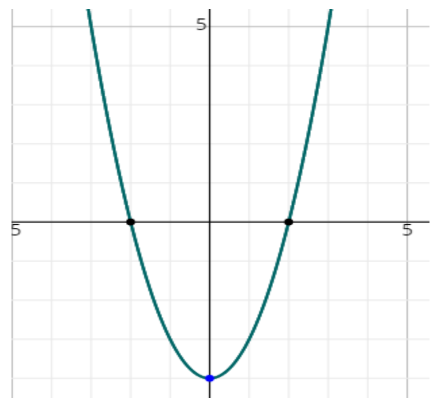
59. Initially, notice that each term in the expression is divisible by 4. Therefore, we factor out 4, resulting in $4(x^2-x-2)$. Now, focus on factoring x^2-x-2. We need two numbers that add up to -1 (the coefficient of 'x') and multiply to -2 (the constant term). These numbers are 1 and -2. Thus, the factored form is $4(x+1)(x-2)$.

60. This requires a bit more thought because the coefficients are larger. We seek two numbers that add up to 37 and multiply to $6 \times 6 = 36$. The numbers 36 and 1 fit this criterion. Place these numbers in the expression, giving $6x^2 + 36x + x + 6$. Then, group and factor by grouping: $6x(x+6) + 1(x+6)$. The final factored form is $(6x+1)(x+6)$.

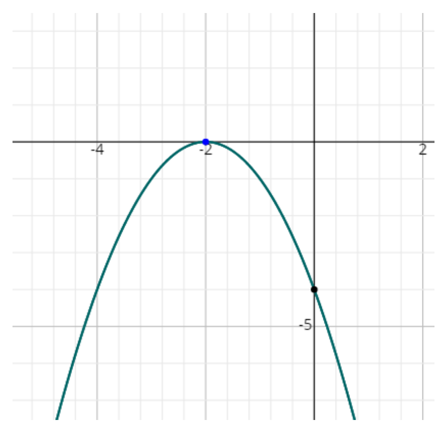
61. Start by noticing all terms are divisible by 4. So, we factor out 4, resulting in $4(4x^2 + 15x - 25)$. Next, factor $4x^2 + 15x - 25$. The numbers we're looking for are those that add to 15 and multiply to $4 \times (-25) = -100$. The suitable numbers are 20 and -5. After rearranging and grouping, the factored form becomes $4(4x^2 + 20x - 5x - 25)$ which simplifies to $4(4x(x+5) - 5(x+5))$, and finally to $4(4x-5)(x+5)$.

62. This one also needs careful consideration. We want two numbers that add up to -17 and multiply to $4 \times 4 = 16$. These numbers are -16 and -1. By splitting the middle term, we get $4x^2 - 16x - x + 4$. Grouping these gives $4x(x-4) - 1(x-4)$. The expression then factors to $(4x-1)(x-4)$.

63. The graph of this polynomial is a parabola opening upwards. The equation $x^2 - 4$ can be rewritten as $(x-2)(x+2)$ using the difference of squares formula. Graphically, this parabola will intersect the x-axis at $x = 2$ and $x = -2$. So, the factored form $(x-2)(x+2)$ directly corresponds to these intercepts, indicating the points where the graph touches the x-axis.



64. This is the graph of a downward-opening parabola because of the negative sign in front. The term $(x+2)^2$ means that the parabola is a shifted version of the basic x^2 graph, moved 2 units to the left. For the roots, we set $(x+2)^2 = 0$. The only solution to this is x=−2, which means the graph touches the x-axis at this point only. Since it's a perfect square, the graph only touches the axis at one point, reflecting the fact that $x = -2$ is a repeated or double root.



65. This is a difference of squares, as both $36x^2$ and 121 are perfect squares. It can be rewritten as $(6x)^2 - 11^2$. The factored form, using the difference of squares formula $a^2 - b^2 = (a-b)(a+b)$, is $(6x-11)(6x+11)$.

66. First, factor out the greatest common factor, which is $4x^2$, giving $4x^2(-9x^2 + 1)$. Notice that $1 - 9x^2$ is also a difference of squares. Thus, it factors further into $4x^2(1-3x)(1+3x)$.

67. Firstly, we notice that both terms, $-36x^2$ and 400, are divisible by 4. However, since our first term is negative, we factor out -4. This step changes the expression to $-4(9x^2 - 100)$. The next step involves recognizing that $9x^2 - 100$ is a difference of two squares. In this case, $9x^2$ is $3x$ squared, and 100 is 10 squared. Therefore, $9x^2 - 100$ factors into $(3x + 10)(3x - 10)$. Now, we substitute this back into our expression to get $-4(3x + 10)(3x - 10)$ or $4(10 + 3x)(10 - 3x)$.

68. This is a perfect square trinomial. It factors into a binomial squared. The square root of $49x^2$ is $7x$, and the square root of 16 is 4. The factored form is $(7x - 4)^2$.

69. This is another difference of squares, where $a = 1$ and $b = x$. It factors to $(1 - x)(1 + x)$.

70. Start by factoring out the greatest common factor, which is $9x^2$, yielding $9x^2(9x^2 - 100)$. The expression inside the parentheses is again a difference of squares. So, it factors to $9x^2(3x - 10)(3x + 10)$.

71. To find the perimeter, you add all four sides: $P = (3x + 6) + (3x + 6) + (x + 2) + (x + 2)$. Simplify by combining like terms: $P = 2(3x + 6) + 2(x + 2) = 6x + 12 + 2x + 4 = 8x + 16$.

72. Add the three sides: $P = (x + 16) + (x + 16) + (2x + 36)$. Combine like terms to simplify: $P = 2(x + 16) + (2x + 36) = 2x + 32 + 2x + 36 = 4x + 68$.

CHAPTER

12 Relations and Functions

Math topics in this chapter:

- ☑ Function Notation and Evaluation
- ☑ Adding and Subtracting Functions
- ☑ Multiplying and Dividing Functions
- ☑ Composition of Functions
- ☑ Evaluate an Exponential Function
- ☑ Match Exponential Functions and Graphs
- ☑ Write Exponential Functions: Word Problems
- ☑ Function Inverses
- ☑ Domain and Range of Relations
- ☑ Rate of Change and Slope
- ☑ Complete a Function Table from an Equation

Practices

Evaluate each function.

1) $f(x) = x - 2$, find $f(-1)$
2) $g(x) = 2x + 4$, find (3)
3) $g(n) = 2n - 8$, find $g(-1)$
4) $h(n) = n^2 - 1$, find $h(-2)$
5) $f(x) = x^2 + 12$, find $f(5)$
6) $g(x) = 2x^2 - 9$, find $g(-2)$
7) $w(x) = 2x^2 - 4x$, find $w(2n)$
8) $p(x) = 4x^3 - 10$, find $p(-3a)$

Perform the indicated operation.

9) $g(x) = x - 2$
$h(x) = 2x + 6$
Find: $(h + g)(3)$
_____

10) $f(x) = 3x + 2$
$g(x) = -x - 6$
Find: $(f + g)(2)$
_____

11) $f(x) = 5x + 8$
$g(x) = 3x - 12$
Find: $(f - g)(-2)$
_____

12) $h(x) = 2x^2 - 10$
$g(x) = 3x + 12$
Find: $(h + g)(3)$
_____

13) $g(x) = 12x - 8$
$h(x) = 3x^2 + 14$
Find: $(h - g)(x)$
_____

14) $h(x) = -2x^2 - 18$
$g(x) = 4x^2 + 15$
Find: $(h - g)(a)$
_____

Perform the indicated operation.

15) $g(x) = x - 5$
$h(x) = x + 6$
Find: $(g.h)(-1)$
____

16) $f(x) = 2x + 2$
$g(x) = -x - 6$
Find: $(\frac{f}{g})(-2)$
____

17) $f(x) = 5x + 3$
$g(x) = 2x - 4$
Find: $(\frac{f}{g})(5)$
____

18) $h(x) = x^2 - 2$
$g(x) = x + 4$
Find: $(g.h)(3)$
____

19) $g(x) = 4x - 12$
$h(x) = x^2 + 4$
Find: $(g.h)(-2)$
____

20) $h(x) = 3x^2 - 8$
$g(x) = 4x + 6$
Find: $(\frac{h}{g})(-4)$
____

Solve.

21) $f(x) = 2x$

$g(x) = x + 3$

Find: $(fog)(2)$

______

24) $h(x) = 2x - 2$

$g(x) = x + 4$

Find: $(goh)(2)$

______

22) $f(x) = x + 2$

$g(x) = x - 6$

Find: $(fog)(-1)$

______

25) $f(x) = 2x - 8$

$g(x) = x + 10$

Find: $(fog)(-2)$

______

23) $f(x) = 3x$

$g(x) = x + 4$

Find: $(gof)(4)$

______

26) $f(x) = x^2 - 8$

$g(x) = 2x + 3$

Find: $(gof)(4)$

______

Use the following function to find: $f(x) = 3x(\frac{1}{2})^{2x+2}$

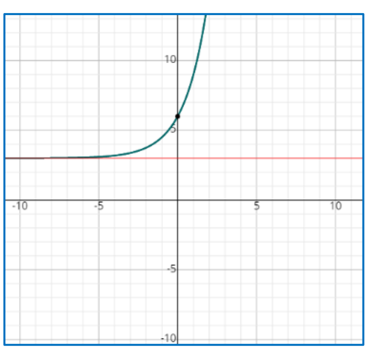
27) $f(2)$

28) $f(4)$

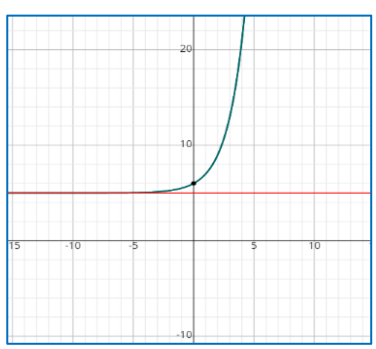
Match each exponential function to its graph.

29) $f(x) = -4(3)^x$, $f(x) = 2^x + 5$, $f(x) = 3(2)^x + 3$

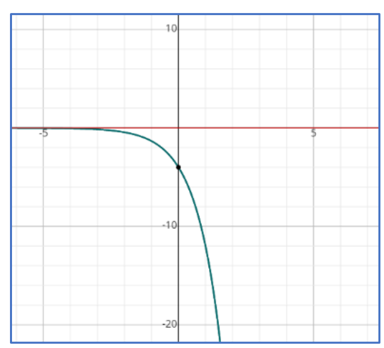
A



B



C



Solve.

30) As of 2019, the world population is 8.716 billion and growing at a rate of 1.2% per year. Write an equation to model population growth, where $p(t)$ is the population in billions of people and t is the time in years.

31) You decide to buy a used car that costs $20,000. You have heard that the car may depreciate at a rate of 10% per year. At this rate, how much will the car be worth in 6 years?

Find the inverse of each function.

32) $f(x) = -\frac{1}{x} - 9$

$f^{-1}(x) =$ ________

33) $g(x) = \sqrt{x} - 2$

$g^{-1}(x) =$ ________

34) $h(x) = -\frac{5}{x+3}$

$h^{-1}(x) =$ ________

35) $f(x) = 6x + 6$

$f^{-1}(x) =$ ________

Find the domain and range of each relation.

36) $\{(1,-1),(2,-4),(0,5),(-1,6)\}$

37) $\{(10,-5),(-16,-8),(-4,19),(16,7),(6,-14)\}$

38) $\{(4,7),(-15,6),(-20,9),(13,8),(7,5)\}$

Solve.

39) Average food preparation time in a restaurant was tracked daily as part of an efficiency improvement program.

Day	Food preparation time (minutes)
Tuesday	45
Wednesday	49
Thursday	32
Friday	15
Saturday	25

According to the table, what was the rate of change between Tuesday and Wednesday?

Complete the table.

40)

$f(x) = 3x - 2$	
x	(x)
-3	
0	
2	

41)

$f(x) = 2x$	
x	$f(x)$
1	
2	
3	

Answers

1) -3
2) 10
3) -10
4) 3
5) 37
6) -1
7) $8n^2 - 8n$
8) $-108a^3 - 10$
9) 13
10) 0
11) 16
12) 29
13) $3x^2 - 12x + 22$
14) $-6a^2 - 33$
15) -30
16) $\frac{1}{2}$
17) $\frac{14}{3}$
18) 49
19) -160
20) -4
21) 10
22) -5
23) 16
24) 6
25) 8
26) 19
27) $\frac{3}{32}$
28) $\frac{3}{256}$
29) $A = 3(2)^x + 3$,

$B = 2^x + 5$,

$C = -4(3)^x$
30) $p(t) = 8.716(1 + 0.012)^t$
31) $A = 20{,}000(1 - 0.1)^6$
32) $-\frac{1}{x+9}$
33) $x^2 + 4x + 4$
34) $-\frac{5}{x} - 3$
35) $\frac{x-6}{6}$

36) $D = (1,2,0,-1)$,

$R = (-1,-4,5,6)$

37) $D = (10,-16,-4,16,6)$,

$R = (-5,-8,19,7,-14)$

38) $D = (4,-15,-20,13,7)$,

$R = (7,6,9,8,5)$

39) 4

40)

$f(x) = 3x - 2$	
x	$f(x)$
-3	-11
0	-2
2	4

41).

$f(x) = 2x$	
x	$f(x)$
1	2
2	4
3	6

Answers and Explanations

1. Replace x with -1 in the function $f(x) = x - 2$. So, $f(-1) = -1 - 2 = -3$. This means when x is -1, the value of the function is -3.

2. Substitute x with 3 in $g(x) = 2x + 4$. This gives $g(3) = 2 \times 3 + 4 = 10$. Thus, when x is 3, the function evaluates to 10.

3. Here, replace n with -1 in $g(n) = 2n - 8$. This results in $g(-1) = 2 \times -1 - 8 = -10$. So, the function value is -10 when n is -1.

4. For $h(-2)$, substitute n with -2 in $h(n) = n^2 - 1$. We get $h(-2) = -2^2 - 1 = 4 - 1 = 3$. Hence, $h(-2)$ is 3.

5. Replace x with 5 in $f(x) = x^2 + 12$. So, $f(5) = 5^2 + 12 = 25 + 12 = 37$. The function value is 37 when x is 5.

6. In $g(x) = 2x^2 - 9$, substitute x with -2, giving $g(-2) = 2 \times (-2)^2 - 9 = 8 - 9 = -1$. Therefore, $g(-2)$ is -1.

7. For $w(2n)$, replace x with $2n$ in $w(x) = 2x^2 - 4x$. This leads to $w(2n) = 2 \times (2n)^2 - 4 \times 2n = 8n^2 - 8n$. Thus, $w(2n)$ equals $8n^2 - 8n$.

8. Substitute x with $-3a$ in $p(x) = 4x^3 - 10$. We get $p(-3a) = 4(-3a)^3 - 10 = -108a^3 - 10$. So, $p(-3a)$ is $-108a^3 - 10$.

9. We add g and h and then substitute $x = 3$: $(h + g)(x) = (2x + 6) + (x - 2)$, which simplifies to $3x + 4$. Substituting $x = 3$ gives us $3(3) + 4 = 9 + 4 = 13$.

10. We add f and g and substitute $x = 2$: $(f + g)(x) = (3x + 2) + (-x - 6)$, simplifying to $2x - 4$. Substituting $x = 2$ gives us $2(2) - 4 = 4 - 4 = 0$.

11. We subtract g from f and substitute $x = -2$: $(f - g)(x) = (5x + 8) - (3x - 12)$, simplifying to $2x + 20$. Substituting $x = -2$ gives us $2(-2) + 20 = -4 + 20 = 16$.

12. We add h and g and substitute $x = 3$: $(h + g)(x) = (2x^2 - 10) + (3x + 12)$, which simplifies to $2x^2 + 3x + 2$. Substituting x=3 gives us $2(3)^2 + 3(3) + 2 = 18 + 9 + 2 = 29$.

13. We subtract g from h without substituting any value for x: $(h - g)(x) = (3x^2 + 14) - (12x - 8)$, simplifying to $3x^2 - 12x + 22$.

14. We subtract g from h and substitute x with a: $(h - g)(x) = (-2x^2 - 18) - (4x^2 + 15)$, simplifying to $-6x^2 - 33$. When we substitute x with a, the expression remains $-6a^2 - 33$.

15. First, we find $g(-1)$, which is $-1 - 5 = -6$. Then, we find $h(-1)$, so $h(-1) = -1 + 6 = 5$. Therefore, $(g.h)(-1) = -6 \times 5 = -30$.

16. We calculate $f(-2)$ as $2\times(-2)+2=-4+2=-2$. Similarly, $g(-2)$ is $-(-2)-6=2-6=-4$. We then divide these results: $\frac{-2}{-4}=\frac{1}{2}$.

17. Here, $f(5)=5\times 5+3=25+3=28$. For $g(5)$, we get $(2\times 5)-4=10-4=6$. Dividing, $\frac{28}{6}=\frac{14}{3}$. Thus, $\left(\frac{f}{g}\right)(5)=\frac{14}{3}$.

18. First, calculate $h(3)=3^2-2=9-2=7$. Then, $g(3)=3+4=7$. So, $(g.h)(3)=7\times 7=49$.

19. We start with $h(-2)=(-2)^2+4=4+4=8$. Then, $g(-2)=4\times(-2)-12=-8-12=-20$. Therefore, $(g.h)(-2)=8\times(-20)=-160$.

20. Calculating $h(-4)=3\times(-4)^2-8=(3\times 16)-8=48-8=40$. For $g(-4)$, it's

$4\times(-4)+6=-16+6=-10$. Dividing these gives $\frac{40}{-10}=-4$. Thus, $(\frac{h}{g})(-4)=-4$.

21. First, we find $g(2)$, which is $2+3=5$. Then, we use this value in $f(x)$, so $f(5)=2\times 5=10$. Therefore, $(fog)(2)=10$.

22. We calculate $g(-1)$ as $-1-6=-7$. Then, we substitute this into $f(x)$, so $f(-7)=-7+2=-5$. Hence, $(fog)(-1)=-5$.

23. First, find $f(4)$, which is $3\times 4=12$. Next, use this value in $g(x)$, resulting in $g(12)=12+4=16$. Thus, $(gof)(4)=16$.

24. Compute $h(2)$ by substituting 2 into $h(x)$, giving $h(2)=(2\times 2)-2=4-2=2$. Then, use this result in $g(x)$, so $g(2)=2+4=6$. Therefore, $(goh)(2)=6$.

25. Calculate $g(-2)$ as $-2+10=8$. Next, use this value in $f(x)$, resulting in $f(8)=2\times 8-8=16-8=8$. So, $(fog)(-2)=8$.

26. Start by finding $f(4)$, which is $4^2-8=16-8=8$. Then, substitute this result into $g(x)$, giving $g(8)=(2\times 8)+3=16+3=19$. Hence, $(gof)(4)=19$.

27. To evaluate $f(2)$, we substitute $x=2$ into the function $f(x)=3x(\frac{1}{2})^{2x+2}$. This becomes $3\times 2(\frac{1}{2})^{2\times 2+2}=6\times(\frac{1}{2})^6$. The expression $(\frac{1}{2})^6$ represents $\frac{1}{2}$ multiplied by itself 6 times, which equals $\frac{1}{64}$. Thus, $f(2)=6\times\frac{1}{64}=\frac{6}{64}=\frac{3}{32}$.

28. For $f(4)$, we substitute $x=4$ into the same function, resulting in $3\times 4(\frac{1}{2})^{2\times 4+2}=12\times(\frac{1}{2})^{10}$. The term $(\frac{1}{2})^{10}$ is $\frac{1}{2}$ raised to the 10th power, which equals $\frac{1}{1024}$. Therefore, $f(4)=12\times\frac{1}{1024}=\frac{12}{1024}=\frac{3}{256}$.

29. $f(x) = -4(3)^x$: This function has a negative coefficient in front of the base, which means it will reflect across the x-axis, resulting in a graph that decreases as x increases. The base is greater than 1, so the reflection will still show an exponential decay. Therefore, we're looking for a graph that goes downwards as we move from left to right. This matches with graph C.

$f(x) = 2^x + 5$: This function is a standard exponential growth function with a vertical shift upwards by 5 units. Since there's no negative sign or coefficient greater than 1 in front of the base, the graph will increase as x increases and will be above the standard 2^x graph by 5 units. This is represented by graph B.

$f(x) = 3(2)^x + 3$: Similar to the second function, this one also represents exponential growth with a base of 2. However, it has a larger vertical shift than the second function due to the "+3" outside the exponent, moving it further up. The coefficient of 3 in front of the 2^x will make the graph grow faster than the standard 2^xgraph. This is shown by graph A, which grows faster and starts higher due to the vertical shift compared to graph B.

30. To model the world population growth based on the given data, we can use an exponential growth equation. The standard form for exponential growth is $p(t) = p_0(1 + r)^t$, where p_0 is the initial amount (in this case, the initial population), r is the growth rate (as a decimal), and t is the time in years. Here, $p_0 = 8.716$ billion (the population in 2019). The growth rate $r = 1.2\% = 0.012$ (converted from a percentage to a decimal), t will be the number of years since 2019. So, the equation to model the population growth is $p(t) = 8.716(1 + 0.012)^t$.

31. For the depreciation of the car, we'll use an exponential decay model since the car's value decreases over time. The formula for exponential decay is similar to growth: $A_t = A_0(1 - r)^t$, where A_0 is the initial value, r is the rate of decay, and t is the time in years. The initial value of the car, A_0 is \$20,000, the annual depreciation rate $r = 10\% = 0.1$. For $t = 6$ years, the equation becomes $A(6) = 20{,}000(1 - 0.1)^6$.

32. First, replace $f(x)$ with y: $y = -\frac{1}{x} - 9$. To find the inverse, swap x and y: $x = -\frac{1}{y} - 9$. Next, isolate y. Start by adding 9 to both sides: $x + 9 = -\frac{1}{y}$. Multiply both sides by $-y$ to get rid of the fraction: $-y(x + 9) = 1$. Finally, solve for y by dividing by $-(x + 9)$: $y = -\frac{1}{x+9}$. Therefore, $f^{-1}(x) = -\frac{1}{x+9}$.

33. Write the function as $y = \sqrt{x} - 2$. To find its inverse, exchange x and y: $x = \sqrt{y} - 2$. Rearrange to solve for y. First, add 2 to both sides: $x + 2 = \sqrt{y}$. Then square both sides to eliminate the square root: $(x + 2)^2 = y$. Thus, $g(x)^{-1} = (x + 2)^2 = x^2 + 4x + 4$.

34. Start with $y = -\frac{5}{x+3}$. Swap x and y to get $x = -\frac{5}{y+3}$. Rearrange to find y. Multiply both sides by $y + 3$ and then by $-x(y + 3) = 5$. Divide by $-x$ to get $y + 3 = -\frac{5}{x}$. Subtract 3 to isolate y: $y = -\frac{5}{x} - 3$. Therefore, $h(x)^{-1} = -\frac{5}{x} - 3$.

35. Represent the function as $y = 6x + 6$. Exchange x and y: $x = 6y + 6$. Now, solve for y. Subtract 6 from both sides: $x - 6 = 6y$. Divide everything by 6: $y = \frac{x-6}{6}$. Hence, $f^{-1}(x) = \frac{x-6}{6}$.

36. Look at the first numbers in each pair: $1, 2, 0, -1$. These represent the x-values. So, the domain is $\{1, 2, 0, -1\}$. Now look at the second numbers: $-1, -4, 5, 6$. These are the y-values. Therefore, the range is $\{-1, -4, 5, 6\}$.

37. The first numbers are $10, -16, -4, 16, 6$. These form the domain: $\{10, -16, -4, 16, 6\}$. The second numbers are $-5, -8, 19, 7, -14$. These make up the range: $\{-5, -8, 19, 7, -14\}$.

38. The first elements are $4, -15, -20, 13, 7$. So, the domain is $\{4, -15, -20, 13, 7\}$. The second elements are $7, 6, 9, 8, 5$. Thus, the range is $\{7, 6, 9, 8, 5\}$.

39. To find the rate of change, we subtract the value at the starting point from the value at the ending point, and then divide by the time passed. $Rate\ of\ Change = \frac{Wednesday's\ time - Tuesday's\ time}{Time\ passed} = \frac{49-45}{1} = 4$.

40. For the first function, $f(x) = 3x - 2$: When $f(-3) = 3(-3) - 2 = -9 - 2 = -11$.

When $f(0) = 3(0) - 2 = 0 - 2 = -2$. When $f(2) = 3(2) - 2 = 6 - 2 = 4$.

$f(x) = 3x - 2$	
x	$f(x)$
-3	-11
0	-2
2	4

41. For the second function, $f(x) = 2x$: When $f(1) = 2(1) = 2$. When $f(2) = 2(2) = 4$. When $f(3) = 2(3) = 6$.

$f(x) = 2x$	
x	$f(x)$
1	2
2	4
3	6

CHAPTER 13

Radical Expressions

Math topics in this chapter:

- ☑ Simplifying Radical Expressions
- ☑ Adding and Subtracting Radical Expressions
- ☑ Multiplying Radical Expressions
- ☑ Rationalizing Radical Expressions
- ☑ Radical Equations
- ☑ Domain and Range of Radical Functions
- ☑ Simplify Radicals with Fractions

Practices

Evaluate.

1) $\sqrt{49} =$ ________

2) $\sqrt{4} \times \sqrt{81} =$ ________

3) $\sqrt{16} \times \sqrt{4x^2} =$ ________

4) $\sqrt{289} =$ ________

5) $\sqrt{25 \quad ^{4}} =$ ________

6) $\sqrt{9} \times \sqrt{x^2} =$ ________

Simplify.

7) $\sqrt{6} + 6\sqrt{6} =$

8) $9\sqrt{8} - 6\sqrt{2} =$

9) $-\sqrt{7} - 5\sqrt{7} =$

10) $10\sqrt{2} + 3\sqrt{18} =$

11) $\sqrt{12} - 6\sqrt{3} =$

12) $-2\sqrt{x} + 6\sqrt{x} =$

Evaluate.

13) $\sqrt{4} \times 2\sqrt{9} =$

14) $\sqrt{5} \times 3\sqrt{20y} =$

15) $-6\sqrt{4} \times 3\sqrt{4} =$

16) $-9\sqrt{3b^2} \times \left(-\sqrt{6}\right) =$

Simplify.

17) $\frac{1+\sqrt{5}}{1-\sqrt{3}} =$

18) $\frac{2+\sqrt{6}}{\sqrt{2}-\sqrt{5}} =$

19) $\frac{\sqrt{7}}{\sqrt{6}-\sqrt{3}} =$

20) $\frac{\sqrt{8a}}{\sqrt{a^5}} =$

Solve for *x* in each equation.

21) $2\sqrt{2x - 4} = 8$

22) $9 = \sqrt{4x - 1}$

23) $\sqrt{x} + 6 = 11$

24) $\sqrt{5x} = \sqrt{x + 3}$

Identify the domain and range of each function.

25) $= \sqrt{x+1}$

26) $y = \sqrt{x-2} + 6$

27) $y = \sqrt{x} - 1$

28) $y = \sqrt{x-4}$

Simplify.

29) $\sqrt{\frac{625}{36}}$

30) $\sqrt{\frac{1296}{25}}$

31) $\sqrt{\frac{147}{64}}$

32) $\sqrt{\frac{98}{18}}$

Answers

1) 7
2) 18
3) 8
4) 17
5) $5b^2$
6) $3x$
7) $7\sqrt{6}$
8) $12\sqrt{2}$
9) $-6\sqrt{7}$
10) $19\sqrt{2}$
11) $-4\sqrt{3}$
12) $4\sqrt{x}$
13) 12
14) $30y$
15) -72
16) $27b\sqrt{2}$
17) $-\frac{(1+\sqrt{5})(1+\sqrt{3})}{2}$
18) $-\frac{2\sqrt{2}+2\sqrt{5}+2\sqrt{3}+\sqrt{30}}{3}$
19) $\frac{\sqrt{7}(\sqrt{6}+\sqrt{3})}{3}$
20) $\frac{2\sqrt{2}}{a^2}$
21) $x = 10$
22) $x = 20.5$
23) $x = 25$
24) $x = \frac{3}{4}$
25) $x \geq -1, y \geq 0$
26) $x \geq 2, \geq 6$
27) $x \geq 0, y \geq -1$
28) $x \geq 4, y \geq 0$
29) $\frac{25}{6}$
30) $\frac{36}{5}$
31) $\frac{7\sqrt{3}}{8}$
32) $\frac{7}{3}$

Answers and Explanations

1. The square root of 49 is the number that when multiplied by itself equals 49. This number is 7 because $7 \times 7 = 49$.

2. The square root of 4 is 2, and the square root of 81 is 9. When you multiply these together, 2×9, you get 18.

3. The square root of 16 is 4, and the square root of $4x^2$ is $2x$. Multiplying these together, $4 \times 2x$, gives $8x$.

4. The square root of 289 is the number that when squared equals 289. This number is 17 because $17 \times 17 = 289$.

5. The square root of $25b^4$ is the product of the square root of 25, which is 5, and the square root of b^4, which is b^2. So, $5 \times \ b^2 = 5b^2$.

6. The square root of 9 is 3, and the square root of x^2 is x. Multiplying these gives $3 \times x$, or $3x$.

7. Both terms have the square root of 6, so they can be combined just like you would combine like terms such as $1x + 6x$. Therefore, $\sqrt{6} + 6\sqrt{6}$ simplifies to $7\sqrt{6}$.

8. Here, we can simplify $\sqrt{8}$ to $2\sqrt{2}$ (since 8 is 4 times 2, and the square root of 4 is 2). This gives us $9 \times 2\sqrt{2} - 6\sqrt{2}$, which simplifies to $18\sqrt{2} - 6\sqrt{2}$, and further to $12\sqrt{2}$.

9. These are like terms since both involve the square root of 7. Combining them like regular numbers, we get $-1\sqrt{7} - 5\sqrt{7}$ which simplifies to $-6\sqrt{7}$.

10. We can simplify $\sqrt{18}$ to $3\sqrt{2}$ (since 18 is 9 times 2, and the square root of 9 is 3). This gives us $10\sqrt{2} + 3 \times 3\sqrt{2}$, which simplifies to $10\sqrt{2} + 9\sqrt{2}$, and further simplifies to $19\sqrt{2}$.

11. We can simplify $\sqrt{12}$ to $2\sqrt{3}$ (since 12 is 4 times 3, and the square root of 4 is 2). This gives us $2\sqrt{3} - 6\sqrt{3}$, which simplifies to $-4\sqrt{3}$.

12. These are like terms since both involve the square root of x. We combine them just like we would with similar variables, $-2\sqrt{x} + 6\sqrt{x}$, which simplifies to $4\sqrt{x}$.

13. First, evaluate the square roots: $\sqrt{4}$ is 2, and $\sqrt{9}$ is 3. Now multiply the numbers outside the square roots: 2 times 2, which equals 4. Then multiply this by the square root we found earlier, which is 3, to get $4 \times 3 = 12$.

14. We can't simplify $\sqrt{5}$, but we can simplify $\sqrt{20}$. Since 20 is 4 times 5 and the square root of 4 is 2, $\sqrt{20}$ is $2\sqrt{5}$. Now multiply the coefficients (numbers outside the square roots): 3

times 2 equals 6. Finally, multiply this by the $\sqrt{5}$ we left untouched earlier to get $6\sqrt{5} \times \sqrt{5}$, which simplifies to $6 \times 5 = 30$ since $\sqrt{5} \times \sqrt{5} = 5$. So, the expression becomes $30y$.

15. The square root of 4 is 2, so $\sqrt{4}$ is 2. Multiply -6 by 2 to get -12, and 3 by 2 to get 6. Then multiply -12 by 6, which equals -72.

16. We'll simplify $\sqrt{(3b^2)}$. Since b^2 is just b times b, and the square root of a squared number is the number itself, $\sqrt{(3b^2)}$ is $b\sqrt{3}$. Now, multiply -9 by $b\sqrt{3}$ to get $-9b\sqrt{3}$. Finally, multiply this by $-\sqrt{6}$ to get $-9b\sqrt{3} \times -\sqrt{6}$, which simplifies to $9b\sqrt{18}$. The square root of 18 can be simplified further to $3\sqrt{2}$ (since 18 is 9 times 2 and the square root of 9 is 3), resulting in $9b \times 3\sqrt{2} = 27b\sqrt{2}$.

17. To simplify, we multiply the numerator and denominator by the conjugate of the denominator. The conjugate of $1 - \sqrt{3}$ is $1 + \sqrt{3}$. This removes the square root from the denominator. $\frac{1+\sqrt{5}}{1-\sqrt{3}} \times \frac{1+\sqrt{3}}{1+\sqrt{3}}$. Multiply out both the numerator and the denominator: $\frac{(1+\sqrt{5})(1+\sqrt{3})}{1-3} = -\frac{(1+\sqrt{5})(1+\sqrt{3})}{2}$.

18. Here again, we use the conjugate to eliminate the square root from the denominator. The conjugate of $\sqrt{2} - \sqrt{5}$ is $\sqrt{2} + \sqrt{5}$. $\frac{2+\sqrt{6}}{\sqrt{2}-\sqrt{5}} \times \frac{\sqrt{2}+\sqrt{5}}{\sqrt{2}+\sqrt{5}}$. Multiply and simplify: $\frac{(2+\sqrt{6})(\sqrt{2}+\sqrt{5})}{2-5}$. Expand and simplify: $-\frac{2\sqrt{2}+2\sqrt{5}+2\sqrt{3}+\sqrt{30}}{3}$.

19. Use the conjugate technique again. Multiply the expression by $\frac{\sqrt{6}+3}{\sqrt{6}+3}$. $\frac{\sqrt{7}}{\sqrt{6}-\sqrt{3}} \times \frac{\sqrt{6}+3}{\sqrt{6}+3}$. Simplify: $\frac{\sqrt{7}(\sqrt{6}+\sqrt{3})}{6-3} = \frac{\sqrt{7}(\sqrt{6}+\sqrt{3})}{3}$.

20. Simplify each square root separately. $\sqrt{8a} = \sqrt{2 \times 4 \times a} = 2\sqrt{2a}$ and $\sqrt{a^5} = a^2\sqrt{a}$. Now divide: $\frac{2\sqrt{2a}}{a^2\sqrt{a}}$. Simplify the square roots: $\frac{2}{a^2} \times \sqrt{\frac{2a}{a}} = \frac{2\sqrt{2}}{a^2}$.

21. To solve, first divide both sides by 2: $\sqrt{2x - 4}$. Now, square both sides to eliminate the square root: $2x - 4 = 16$. Next, add 4 to both sides: $2x = 20$. Finally, divide by 2: $x = 10$.

22. Start by squaring both sides to remove the square root: $81 = 4x - 1$. Then, add 1 to both sides: $82 = 4x$. Divide by 4 to isolate $x = 20.5$.

23. First, isolate the square root by subtracting 6 from both sides: $\sqrt{x} = 5$. Now, square both sides to eliminate the square root: $x = 25$.

24. Square both sides to remove the square roots: $5x = x + 3$. Next, subtract x from both sides: $4x = 3$. Finally, divide by 4: $x = \frac{3}{4}$.

25. The expression inside the square root, $x+1$, must be greater than or equal to 0. Solving $x+1 \geq 0$ gives us $x \geq -1$. So, the domain is all real numbers x such that $x \geq -1$. Since square roots always yield non-negative results, y will be 0 or positive. Therefore, the range is all real numbers y such that $y \geq 0$.

26. For $x-2$ to be non-negative, $x \geq 2$. Thus, the domain is $x \geq 2$. The smallest value of $\sqrt{(x-2)}$ is 0 (when $x=2$), and since we add 6 to it, the smallest value of y is 6. Hence, the range is $y \geq 6$.

27. The square root of x requires x to be non-negative, so $x \geq 0$. This makes the domain $x \geq 0$. The smallest value $\sqrt{x}$ can take is 0 (when $x=0$), and subtracting 1 from it gives -1. So, the range starts from -1 and goes upwards, making it $y \geq -1$.

28. For $x-4$ to be non-negative, x must be at least 4. Therefore, the domain is $x \geq 4$. The square root function yields non-negative outputs. Thus, the smallest value for y is 0 (when $x=4$), making the range $y \geq 0$.

29. The square root of 625 is 25 (because $25 \times 25 = 625$), and the square root of 36 is 6 (since $6 \times 6 = 36$). So, $\sqrt{\frac{625}{36}}$ becomes $\frac{25}{6}$.

30. Here, the square root of 1296 is 36 (as $36 \times 36 = 1296$), and the square root of 25 is 5 ($5 \times 5 = 25$). Thus, $\sqrt{\frac{1296}{25}}$ simplifies to $\frac{36}{5}$.

31. For this, the square root of 147 is not a whole number, but we can simplify it. Let's find the largest perfect square that divides into 147. The factors of 147 include $1, 3, 7, 21, 49, 147$. The largest perfect square among these is 49. So, 147 can be written as 49×3. The square root of 49 is 7, so the square root of 147 simplifies to $7\sqrt{3}$. The square root of 64 is 8 ($8 \times 8 = 64$). So, $\sqrt{\frac{147}{64}}$ becomes $\frac{7\sqrt{3}}{8}$.

32. The number 98 doesn't have a perfect square root, but it can be simplified. Let's find the largest perfect square that divides into 98. The factors of 98 include $1, 2, 7, 14, 49, 98$. The largest perfect square among these is 49. So, 98 can be written as 49×2. The square root of 49 is 7, so the square root of 98 simplifies to $7\sqrt{2}$. The number 18 is not a perfect square, so its square root does not simplify to a whole number. The factors of 18 include $1, 2, 3, 6, 9, 18$. The largest perfect square is 9. So, 18 can be written as 9×2, and the square root of 18 simplifies to $3\sqrt{2}$. Combining these, $\sqrt{\frac{98}{18}}$ simplifies to $\frac{7\sqrt{2}}{3\sqrt{2}}$. When you simplify this fraction, the $\sqrt{2}$ in the numerator and denominator cancel out, leaving $\frac{7}{3}$.

CHAPTER

14 Geometry and Solid Figures

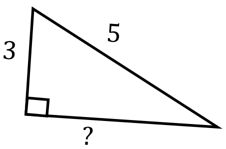
Math topics in this chapter:

- ☑ The Pythagorean Theorem
- ☑ Complementary and Supplementary angles
- ☑ Parallel lines and Transversals
- ☑ Triangles
- ☑ Special Right Triangles
- ☑ Polygons
- ☑ Circles
- ☑ Trapezoids
- ☑ Cubes
- ☑ Rectangle Prisms
- ☑ Cylinder

Practices

Find the missing side?

1)



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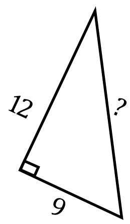

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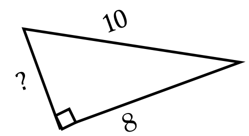

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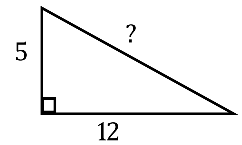
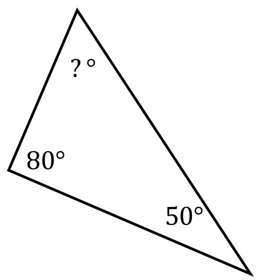


Find the measure of the unknown angle in each triangle.

5)



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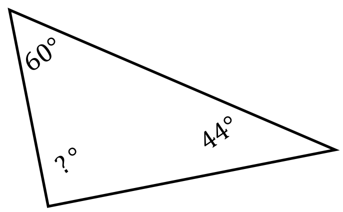

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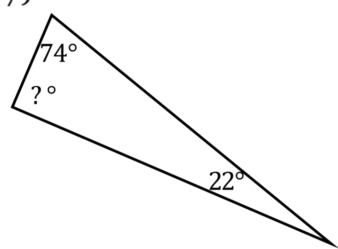

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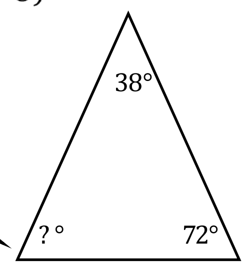
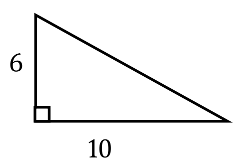


Find the area of each triangle.

9)



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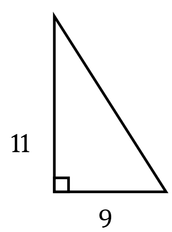

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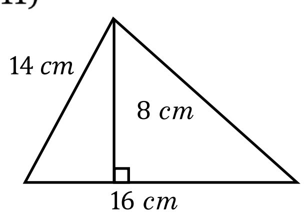

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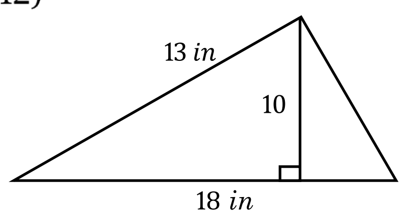
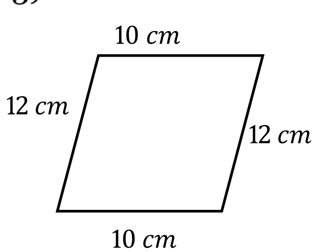


Find the perimeter or circumference of each shape.

13)



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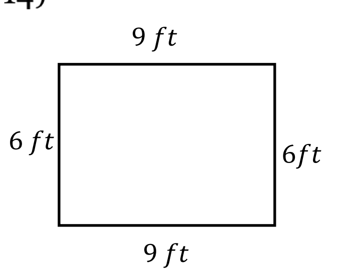

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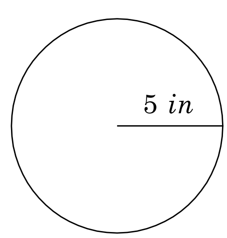
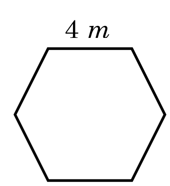
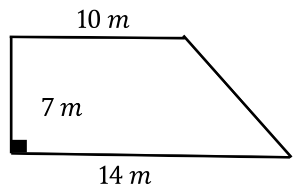


16) *regular hexagon*



Find the area of each trapezoid.

17)



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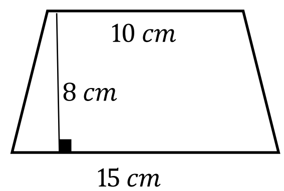

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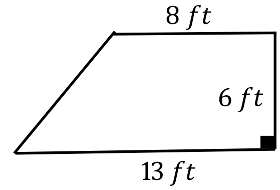

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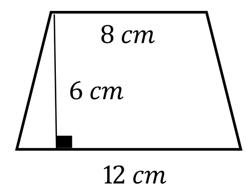
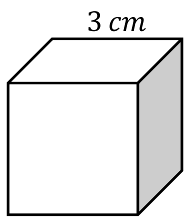


Find the volume of each cube.

21)



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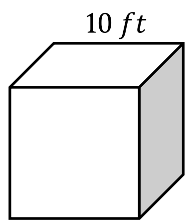

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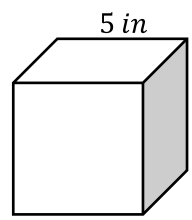

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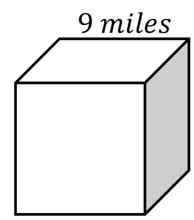
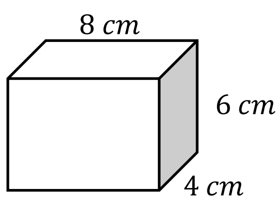
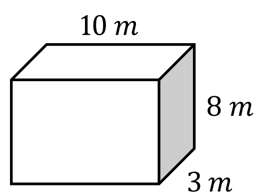


Find the volume of each Rectangular Prism.

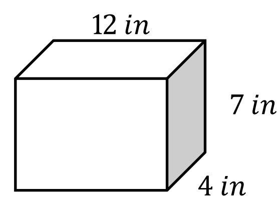
25)



26)

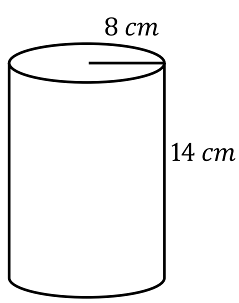


27)



Find the volume of each Cylinder. Round your answer to the nearest tenth. ($\pi = 3.14$)

28)



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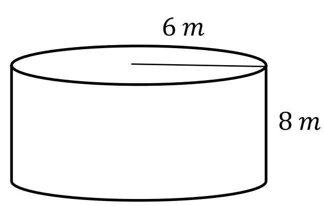

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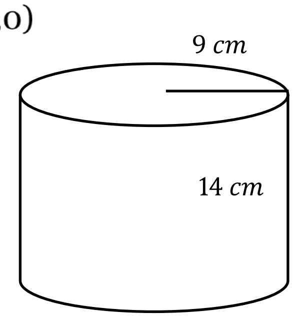
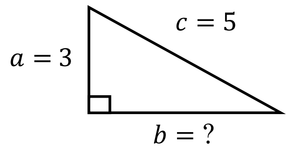


Answers

1) 4
2) 15
3) 6
4) 13
5) 50
6) 76
7) 84
8) 70
9) 30
10) 49.5
11) $64\ cm^2$
12) $90\ in^2$
13) $44\ cm$
14) $30\ ft$
15) $10\ \pi \approx 31.4\ n$
16) $24\ m$
17) $84\ m^2$
18) $100\ cm^2$
19) $63\ ft^2$
20) $60\ cm^2$
21) $27\ cm^3$
22) $1{,}000\ ft^3$
23) $125\ in^3$
24) $729\ mi^3$
25) $192\ cm^3$
26) $240\ m^3$
27) $336\ ^3$
28) $2{,}813.44\ cm^3$
29) $904.32\ m^3$
30) $3{,}560.76\ cm^3$

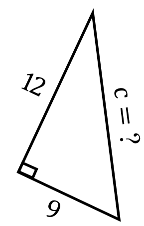
Answers and Explanations

1. The first triangle is a right triangle, which means we can use the Pythagorean theorem. This theorem states that in a right triangle, $a^2 + b^2 = c^2$. Use this formula to find side b: $5^2 = 3^2 + b^2 \Rightarrow 25 = 9 + b^2$.

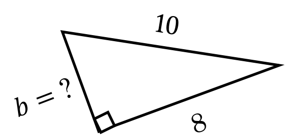


From this, $b^2 = 16$. Taking the square root of both sides, b is approximately 4.

2. Using the Pythagorean theorem: $c^2 = 9^2 + 12^2$. This simplifies to $c^2 = 81 + 144$, which is $c^2 = 225$. Taking the square root, $c = 15$.

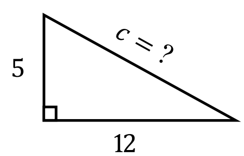


3. Applying the Pythagorean theorem: $10^2 = 8^2 + b^2$.



This results in $100 = 64 + b^2$. From this, $b^2 = 36$, and taking the square root, $b = 6$.

4. Using the theorem: $c^2 = 5^2 + 12^2$. From this equation, $25 + 144 = c^2$. This leads to $c = 13$.



5. In any triangle, the total of all angles is 180°. For the triangle shown: given angles: 80° and 50°. To find the unknown angle, subtract the sum of the given angles from 180°.

$$180^\circ - (80^\circ + 50^\circ) = 50^\circ$$

6. Subtract the sum of 60° and 44° from 180°. By performing the calculation, $180^\circ - (60^\circ + 44^\circ) = 76^\circ$. Thus, the unknown angle is 76°.

7. Find the difference between 180° and the total of 74° and 22°. After doing the math, $180^\circ - (74^\circ + 22^\circ) = 84^\circ$. This means our missing angle measures 84°.

8. To determine the third angle: deduct the sum of 38° and 72° from 180°. Working it out gives us: $180^\circ - (38^\circ + 72^\circ) = 70^\circ$.

9. The area of a triangle is given by the formula: $Area = 0.5 \times base \times height$. Here, the base is 10 units, and the height is 6 units. $Area = 0.5 \times 10 \times 6 = 30$ square units.

10. To find the area, multiply the base and height together and then divide by 2:

$Area = (9 \times 11) \div 2 = 49.5$ square units

11. In this triangle, the base measures $16\ cm$ and the height is $8\ cm$. Find the area by taking the base and height, multiplying them, and then halving the result.

$$Area = 0.5 \times 16\ cm \times 8\ cm = 64\ cm^2$$

12. To determine the area, we combine the base and height values, take their product, and then split it in two: $Area = 0.5 \times 18\ in \times 10\ in = 90\ in^2$.

13. For a parallelogram, the perimeter is the sum of all its sides. Here, two sides are $10\ cm$ each and two sides are $12\ cm$ each.

$$Perimeter = (2 \times 10\ cm) + (2 \times 12\ cm) = 20\ cm + 24\ cm = 44\ cm$$

14. For a square, all four sides are equal. Given two sides are $9\ ft$ each, and two sides are $6\ ft$ each. Therefore, $Perimeter = (2 \times 9\ ft) + (2 \times 6\ ft) = 30\ ft$.

15. Circles have a unique calculation called circumference, determined using the formula: $Circumference = 2\pi r$. Here, the radius is $5\ in$. So, we have:

$$Circumference = 2 \times \pi \times 5\ in = 10\pi\ in \approx 10 \times 3.14\ in = 31.4\ in$$

16. A regular hexagon has six equal sides. With each side being $4\ m$,

$$Perimeter = 6 \times 4\ m = 24\ m$$

17. A trapezoid's area is calculated as half the sum of its parallel sides (bases) multiplied by its height. The formula is: $Area = \frac{1}{2} \times (sum\ of\ bases) \times height$. For this shape, the bases are $10\ m$ and $14\ m$. The distance between them (height) is $7\ m$.

$$Area = 0.5 \times (10\ m + 14\ m) \times 7m = 0.5 \times 24\ m \times 7\ m = 84\ m^2$$

18. For this trapezoid, the bases are $10\ cm$ and $15\ cm$, and the height is $8\ cm$.

$$Area = 0.5 \times (10\ cm + 15\ cm) \times 8\ cm = 0.5 \times 25\ cm \times 8\ cm = 100\ cm^2$$

19. Here, the bases are $8\ ft$ and $13\ ft$, with a height of $6\ ft$.

$$Area = 0.5 \times (8\ ft + 13\ ft) \times 6\ ft = 0.5 \times 21\ ft \times 6\ ft = 63\ ft^2$$

20. For this last trapezoid, the bases are $8\ cm$ and $12\ cm$, and the height is $6\ cm$.

$$Area = 0.5 \times (8\ cm + 12\ cm) \times 6\ cm = 0.5 \times 20\ cm \times 6\ cm = 60\ cm^2$$

21. The volume of a cube is calculated by raising the length of one side to the third power. For the first cube with a side length of $3\ cm$: $Volume = 3\ cm \times 3\ cm \times 3\ cm = 27\ cm^3$.

22. For the second cube with a side length of $10\ ft$:

$$Volume = 10\ ft \times 10\ ft \times 10\ ft = 1{,}000\ ft^3$$

23. For the third cube, which is $5\ in$ on each side: $Volume = 5\ in \times 5\ in \times 5\ in = 125\ in^3$.

24. For the cube that's 9 miles on each side: $Volume = 9\ mi \times 9\ mi \times 9\ mi = 729\ mi^3$.

25. The volume of a rectangular prism is determined by multiplying its length, width, and height. For the first prism, with dimensions $8\ cm$ by $6\ cm$ by $4\ cm$: $Volume = 8\ cm \times 6\ cm \times 4\ cm = \ 192\ cm^3$.

26. $Volume = 10\ m \times 8\ m \times 3\ m = 240\ m^3$.

27. For the prism, having dimensions $12\ in$ by $7\ in$ by $4\ in$: $Volume = 12\ in \times 7\ in \times 4\ in = 336\ in^3$.

28. To determine the volume of a cylinder, we utilize the formula:

$$Volume\ of\ \text{a}\ Cylinder = \pi(radius)^2 \times height$$

$$Volume = 3.14 \times (8\ cm \times 8\ cm) \times 14\ cm = 3.14 \times 64\ cm^2 \times 14\ cm = 2{,}813.4\ cm^3$$

29. $Volume = 3.14 \times (6\ m \times 6\ m) \times 8\ m = 3.14 \times 36\ m^2 \times 8\ m = 904.3\ m^3$.

30. $Volume = 3.14 \times (9\ cm \times 9\ c) \times 14\ cm = 3.14 \times 81\ cm^2 \times 14\ cm = 3{,}560.8\ cm^3$.

CHAPTER

15 Statistics

Math topics in this chapter:

- ☑ Mean, Median, Mode, and Range of the Given Data
- ☑ Pie Graph
- ☑ Probability Problems
- ☑ Permutations and Combinations

Practices

Find the values of the Given Data.

1) 6, 11, 5, 3, 6

Mode: _____ Range: _____

Mean: _____ Median: _____

2) 4, 9, 1, 9, 6, 7

Mode: _____ Range: _____

Mean: _____ Median: _____

3) 10, 3, 6, 10, 4, 15

Mode: _____ Range: _____

Mean: _____ Median: _____

4) 12, 4, 8, 9, 3, 12, 15

Mode: _____ Range: _____

Mean: _____ Median: _____

The circle graph below shows all Bob's expenses for last month. Bob spent $790 on his Rent last month.

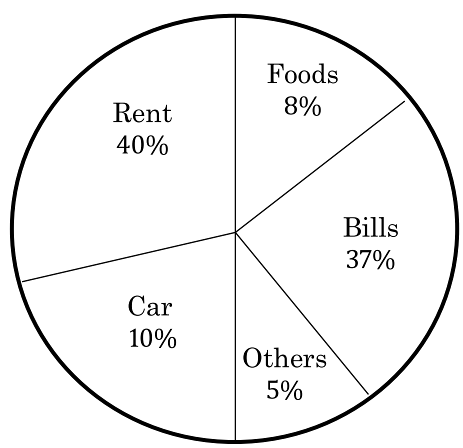
5) How much did Bob's total expenses last month? ________

6) How much did Bob spend for foods last month? ________

7) How much did Bob spend for his bills last month? ________

8) How much did Bob spend on his car last month? ________

Bob's last month expenses



Solve.

9) Bag A contains 8 red marbles and 6 green marbles. Bag B contains 5 black marbles and 7 orange marbles. What is the probability of selecting a green marble at random from bag A? What is the probability of selecting a black marble at random from Bag B?

_______________ _______________

Solve.

10) Susan is baking cookies. She uses sugar, flour, butter, and eggs. How many different orders of ingredients can she try? _______________

11) Jason is planning for his vacation. He wants to go to museum, go to the beach, and play volleyball. How many different ways of ordering are there for him? _______________

12) In how many ways can a team of 6 basketball players choose a captain and co-captain? _______________

13) How many ways can you give 5 balls to your 8 friends? _______________

14) A professor is going to arrange her 5 students in a straight line. In how many ways can she do this? _______________

15) In how many ways can a teacher chooses 12 out of 15 students? _______________

Answers

1) Mode: 6, Range: 8, Mean: 6.2, Median: 6
2) Mode: 9, Range:8, Mean: 6, Median: 6.5
3) Mode: 10, Range: 12, Mean: 8, Median: 8
4) Mode: 12, Range: 12, Mean: 9, Median: 9
5) $1,975
6) $158
7) $730.75
8) $197.50
9) $\frac{3}{7}, \frac{5}{12}$
10) 24
11) 6
12) 30 (it's a permutation problem)
13) 56 (it's a combination problem)
14) 120
15) 455 (it's a combination problem)

Answers and Explanations

1. The mode is the number that appears most frequently. For this set, 6 appears twice which is more than any other number. Thus, the mode is 6.

The range is found by subtracting the smallest number from the largest number. Here, the smallest is 3 and the largest is 11. Range: $11 - 3 = 8$.

To find the mean, sum up all the numbers and then divide by the number of numbers. In this case, the total sum is 31 and there are 5 numbers. Mean: $31 \div 5 = 6.2$.

The median is the middle value when numbers are in order. As there are 5 numbers, the third number when sorted is the median. The sorted set is 3, 5, 6, 6, 11. Median: 6.

2. Here, the number 9 appears twice, which is more frequent than any other number. Mode: 9.

With 9 as the largest and 1 as the smallest, range: $9 - 1 = 8$.

Total sum is 36, and there are 6 numbers. Mean: $36 \div 6 = 6$.

With 6 numbers, the median is the average of the third and fourth numbers when sorted. Our sorted set is 1, 4, 6, 7, 9, 9. Median: $(6 + 7) \div 2 = 6.5$.

3. The number 10 appears twice. Mode: 10.

Largest is 15 and smallest is 3. Range: $15 - 3 = 12$.

Sum is 48, and there are 6 numbers. Mean: $48 \div 6 = 8$.

Like before, it's the average of third and fourth numbers when sorted. Our sorted list is 3, 4, 6, 10, 10, 15. Median: $(6 + 10) \div 2 = 8$.

4. 12 appears twice here. Mode: 12.

Largest is 15, smallest is 3. Range: $15 - 3 = 12$.

Mean: Sum is 63 and there are 7 numbers. Mean: $63 \div 7 = 9$.

Median: As there are 7 numbers, the fourth number when sorted is the median. Sorted, it's 3, 4, 8, 9, 12, 12, 15. Median: 9.

5. Let x be Bob's total expenses. Therefore, 40% of $x = \$790$. To represent 40% in decimal form, it is (0.40). So, $0.40x = \$790 \Rightarrow x = \$790 \div 0.40 \Rightarrow x = \$1{,}975$. Bob's total expenses last month were $\$1{,}975$.

6. 8% of $\$1{,}975$ is the amount spent on food. To find 8% in decimal form, it's 0.08: $0.08 \times \$1{,}975 = \$158$. Bob spent $\$158$ on food last month.

7. 37% of $\$1{,}975$ is the amount spent on bills. To convert 37% to decimal form, it's 0.37: $0.37 \times \$1{,}975 = \$730.75$. Bob spent $\$730.75$ on bills last month.

8. 10% of $\$1{,}975$ is the amount spent on the car. 10% in decimal form is 0.10: $0.10 \times \$1975 = \$197.50$. Bob spent $\$197.50$ on his car last month.

9. There are a total of $8 + 6 = 14$ marbles in Bag A. The probability of an event happening is given by the formula: $Probability = \frac{number\ of\ desired\ outcomes}{number\ of\ total\ outcomes}$. For selecting a green marble from Bag A: $p(green) = \frac{6}{14} = \frac{3}{7}$. So, the probability of selecting a green marble from Bag A is $\frac{3}{7}$.

There are a total of $5 + 7 = 12$ marbles in Bag B. For selecting a black marble from Bag B: $p(black) = \frac{5}{12}$. So, the probability of selecting a black marble from Bag B is $\frac{5}{12}$.

10. The formula for the number of permutations of n items taken k at a time is ${}_nP_k = \frac{n!}{(n-k)!}$. Using the permutation formula, we get: $P(4,4) = \frac{4!}{(4-4)!} = \frac{4!}{0!} = \frac{4\times3\times2\times1}{1} = 24$.

11. Consider Jason's activities as positions in a sequence: first, second, and third. For the first position, he can choose any of the 3 activities (museum, beach, or volleyball). Once he's picked the first activity, he has 2 remaining options for the second position. And for the third position, he's left with just the last unchosen activity. To find the total number of possible sequences, multiply the choices for each position together: 3 (choices for first) $\times$ 2 (choices for second) $\times$ 1 (choice for third) $=$ 6 different sequences or orders for his activities.

12. When the basketball team decides to choose a captain, any of the 6 players could potentially be selected. So, there are 6 possibilities for the captain. After the captain has been chosen, only 5 players remain to be selected as the co-captain. Therefore, the total number of ways to choose both a captain and a co-captain from the team is given by multiplying the choices for each position: 6 (choices for captain) $\times$ 5 (choices for co-captain) $=$ 30 different ways.

13. The number of ways to arrange these balls and dividers is the combination of 12 items, choosing 5 to be balls, or $C(8,5)$. The number of ways is thus $C(8,5) = \frac{8!}{5!3!} = \frac{8\times7\times6\times5!}{5!\times3\times2\times1} = 56$. So, there are 792 ways to distribute the 5 balls among the 8 friends.

14. When the professor arranges her 5 students in a straight line, the order in which they are placed matters. This is a permutation problem. For the first position, the professor has 5 choices (any of the 5 students). Once a student is placed in the first position, 4 students remain for the second position. For the third position, there are 3 students left. Then, 2 students for the fourth position, and finally, 1 student for the last position. To find the total number of arrangements, multiply the choices for each position:

5 (Choices for the first position) $\times$ 4 (choices for the second) $\times$ 3 (for the third) $\times$ 2 (for the fourth) $\times$ 1 (for the last) $= 5! \Rightarrow$ (5 factorial) $= 120$. Thus, the professor can arrange her 5 students in 120 different ways.

15. The number of ways to choose 12 students out of 15 is given by the combination formula: ${}_{n}C_{r} = \frac{n!}{(n-r)!}$. Plugging in our values: ${}_{n}C_{r} = C(15,12) = \frac{15!}{12!(15-12)!} = \frac{15!}{12!3!}$. When you compute this, you get: $\frac{15\times14\times13}{3\times2\times1} = 455$. So, the teacher can choose 12 students out of 15 in 455 different ways.

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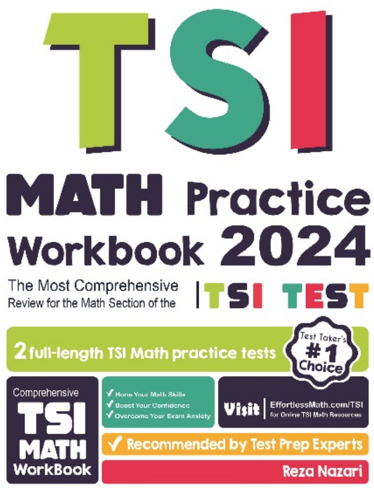

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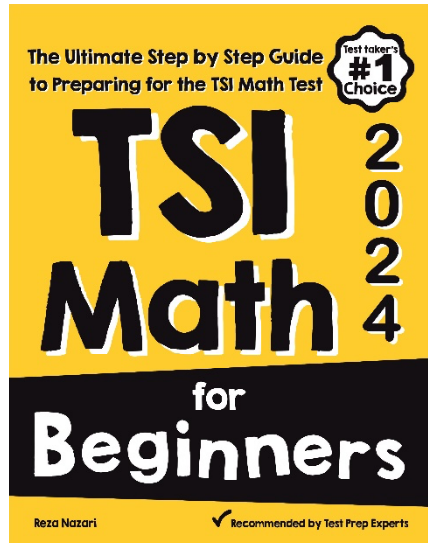

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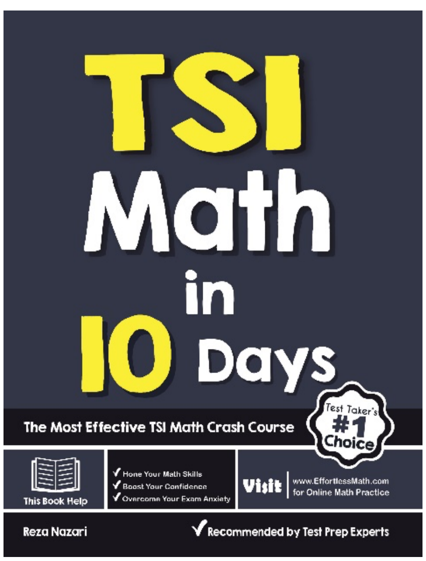

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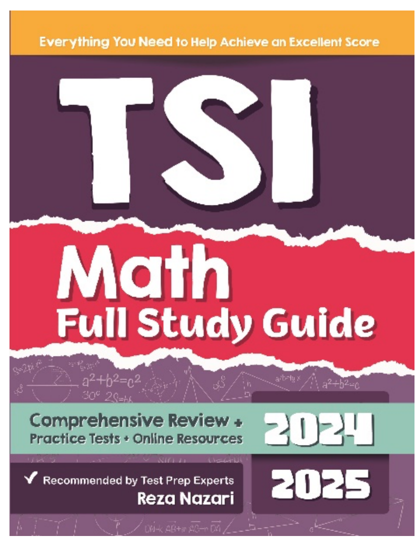

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